# CS 567 COMPUTATIONAL STATISTICS

# Project 1

# Life Insurance using R

Authors
Chao Huang Lin
41716747
Hermann Yepdjio
140917845
Lubna Alzamil
4144208

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# 1 Introduction

This our first project. We've been asked to implement the formulas used by life insurance companies using R programming language. Implementing the formulas into the computer will make it easier and more efficient to calculate and visualize several types of products offered by the insurance company.

# 2 Methodology

These steps have been followed to reach the goal of this project:

- 1. A mortality table is downloaded and saved it as compare.csv.
- 2. We started to code the necessary formulas in Life-table.R
- After that, we called the formulas file into our Project1.R to calculate The Net Single Premium for one individual and for 10000 individuals with random ages and benefits.
- 4. Then, Probability of Death is Calculated.
- 5. All the output data is stored in BusinessData.csv.
- 6. Finally, data in BusinessData.csv is visulized using ggplot library.

# 3 The Code

#### 3.1 Life-table.R.

As mentioned earlier, We coded the necessary formulas in Life-table.R.

Listing 1: Life Table File

Listing 1 Line 1 stores the start time when the code is started to execute. At the end of code execution, the total time will be printed. Line 2 sets the working directory. Line 3 read the inputs values from the life-table-inputs.txt file and store them in *inputs*. Line 4 gets the value of inputMortalityFile row from *inputs* data frame which is compare.csv and stores it in *inputMortalityFile*.Line 5 prints the value of *inputMortalityFile*. Line 6 reads the csv data of inputMortalityFile and store them in *data*. stringsAsFactors=FALSE means that do not covert the string data to factors. Line 7 counts the number of rows in *data*.

#### Listing 2: Life Table File

```
ages = as.numeric(data[beginRowData:dataTotalRow, 1])
mortality = as.numeric(data[beginRowData:dataTotalRow, 2])

lifeTable <- data.frame(ages,mortality)

lifeTableTotalRow = nrow(lifeTable)
lifeTable$t <- lifeTable$ages + 1
lifeTable$q <- lifeTable$mortality
lifeTable$p <- 1 - lifeTable$q</pre>
```

in Listing 2, line 1 starts to loop inside the first column (ages column) of data data frame from 1 until the total rows of data then convert the ages into numeric values and store them in ages. Line 2 does the same with the second column ,which contains the mortality, storing them as numeric values in mortality. Line 4 creates a data frame called lifeTable which contains for now two columns ages and mortality. Line 6 counts the number of rows in lifeTable and store them in lifeTableTotalRow. Line 7 adds new column to the data frame lifeTable the column is called t and contains the values t ages+1. Line 8 adds new column to the data frame t as t and t are t and t are t and t and t are t and t are t and t and t are t and t are t and t are t and t are t and t and t are t are t and t are t and t are t and t are t are t and t are t are t and t are t are

Listing 3: Life Table File

```
#calculate v^t
interest <- as.numeric(inputs[inputs$label == "interest", c("value")])
lifeTable$vt <- 1/(1+interest)^lifeTable$t

#calculate t_p_x | x=0
lifeTable$t_p_x0[1] <- lifeTable$p[1]
for (i in 2:lifeTableTotalRow) {
    lifeTable$t_p_x0[i] <- lifeTable$t_p_x0[i-1] * lifeTable$p[i]
}</pre>
```

In Listing 3, lines 1 gets the value of interest from *inputs* and stores it in *interest*. Line 2 add new column to the data frame *lifeTable* the column is called  $v^t$  which contains the value of  $v^t = (1+i)^- t$ . Lines 6-8 add new column to the data frame

life Table the column is called  $_tp_{x0}$  that contains the value of  $_tp_x = lx_n/lx_{n-1} = p_x *_{t-1}p_{t=0}$ .

Listing 4: Life Table File

```
#calculate t_E_x | x=0
lifeTable$t_E_x0 <- lifeTable$vt * lifeTable$t_p_x0

#calculate a_x
lifeTable$a_x[1] <- 1 + sum(lifeTable$t_E_x0)

for (i in 2:lifeTableTotalRow) {
    lifeTable$a_x[i] <- 1 +
        sum(lifeTable$t_E_x0[i:lifeTableTotalRow])/lifeTable$t_E_x0[i-1]
}

#calculate A_x
d = interest/(1 + interest)
lifeTable$A_x <- 1 - d * lifeTable$a_x</pre>
```

In Listing 4, line 2 adds new column to the data frame lifeTable the column is called  ${}_{t}E_{x0}$  and contains the value  ${}_{t}E_{x} = v^{t} * {}_{t}p_{x0}$ . Lines 5-7 add new column to the data frame lifeTable the column is called ax and its values are  $ax = 1 + \Sigma_{t}E_{x}$ . The first value in ax column is=  $1 + \Sigma_{t}E_{x0}$ . Lines 11-12 add new column to the data frame lifeTable the column is called  $A_{x}$  and contains the value of  $A_{x} = 1 - d * ax$ .

# 3.2 Projec1.R

In this file, we are calling the formulas located in life-table.R using the source function.

Listing 5: Project1 File

```
setwd(C://Desktop//Statis//GroupProject1//3R)
   inputsProject1 <- read.delim("project1_inputs.txt", header = TRUE, sep =</pre>
      "\t", dec = ".", stringsAsFactors=FALSE) #read the inputs values from
      the project1_inputs.txt file
g print (inputsProject1)
4 source("life_table.R")
5 #install library if not exist
6 if (!require(ggplot2)) install.packages('ggplot2')
7 if (!require(reshape)) install.packages('reshape')
8 library(ggplot2)
9 library(reshape)
10 x11()
myGraph <- ggplot(lifeTable, aes(ages, A_x))</pre>
  myGraph <- myGraph + geom_point()</pre>
print(myGraph)
ggsave(filename = "images/A_x Vs Ages.png", plot = myGraph)
```

In Listing 5, line 1 sets the working directory to the path of where the project and needed files are stored. Line 2 reads the inputs values from the project1-inputs.txt file and stores them in inputsProject1. Line 3 prints the values in inputsProject1. Line 4 calls Life-table.R file that contains the necessary formulas. Line 5-6 install the needed packages and call them with library function. Here we are calling ggplot2 and reshape libries in lines 8-9. Reshape library is used to cast a data frame into the reshaped form. The library ggplot2 is used for creating plots. Line 10 opens a new graph window before creating a new graph to avoid overwriting of previous plotted graphs. X11() function is used in Unix platform, windows() is used in Windows platform and quartz() is used in Mac platform. Line 11 myGraph contains plots the ages and  $A_x$  cloumns from lifeTable data frame. Line 12 geom point function is used to create scatter points displaying the relationship between ages and  $A_x$ . Line 13 shows the plot on the screen. Line 14 saves the plot on images folder. Figure 1 shows the result of Line 12.

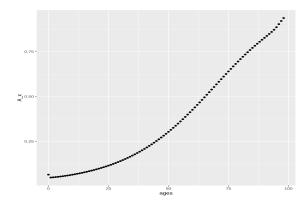


Figure 1: The Relationship between  $A_x$  and Ages

#### Listing 6: Project1 File

In Listing 6, lines 1-2 get the values of inputAges and inputBenefit columns in *input-sProject1* convert them to numeric and store them in *inputAges* and *inputBenefit*. Line 3 extracts the corresponding  $A_x$  of the *inputAges* which is 12 and multiples it by the *inputBenefit* which is 1000 storing the result in nsp. The formula used is  $NetSinglePremium = Benefit * A_x$ . Line 4 prints the age. Line 5 prints the result of Net Single Premium.

### Listing 7: Project1 File

```
#-----Calculating Probability of Dead Curve-----
   CalculateFinalAge <- function(inputAge) {</pre>
     #calculate probability of (x) lives t years t_p_x where x = inputAges
     previousAgeP <- lifeTable[ages == (inputAge - 1), c("t_p_x0")]</pre>
     if (inputAge > 0) {
      pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]/previousAgeP
     } else {
      pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]
9
10
     pLivesAges <- lifeTable[ages >= inputAge, c("ages")]
11
     pCurveX <- data.frame(pLivesAges, pLives)</pre>
13
     #calculate probability tl1_q_x that x survives t years and dies within
14
        1 year
     pCurveXRow <- nrow(pCurveX)</pre>
15
16
     #probability of dead based on input age
     pCurveX$tl1_q_x[1] <- 1 - pCurveX$pLives[1] # firt data important,,, to
        avoid bug
     if(pCurveXRow > 1) { #bug -fixed, avoid enter for when nrow(pCurveX) ==
19
        O, this occur when inputAge is the last value of the tablelife
       for (j in 2:(pCurveXRow)) {
        #small bug which does not consider the 0|1_q_x data.... also add
22
            q_x of 1 in the last year automatically?
        pCurveX$tl1_q_x[j] <- pCurveX$pLives[j-1] - pCurveX$pLives[j]</pre>
23
         #pCurveX$tl1_q_x[j] <- pCurveX$pLives[j] - pCurveX$pLives[j+1] #i =</pre>
24
            u in the pdf 1 = t in pdf
       }
25
     }
26
     #generate random dies based on input age
27
     finalAge <- 0
2.8
     if(pCurveXRow > 1) { #bug -fixed, avoid enter when nrow(pCurveX) == 0,
29
        this occur when inputAge is the last value of the tablelife
       finalAge <- sample(pCurveX$pLivesAges, 1, replace = T,</pre>
30
          pCurveX$tl1_q_x)
     }else
31
32
       finalAge <- pCurveX$pLivesAges[1] # only has 1 probability</pre>
33
     return (finalAge)
35
   }
36
```

Listing 7 Calculates Probability of Death using the defined function CalculateFinalAge.

```
CalculateFinalAgePerfectData <- function(inputAge) { #---only for testing</pre>
     #calculate probability of (x) lives t years t_p_x where x = inputAges
     previousAgeP <- lifeTable[ages == (inputAge - 1), c("t_p_x0")]</pre>
     if (inputAge > 0) {
      pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]/previousAgeP
     } else {
      pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]
9
10
    pLivesAges <- lifeTable[ages >= inputAge, c("ages")]
11
    pCurveX <- data.frame(pLivesAges, pLives)</pre>
     #calculate probability tl1_q_x that x survives t years and dies within
14
        1 year
    pCurveXRow <- nrow(pCurveX)</pre>
15
16
     #probability of dead based on input age
     pCurveX$tl1_q_x[1] <- 1 - pCurveX$pLives[1] # firt data important,,, to
        avoid bug
     for (j in 2:(pCurveXRow)) {
19
20
      #small bug which does not consider the 0|1_q_x data..... also add q_x
          of 1 in the last year automatically?
      pCurveX$tl1_q_x[j] <- pCurveX$pLives[j-1] - pCurveX$pLives[j]</pre>
22
      #pCurveX$tl1_q_x[j] <- pCurveX$pLives[j] - pCurveX$pLives[j+1] #i = u</pre>
23
          in the pdf 1 = t in pdf
24
     26
     #bFAge <- sample(pCurveX$pLivesAges, inputNumberClients, replace = T,
27
        pCurveX$tl1_q_x)
28
     #----generate perfect pCurve data
29
     perfectData <- vector(mode="numeric", length=inputNumberClients) # it</pre>
        is better to initialize variable to make it faster
31
     bIndex <- 1 #begin index
32
     fIndex <- 0 #final index
33
     counter <- 0
34
     for(k in 1:pCurveXRow){
36
      nRepeat <- pCurveX$tl1_q_x[k]*inputNumberClients</pre>
37
      if(nRepeat%%1 >= 0.5){ # use this to avoid number of data > pCurveXRow
38
        if(counter\%2 == 0){
39
          nRepeat <- round(nRepeat)</pre>
        }else{
41
          nRepeat <- trunc(nRepeat) # use trunc to avoid number of data >
```

```
pCurveXRow
         }
43
         counter <- counter +1</pre>
44
       }else{
45
         nRepeat <- trunc(nRepeat)</pre>
46
       fIndex <- bIndex + nRepeat - 1
49
       perfectData[bIndex:fIndex] <- pCurveX$pLivesAges[k]</pre>
50
       bIndex <- bIndex + nRepeat
51
52
     #nData <- length(perfectData)</pre>
     nFill <- inputNumberClients - fIndex
55
     #fill residual round values from nData to numberclient
56
     perfectData[(fIndex+1):inputNumberClients] <-</pre>
57
         sample(pCurveX$pLivesAges, nFill, replace = T, pCurveX$tl1_q_x)
     #perfectData[(nData+1):inputNumberClients] <- perfectData[nData]</pre>
     #return a vector
     return(perfectData)
61
62
```

Listing 8 is just for testing purposes, it generates final age based on a perfect probability curve of death.

```
CalculateFinalAgeSemiPerfectData <- function(inputAge) { #---only for</pre>
       testing
     #calculate probability of (x) lives t years t_p_x where x = inputAges
     previousAgeP <- lifeTable[ages == (inputAge - 1), c("t_p_x0")]</pre>
     if (inputAge > 0) {
       pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]/previousAgeP
     } else {
       pLives <- lifeTable[ages >= inputAge, c("t_p_x0")]
9
10
     pLivesAges <- lifeTable[ages >= inputAge, c("ages")]
11
     pCurveX <- data.frame(pLivesAges, pLives)</pre>
13
     #calculate probability tl1_q_x that x survives t years and dies within
14
     pCurveXRow <- nrow(pCurveX)</pre>
     #probability of dead based on input age
     pCurveX$tl1_q_x[1] <- 1 - pCurveX$pLives[1] # firt data important,,, to
18
        avoid bug
     for (j in 2:(pCurveXRow)) {
19
20
       #small bug which does not consider the 0|1_q_x data..... also add q_x
21
          of 1 in the last year automatically?
       pCurveX$tl1_q_x[j] <- pCurveX$pLives[j-1] - pCurveX$pLives[j]</pre>
22
       #pCurveX$tl1_q_x[j] <- pCurveX$pLives[j] - pCurveX$pLives[j+1] #i = u</pre>
23
          in the pdf 1 = t in pdf
     }
     #note: return a vector
25
     bFAge <- sample(pCurveX$pLivesAges, inputNumberClients, replace = T,
26
        pCurveX$tl1_q_x)
27
28
     return(bFAge)
   }
30
```

Listing 9 calulates the probability of (x) lives t years using the defined function CalculateFinalAgeSemiPerfectData. It finds the probability  $_tp_{x0}$  of inputAges - 1 and stores the result in previousAgeP. Then checks whether the inputAges is greater than zero or not. If it is, it will get all the probabilities  $_tp_{x0}$  of all ages and divide by previousAgeP and store the result in pLives. If the inputAges is not greater than zero, it will get all the probabilities  $_tp_{x0}$  of all ages and store them in pLives. Then stores all the ages that are greater than or equal to inputAges in pLivesAges. After that, it creates a data frame called pCurveX that has two columns pLivesAges and pLives. After that, calculate  $_{t1}q_x$  which is the probability that x survives t years and dies within 1 year. The general formula used to calculate this probability is

 $u|tq_x =_u p_x - u + tp_x$ . pCurveXRow stores the number of rows in pCurveX data frame. It iterates from 1 until pCurveXRow-1. Adds new column to the data frame pCurveX the column is called  $t_1q_x$  and contains the values of the formula  $u|tq_x =_u p_x - u + tp_x$ . Finally, it stores the  $u|tq_x$  of the last row in pCurveX data frame.

## Listing 10: Project1 File

```
#now creating a block of 10000 people who are in different ages and want
      different benefits
  BussinessBlock <- function(rAge, rBenefit) {</pre>
    netSinglePremium <- lifeTable[lifeTable$ages == rAge, c("A_x")]*rBenefit</pre>
     return(netSinglePremium)
  }
6
   inputNumberClients <- as.numeric(inputsProject1[inputsProject1$label ==</pre>
      "inputNumberClients", c("value")])
  bAge <- vector(mode="integer", length=inputNumberClients) #initialize</pre>
      variables
  bBen <- vector(mode="integer", length=inputNumberClients)
  bNps <- vector(mode="numeric", length=inputNumberClients)</pre>
  bFAge <- vector(mode="integer", length=inputNumberClients)
  maxAges <- max(lifeTable$ages)</pre>
   lifeTableAges <- lifeTable$ages[ages < maxAges]# avoid picking max ages</pre>
  #looping 10000
16
  lifeTableAges <- data.frame(Age=ages) # creating a data frame that</pre>
      contains the ages
  print("-----")
  print(paste("Number of clients: ", inputNumberClients))
  lifeTableAsVector = as.vector(lifeTable[1,], mode = 'numeric') #create a
      vector representation of lifeTable so I can pass it to the c function
  for (i in 2:lifeTableTotalRow)
    lifeTableAsVector = c(lifeTableAsVector, as.vector(lifeTable[i,], mode
        = 'numeric'))
24
  dim = c(length(lifeTableAges$Age), 10, length(lifeTableAges$Age)) #bug if
      only put lifeTableAges -> length =1, 99 instead of 100
  dyn.load("c_code.dll")
  res = .C("for_loop", lifeTable=as.numeric(lifeTableAsVector), dim =
      as.integer(dim), bAge = as.numeric(bAge), bBen=as.integer(bBen), bNps
      = as.numeric(bNps), bFAge = as.numeric(bFAge), lifeTableAges =
      as.integer(lifeTableAges), inputNumberClients
      =as.integer(inputNumberClients))
bFAge = res[6]$bFAge
bAge = res[3]$bAge
30 bBen= res[4]$bBen
31 bNps= res[5]$bNps
```

Listing 10 creates a function named BussinessBlock that receives two parameters -age and benefit- and calculates and returns the Net Single Premium according to the formula  $NetSinglePremium = Benefit * A_x$ . Line 7 gets the population value which is 10000 from inputsProject1 and stores it in inputNumberClients. Lines 9-12 creates new variables (arrays) and set them to zeros with same length of inputNumberClients for later use. Line 11 gets the maximum age from lifeTable data frame and stores in maxAges. Line 14 stores all the ages that are less than maxAges in lifeTableAges. Line 17 stores all the ages in lifeTableAges. The loop is made in a c code and called within R to reduce the time needed to loop over all 10000 clients.

#### Listing 11: Project1 File

```
bDataFrame <- data.frame(Age = bAge, Benefit = bBen, NetSinglePremium =
       bNps, Die = bFAge) #Creating the final dataframe
   bDataFrame$SurviveYears <- bDataFrame$Die - bDataFrame$Age
    # creating the CSV file, each time we run the code new file will be
        created and the older file will be replaced
   write.table(bDataFrame, file="BusinessData.csv", row.names=F,
       col.names=T, append=F, sep=",")
   paymentTable <- bDataFrame[c("SurviveYears", "Benefit")]</pre>
   paymentTable <- aggregate(. ~SurviveYears, data=paymentTable, sum, na.rm</pre>
       = TRUE)
   investmentInterest <- as.numeric(inputsProject1[inputsProject1$label ==</pre>
       "investmentInterest", c("value")])
   surviveYears <- 0</pre>
   money <- sum(bDataFrame$NetSinglePremium)</pre>
   earnInterest <- money*investmentInterest</pre>
13
   benefitPayment <- 0
14
   #find payment of the year
   payment <- paymentTable[paymentTable$SurviveYears == 0, c("Benefit")]</pre>
   if(length(payment) == 0){ #sometime there is no payment for specific
       year, so convert numeric(0) to 0
     benefitPayment <- 0
18
   }else{
19
     benefitPayment <- payment
21
   nYears <- max(paymentTable$SurviveYears)</pre>
   for (i in 1:nYears) {
     surviveYears[i+1] <- i</pre>
24
     money[i+1] <- money[i] + earnInterest[i] - benefitPayment[i]</pre>
     if(money[i+1] > 0){ #only calculate earnInterest when money > 0
       earnInterest[i+1] <- money[i+1]*investmentInterest</pre>
27
     }else {
28
       earnInterest[i+1] <- 0</pre>
29
30
31
```

```
#find payment of next year
32
     payment <- paymentTable[paymentTable$SurviveYears == i, c("Benefit")]</pre>
33
     if(length(payment) == 0){ #sometime there is no payment for specific
34
         year, so convert numeric(0) to 0
       benefitPayment[i+1] <- 0</pre>
35
     }else{
36
       benefitPayment[i+1] <- payment</pre>
     }
38
   }
39
   #benefitPayment[i+1] <- 0 #last year payment = 0</pre>
   profitTable <- data.frame(surviveYears, money, earnInterest,</pre>
       benefitPayment)
```

In Listing 11, line 1 creates a data frame named bDataFrame that contains Age, Benefit, NetSinglePremium and Die columns. Line 2 adds new column to bDataFrame named Survive Years that contains the difference between Die and Age column. Line 4 creates the CSV file, each time we run the code new file will be created and the older file will be replaced. Lines 5-6 creates a data frame named payment Table that contains SurviveYears and Benefit columns from bDataFrame data frame. After that, we are gonna calculate and see whether the insurance company makes money or loses money with interest rate of 5.00%. Line 8 gets the interest value which is 0.05 from inputsProject1 and stores it in investmentInterest. Line 10 sets the SurviveYear to zero which indicates the current year. Line 11 calculates the sum of NetSinglePremium column in bDataFrame data frame and stores the result in money. Line 12 multiplies money with investmentInterest and stores the result in earnInterest. Lines 16-29 calculates the payment of benefits in current year. Line 16 stores the benefits paid in the current year in payment. Lines 17-29 checks if the length of payment is equal to zero or not. If it is, let paymentRec be zore. Else, copy the data of payment to payment Rec. Lines 32-39 calculates the benefit payment of next year. Line 33 gets the benefit that should be paid for i survived years. Lines 34-37 checks if the payment is equal to zero or not. If it is, the benefit payment for the next year will be zero and that meas nobody died this year. Otherwise, the benefit payment for the next year will be equal to payment that contains the benefit. Finally, Line 41 creates a data frame called *profitTable* that has four columns SurviveYear, money, earnInterest and benefitPayment.

As mentioned earlier, x11() function opens a new graph window before creating a new graph to avoid overwriting of previous plotted graphs. The following Listings plots the Age, Benefit, NetSinglePremium, Die and SurviveYears columns from bDataFrame. Each of these column is plotted on its own window. All these plots are plotted as histograms. All the images will be saved in images directory which should be created on the same working directory. Each listing will have its output graph under it.

Listing 12: Project1 File

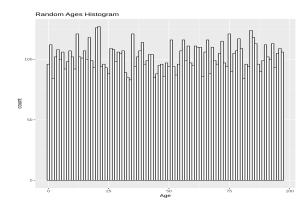


Figure 2: Random Ages Histogram

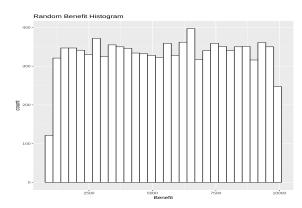


Figure 3: Random Benefit Histogram

# Listing 14: Project1 File

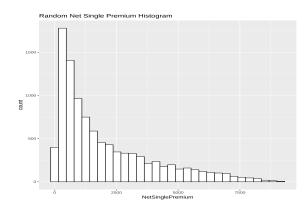


Figure 4: Random Net Single Premium Histogram

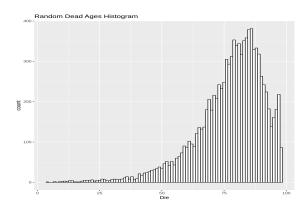


Figure 5: Random Dead Ages Histogram

# Listing 16: Project1 File

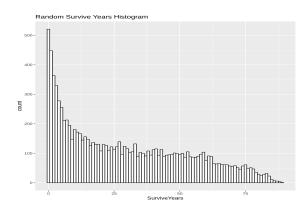


Figure 6: Random Survive Years Histogram

# Listing 17: Project1 File

Line 153 uses one of the reshapes's library functions. Melt function which allows you to plot more that one thing on the same window. As mentioned earlier x11() function opens a new graph window before creating a new graph to avoid overwriting of previous plotted graphs. Line 4 sets the axis of the plot. Line 5 sets the title and the plot will be plotted as lines and sets the legend of the plot. Line 7 displays the graph on the screen. Line 9 saves the graph on images directory which should be created on the same working directory. Line 11 stores the time when the code is ended its execution. Line 12 calculates the total time. Line 13 prints the total time. Figure 7 shows the output of line 7.

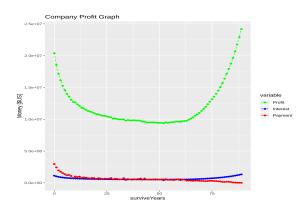


Figure 7: Company Profit Graph