Central Washington University

CS 471 OPTIMIZATION

Spring 2019

Project 3 Report

Author:
Hermann Yepdjio

Supervisor:
Dr. Donald Davendra

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Contents

1 Introduction

For project 3, we were asked to optimize 18 standard benchmark functions namely Schwefel, De Jong 1, Rosenbrock's Saddle, Rastrigin, Griewangk, Sine Envelope Sine Wave, Stretch V Sine Wave, Ackley One, Ackley Two, Egg Holder, Rana, Pathological, Michalewicz, Master's Cosine Wave, Quartic, Levy, Step and Alpine. For this purpose, we've been given 2 optimization algorithms to be implemented then applied to those functions. Those algorithms are: Genetic Algorithm(GA) and Differential Evolution Algorithm (DE). After implementing them, we ran them on each of the 18 functions using randomly generated data. Statistics for each algorithm were computed and stored in a tabular format and they will be discussed then analyzed later on in this report. However, in order to observe how the fitness of each objective function varies across generations, we considered plotting the results for GA for one of the experiments.

2 Results

2.1 Genetic Algorithm

2.1.1 Values used for the parameters

The Genetic Algorithm was run using the following values:

- dimension: 30
- population size: 200
- number of generations: 100
- number of experiments: 50
- crossover rate: 0.8
- elitism rate: 0.2
- mutation range: 0.1
- mutation rate: 0.005
- mutation precision 5

2.1.2 Statistics

Table 1: Statistics for GA (50 experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	2058.26	438.72	1958.65	2067.10	60.68
f2 De Jong 1	3022.68	1433.06	4966.14	2830.41	41.52
f3 Rosenbrock	598618230.00	442278465.18	1922789500.00	499541500.00	59.54
f4 Rastrigin	28924.19	36251.34	150185.80	32706.70	68.70
f5 Griewangk	400.24	82.34	362.36	396.22	78.76
f6 Sine Envelope	-35.95	1.08	4.79	-35.92	118.08
f7 Stretch V Sine	10.15	0.00	0.00	10.15	117.88
f8 Ackley One	40.83	17.84	75.50	40.27	93.34
f9 Ackley Two	151.31	31.50	129.65	150.10	138.06
f10 Egg Holder	-16331.62	1220.14	6204.60	-16425.25	90.74
f11 Rana	-10065.63	485.96	2003.84	-10144.15	142.68
f12 Pathological	-9.76	1.78	8.21	-9.87	125.08
f13 Michalewicz	-26.40	0.60	2.68	-26.36	165.40
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	104.80
f15 Quartic	94064256.00	81955817.35	454582300.00	73253600.00	106.86
f16 Levy	587.22	246.61	1052.16	577.08	117.78
f17 Step	3371.14	1422.10	6273.49	3317.80	43.36
f18 Alpine	29.94	6.52	32.80	29.17	53.40

2.1.3 Statistics analysis

When comparing the statistics of the GA to those obtained for the iterative local search algorithm in the previous assignment, we can see that there is a huge improvement in the fitness of objective functions. For example, Table 1 above shows that in average, the best fitness for the Schwefel function was 2058.26 and for Rosenbrock's Saddle function was $5.98 * 10^9$ while in the previous assignment we respectively got 8524.18 and $3.03 * 10^{10}$. However, Table 1 also shows that some functions such as De Jong 1 produced worse results (3022.68) compared to 0.088 obtained with the iterative local search.

2.1.4 plots

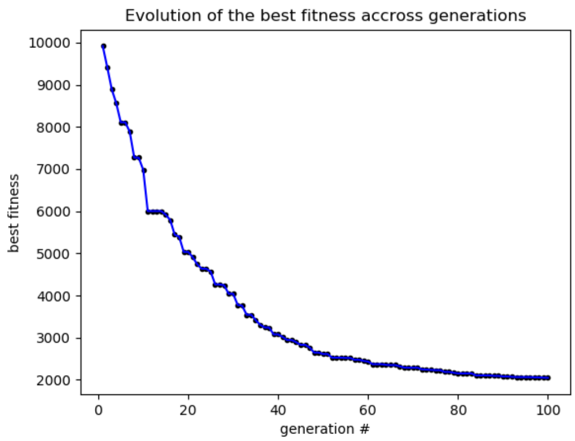

Figure 1: Schwefel (Experiment #1)

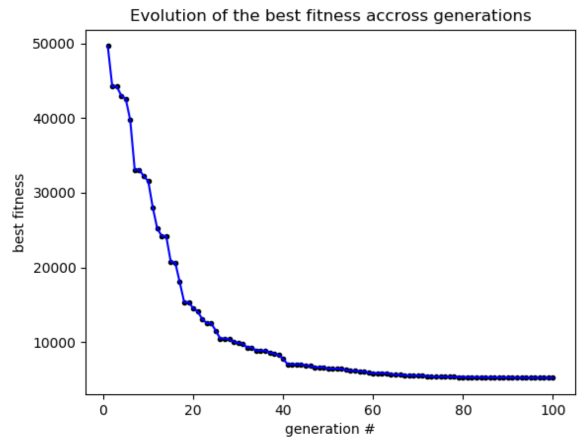

Figure 2: De Jong 1 (Experiment #1)

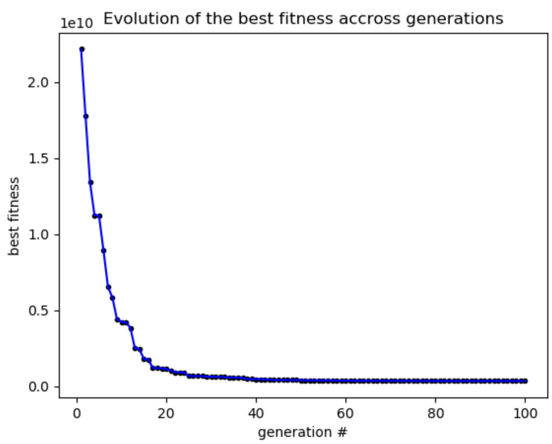

Figure 3: Rosenbrock (Experiment #1)

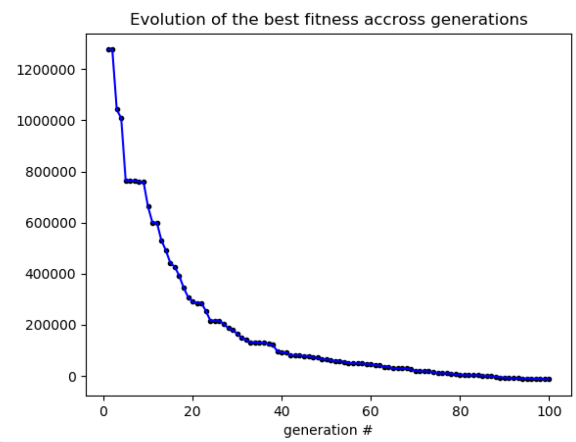

Figure 4: Rastrigin (Experiment #1)

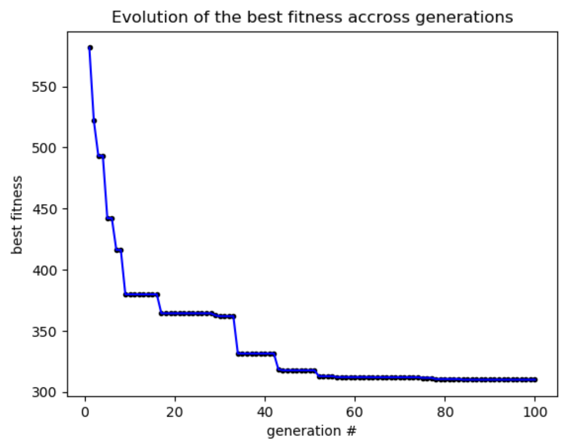

Figure 5: Griewangk (Experiment #1)

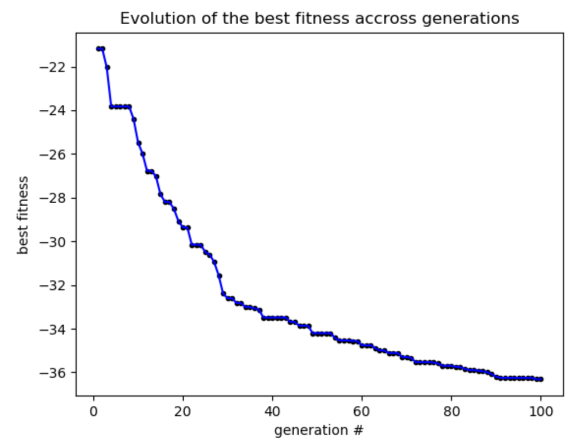

Figure 6: Sine Envelope (Experiment #1)

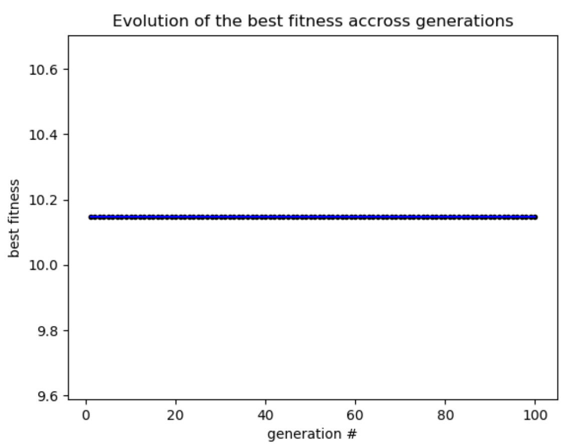

Figure 7: Stretch V Sine (Experiment #1)

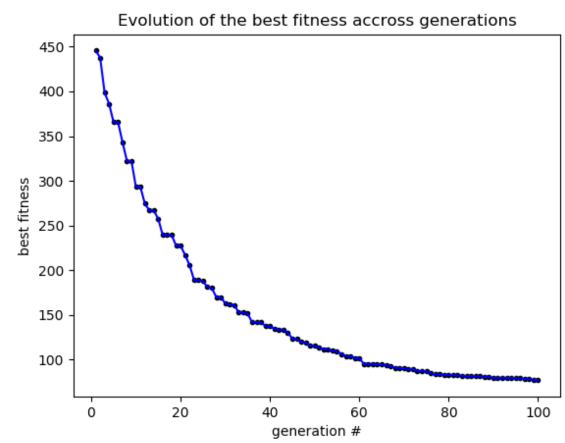

Figure 8: Ackley One (Experiment #1)

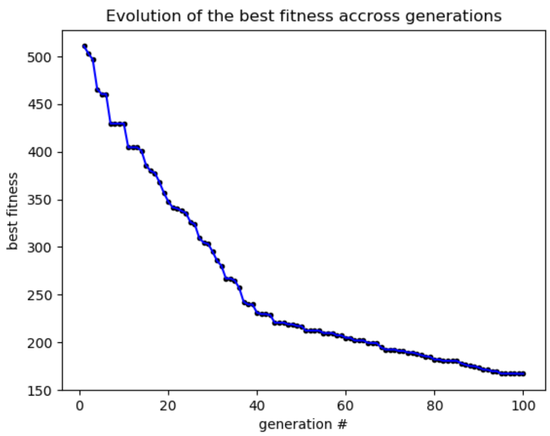

Figure 9: Ackley Two (Experiment #1)

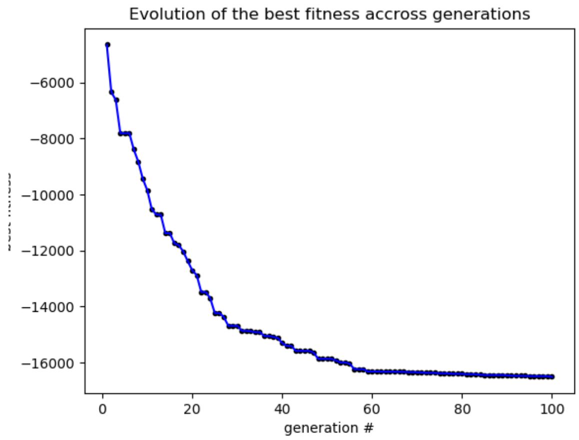

Figure 10: Egg Holder (Experiment #1)

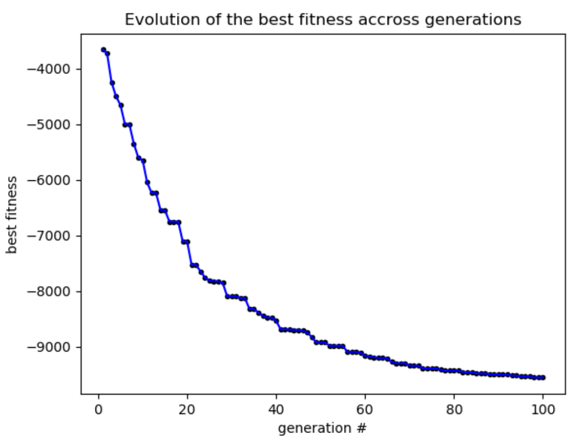

Figure 11: Rana (Experiment #1)

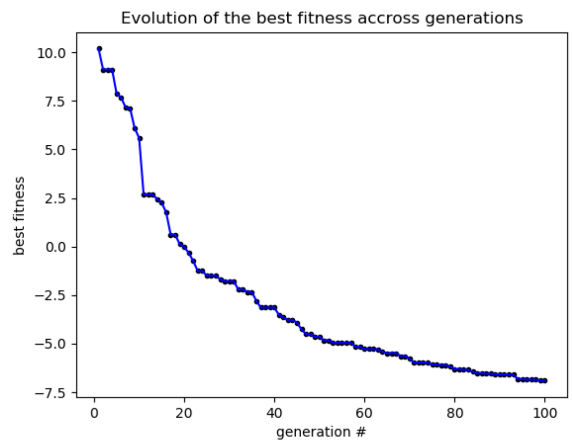

Figure 12: Pathological (Experiment #1)

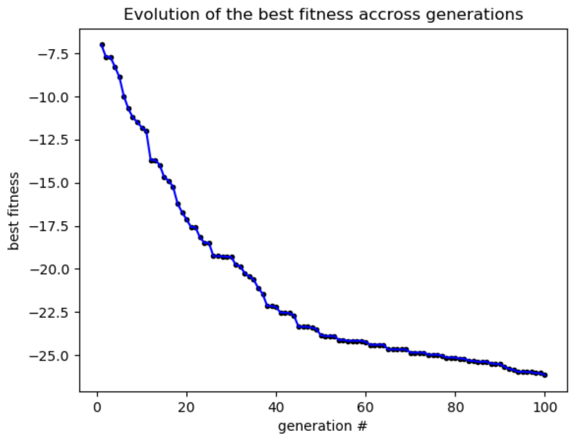

Figure 13: Michalewicz (Experiment #1)

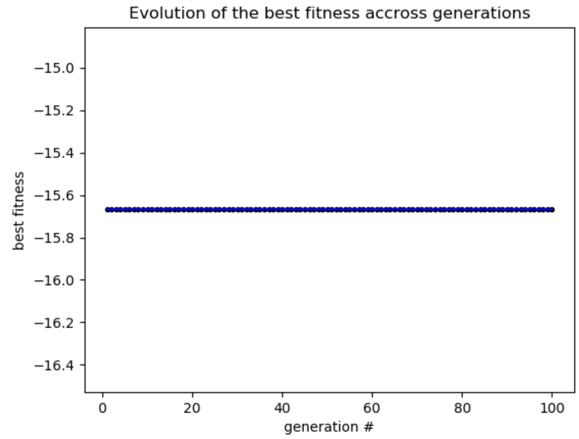

Figure 14: Masters' Cosine (Experiment #1)

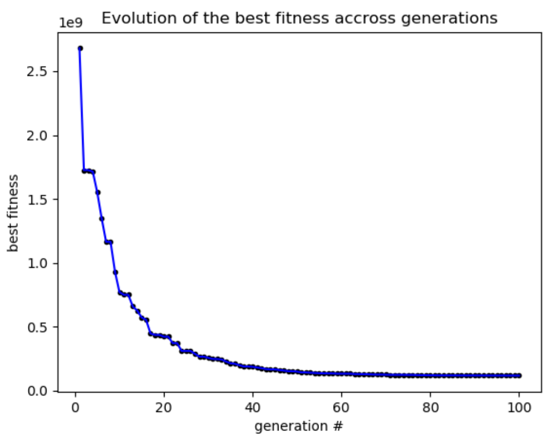

Figure 15: Quartic (Experiment #1)

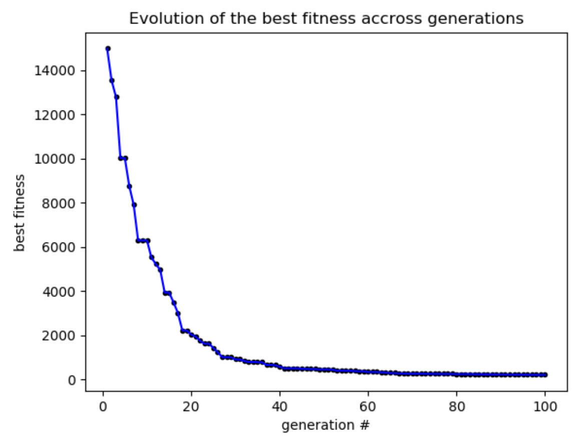

Figure 16: Levy (Experiment #1)

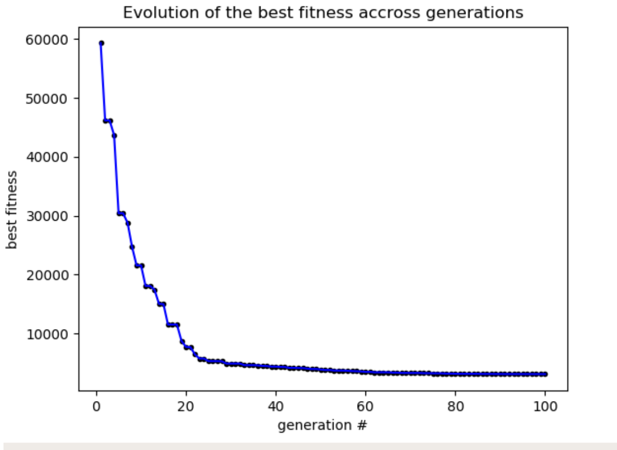

Figure 17: Step (Experiment #1)

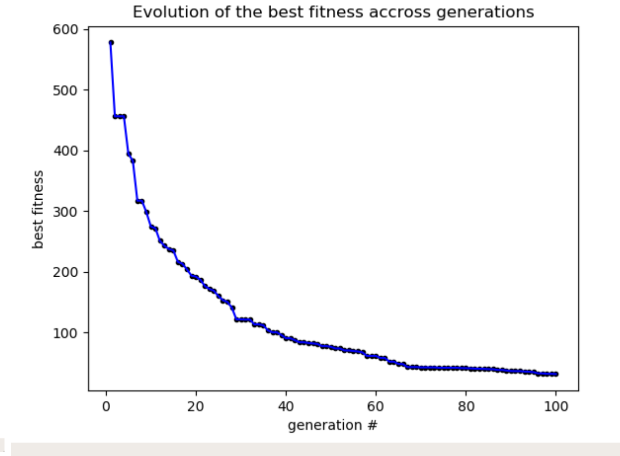

Figure 18: Alpine (Experiment #1)

2.1.5 Plots analysis

When looking at the figures above, we can see that the GA always improve the quality of the solutions as we go through generations except for 2 of the functions (Stretch V Sine (Figure 7) and Master's Cosine(Figure 14)). The reason to this improvement is that after each generation the algorithm gets rid of the bad solutions in both the original population and the new generated population and combines the rest to form the population for the next generation.

2.2 Differential Evolution Algorithm

2.2.1 Values used for the parameters

The Differential Evolution Algorithm was run using the following values:

- dimension: 30
- population size: 200
- number of generations: 100
- number of experiments: 50
- crossover rate: 0.8

- F: 0.5
- λ : 0.5

F and λ were chosen by experimenting with different values for both and picking the ones that performed the best.

2.2.2 Statistics

Table 2: Statistics for DE Strategy #1 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	9337.16	421.15	1763.05	9395.69	56.26
f2 De Jong 1	0.00	0.00	0.00	0.00	25.02
f3 Rosenbrock	23.5500000000	0.4700000000	2.1900000000	23.3600000000	43.68
f4 Rastrigin	-89999.93	0.05	0.10	-89999.90	44.74
f5 Griewangk	228045.76	18953.62	84384.00	227437.50	68.54
f6 Sine Envelope	-26.68	1.05	4.40	-26.55	107.76
f7 Stretch V Sine	10.15	0.00	0.00	10.15	105.06
f8 Ackley One	-13.29	1.52	6.69	-13.71	70.52
f9 Ackley Two	-0.00	0.00	0.00	-0.00	95.60
f10 Egg Holder	-5174.36	599.35	2771.78	-5165.21	100.52
f11 Rana	-3404.17	586.65	2882.95	-3325.24	172.66
f12 Pathological	-7.89	1.21	5.24	-7.66	114.50
f13 Michalewicz	-8.29	0.87	5.31	-8.22	162.26
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	92.22
f15 Quartic	0.00	0.00	0.00	0.00	107.16
f16 Levy	1.76	0.34	1.58	1.74	103.88
f17 Step	7.25	0.00	0.00	7.25	25.68
f18 Alpine	0.00	0.00	0.00	0.00	26.00

Table 3: Statistics for DE Strategy #2 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	9508.19	341.21	1794.45	9489.93	56.20
f2 De Jong 1	0.00	0.00	0.00	0.00	24.76
f3 Rosenbrock	26.4300000000	0.3500000000	1.6000000000	26.4800000000	43.08
f4 Rastrigin	-89999.94	0.05	0.10	-89999.90	45.08
f5 Griewangk	693327.20	19787.04	82785.00	692252.00	51.66
f6 Sine Envelope	-26.29	1.02	4.44	-26.01	107.02
f7 Stretch V Sine	10.15	0.00	0.00	10.15	104.90
f8 Ackley One	-7.99	0.55	2.55	-7.90	71.00
f9 Ackley Two	0.01	0.01	0.02	0.01	103.52
f10 Egg Holder	-4749.70	530.09	2288.92	-4682.90	100.62
f11 Rana	-3147.10	385.98	1576.27	-3131.20	173.36
f12 Pathological	-6.89	1.03	5.98	-6.94	115.64
f13 Michalewicz	-8.26	0.73	3.11	-8.12	162.98
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	93.16
f15 Quartic	0.00	0.00	0.00	0.00	105.40
f16 Levy	2.83	0.47	1.80	2.78	104.56
f17 Step	7.26	0.01	0.02	7.26	26.14
f18 Alpine	0.00	0.00	0.00	0.00	28.46

Table 4: Statistics for DE Strategy #3 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	9282.48	459.01	2341.60	9251.43	59.72
f2 De Jong 1	0.00	0.00	0.00	0.00	30.18
f3 Rosenbrock	24.2000000000	0.2200000000	1.1100000000	24.2700000000	48.26
f4 Rastrigin	-90000.00	0.00	0.00	-90000.00	47.48
f5 Griewangk	428530.68	98709.30	452746.00	417406.00	53.20
f6 Sine Envelope	-27.28	1.01	4.46	-26.83	104.42
f7 Stretch V Sine	10.15	0.00	0.00	10.15	110.26
f8 Ackley One	-10.12	1.43	8.45	-10.15	75.16
f9 Ackley Two	0.00	0.00	0.00	0.00	98.06
f10 Egg Holder	-5297.41	800.25	3447.91	-5161.18	100.54
f11 Rana	-3570.28	664.87	3237.28	-3385.80	168.12
f12 Pathological	-6.41	1.41	5.54	-6.13	117.36
f13 Michalewicz	-7.60	0.63	3.28	-7.55	165.04
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	98.18
f15 Quartic	0.00	0.00	0.00	0.00	110.84
f16 Levy	4.02	0.93	3.60	4.03	108.90
f17 Step	7.25	0.00	0.00	7.25	31.18
f18 Alpine	0.00	0.00	0.00	0.00	31.30

Table 5: Statistics for DE Strategy #4 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	9208.07	421.61	1835.15	9252.95	61.36
f2 De Jong 1	0.00	0.00	0.00	0.00	28.00
f3 Rosenbrock	25.0400000000	0.2900000000	1.2800000000	25.1100000000	47.10
f4 Rastrigin	-89999.92	0.04	0.20	-89999.90	48.44
f5 Griewangk	737636.72	103294.12	572686.00	738333.50	57.60
f6 Sine Envelope	-26.57	0.98	4.63	-26.45	111.74
f7 Stretch V Sine	10.15	0.00	0.00	10.15	109.08
f8 Ackley One	-11.43	1.27	6.76	-11.65	75.52
f9 Ackley Two	0.00	0.00	0.00	0.00	106.48
f10 Egg Holder	-4926.21	721.56	3107.01	-4746.57	105.66
f11 Rana	-3584.02	706.88	3482.63	-3396.12	178.26
f12 Pathological	-6.93	1.20	5.09	-6.67	119.98
f13 Michalewicz	-8.97	0.77	3.49	-8.88	165.86
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	97.76
f15 Quartic	0.00	0.00	0.00	0.00	110.64
f16 Levy	2.55	0.60	2.14	2.58	108.12
f17 Step	7.25	0.00	0.00	7.25	29.44
f18 Alpine	0.00	0.00	0.00	0.00	31.96

Table 6: Statistics for DE Strategy #5 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	9530.00	362.82	1669.31	9581.85	61.74
f2 De Jong 1	0.01	0.01	0.03	0.01	28.46
f3 Rosenbrock	26.7200000000	0.2300000000	1.0100000000	26.7100000000	47.48
f4 Rastrigin	-89970.84	18.77	90.60	-89973.25	50.24
f5 Griewangk	1047711.68	17696.39	90176.00	1045960.00	58.54
f6 Sine Envelope	-26.14	0.88	4.26	-25.93	111.22
f7 Stretch V Sine	10.15	0.00	0.00	10.15	109.58
f8 Ackley One	-7.45	0.93	5.82	-7.23	76.06
f9 Ackley Two	0.05	0.02	0.09	0.05	113.52
f10 Egg Holder	-4772.57	600.81	2870.44	-4753.55	105.70
f11 Rana	-3161.64	377.44	1548.08	-3150.41	179.26
f12 Pathological	-6.35	1.08	4.53	-6.15	119.72
f13 Michalewicz	-8.62	0.72	3.61	-8.58	168.94
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	97.96
f15 Quartic	0.00	0.00	0.00	0.00	110.90
f16 Levy	3.78	0.63	2.44	3.84	109.38
f17 Step	7.30	0.02	0.07	7.30	29.74
f18 Alpine	0.01	0.00	0.03	0.01	35.12

Table 7: Statistics for DE Strategy #6 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	7417.08	650.60	2668.65	7345.21	91.80
f2 De Jong 1	0.00	0.00	0.00	0.00	60.60
f3 Rosenbrock	46.0500000000	71.190000000	359.7300000000	28.480000000	78.90
f4 Rastrigin	-33146.56	6089.82	26704.20	-32740.00	92.12
f5 Griewangk	7244545.80	513801.73	2221270.00	7253925.00	113.96
f6 Sine Envelope	-24.13	0.79	4.32	-24.06	143.76
f7 Stretch V Sine	10.15	0.00	0.00	10.15	139.56
f8 Ackley One	-22.17	3.22	18.67	-22.48	125.76
f9 Ackley Two	0.10	0.04	0.19	0.09	159.84
f10 Egg Holder	-6994.73	862.74	3509.88	-6877.15	139.18
f11 Rana	-5464.64	791.08	3818.87	-5340.44	205.32
f12 Pathological	5.80	1.03	4.65	5.94	151.74
f13 Michalewicz	-12.96	0.83	4.63	-12.95	191.36
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	129.62
f15 Quartic	0.00	0.00	0.00	0.00	138.68
f16 Levy	1.65	0.12	0.51	1.63	144.02
f17 Step	7.28	0.01	0.03	7.28	61.76
f18 Alpine	3.02	6.11	29.18	0.32	81.64

Table 8: Statistics for DE Strategy #7 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	7387.95	361.91	1637.36	7393.95	93.36
f2 De Jong 1	4041.51	643.27	2909.20	4063.03	60.64
f3 Rosenbrock	26391402000.0	5649789703.8	28104100000.0	27218150000.0	83.20
f4 Rastrigin	109457.06	13790.10	61378.10	110926.00	93.10
f5 Griewangk	13854674.00	45412.80	217000.00	13856350.00	116.74
f6 Sine Envelope	-22.87	0.66	3.28	-22.80	145.46
f7 Stretch V Sine	10.15	0.00	0.00	10.15	142.76
f8 Ackley One	108.41	8.76	38.41	109.76	132.72
f9 Ackley Two	368.04	12.83	52.76	368.18	171.78
f10 Egg Holder	-6916.23	435.76	1807.35	-6864.38	140.40
f11 Rana	-4949.63	408.11	1563.37	-4916.11	210.12
f12 Pathological	6.67	0.77	3.87	6.87	153.96
f13 Michalewicz	-9.61	0.45	1.88	-9.60	202.64
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	129.34
f15 Quartic	4113264400.00	931955135.31	3943420000.00	4234680000.00	140.78
f16 Levy	328.56	101.44	470.63	334.64	154.02
f17 Step	3505.00	465.34	2036.62	3527.32	61.82
f18 Alpine	225.62	19.52	97.24	227.74	85.52

Table 9: Statistics for DE Strategy #8 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	7512.93	435.09	2274.57	7525.68	131.70
f2 De Jong 1	0.18	0.09	0.55	0.16	97.96
f3 Rosenbrock	1273.00000000	1732.820000000	8855.34000000	564.970000000	116.32
f4 Rastrigin	-25371.11	6804.06	30661.10	-24912.85	129.42
f5 Griewangk	8477336.80	551493.58	2778570.00	8458935.00	152.28
f6 Sine Envelope	-24.01	0.62	2.85	-23.98	183.70
f7 Stretch V Sine	10.15	0.00	0.00	10.15	180.20
f8 Ackley One	-10.93	5.91	28.52	-11.15	168.90
f9 Ackley Two	3.41	1.67	7.99	3.21	202.68
f10 Egg Holder	-7317.64	549.21	2600.15	-7295.34	177.50
f11 Rana	-5186.27	609.28	3033.31	-5107.62	249.50
f12 Pathological	6.14	1.03	4.64	6.30	192.74
f13 Michalewicz	-9.08	0.55	2.24	-9.12	246.96
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	169.08
f15 Quartic	6.74	7.86	37.36	3.92	175.82
f16 Levy	3720.77	1762.96	7403.88	3302.01	194.86
f17 Step	8.15	0.25	1.30	8.16	98.42
f18 Alpine	50.14	26.58	116.30	43.89	125.30

Table 10: Statistics for DE Strategy #9 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	7989.56	337.86	1490.71	8011.59	122.46
f2 De Jong 1	3823.80	760.63	3652.30	3728.08	85.90
f3 Rosenbrock	6369925800.00	2242623436.37	10721940000.00	6184335000.00	107.80
f4 Rastrigin	106067.78	24503.54	109420.50	105891.00	119.42
f5 Griewangk	12065738.00	199774.63	830400.00	12037500.00	143.86
f6 Sine Envelope	-23.55	0.75	3.10	-23.46	173.84
f7 Stretch V Sine	10.15	0.00	0.00	10.15	169.54
f8 Ackley One	110.69	12.14	54.00	110.08	158.98
f9 Ackley Two	331.37	21.38	104.24	331.48	197.82
f10 Egg Holder	-6837.80	568.91	3016.48	-6828.45	169.24
f11 Rana	-4815.27	464.87	1819.31	-4722.77	237.50
f12 Pathological	6.98	0.58	2.68	6.98	181.36
f13 Michalewicz	-13.29	0.93	4.01	-13.26	219.90
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	159.96
f15 Quartic	842594780.00	256182497.88	1139491000.00	852945000.00	166.42
f16 Levy	74.71	19.62	97.40	72.59	176.64
f17 Step	3364.15	714.64	2906.21	3296.14	87.88
f18 Alpine	227.05	24.13	108.11	225.97	111.82

Table 11: Statistics for DE Strategy #10 (50 Experiments)

	Average	**Std_Dev**	**Range**	**Median**	**Avg_Time**
f1 Schwefel	7778.33	312.46	1510.50	7823.93	124.06
f2 De Jong 1	20574.54	2270.94	10077.50	20996.95	89.16
f3 Rosenbrock	31868718000.0	6328393234.5	26634200000.0	32337750000.0	113.80
f4 Rastrigin	571635.14	59008.57	257106.00	562412.50	121.18
f5 Griewangk	14610540.00	33319.18	185500.00	14612900.00	147.06
f6 Sine Envelope	-22.71	0.62	2.51	-22.55	176.16
f7 Stretch V Sine	10.15	0.00	0.00	10.15	174.00
f8 Ackley One	244.49	15.60	64.30	248.49	161.28
f9 Ackley Two	447.55	11.93	52.65	448.82	200.90
f10 Egg Holder	-7139.87	491.67	2054.78	-7050.31	171.42
f11 Rana	-4999.77	390.24	1570.68	-4949.47	240.28
f12 Pathological	6.31	0.69	2.52	6.48	185.36
f13 Michalewicz	-10.09	0.45	1.81	-10.08	230.62
f14 Masters' Cosine	-15.67	0.00	0.00	-15.67	159.80
f15 Quartic	4434390000.00	1130333159.69	5527710000.00	4486845000.00	170.52
f16 Levy	3116.75	565.63	2600.20	3089.74	187.90
f17 Step	19514.28	2184.96	9502.10	19467.35	90.74
f18 Alpine	334.03	23.52	92.20	337.74	114.66

2.2.3 Statistics Analysis

Tables 2-10 above show the statistics for strategies #1 to #10 of the DE algorithm. The first observation that can be made when looking at those tables is that each of the 10 strategies produces better results for most functions than both the iterative local search implemented in project2 and the GA. Another observation is that each of the strategies works, better on some of the objective functions and worse on the others, than other strategies. For example, strategy #4 (Table 5) has an average fitness of 0 for De Jong 1 and 9208.7 for Schwefel while strategy #10 (Table 11) has an average fitness of 3823.80 for De Jong 1 and 7989.56 for Schwefel. Similarly strategy #3 (Table 4) has an average fitness of 24.2 for Rosenbrock and -90000 for Rastrigin while strategy #8 (Table 9) has an average fitness of 1273 for Rosenbrock and -25371.11 for Rastrigin.

Among all 10 strategies,

- strategy #1 produced the best results for Rosenbrock's Saddle (average fitness: 23.55), Griewangk (average fitness: 228045.76), Masters Cosine Wave(average fitness: -15.67), Step (average fitness: 43.36) and Alpine (average fitness: 507.1146) which are all also better than the GA results for the same functions respectively except for Griewangk where an average fitness of 400.23 was found with the GA.

- Strategy #2 produced the best results for De Jong 1 (average fitness: 0), Egg Holder (average fitness: Egg Holder), Rana (average fitness: -3147.1), Quartic (average fitness: 0) which are all better than the results obtained with GA for the same functions respectively.
- Strategy #3 produced the best results for Sine Envelope Sine Wave (average fitness: -22.71), Stretch V Sine Wave (average fitness: 10.15), Ackley Two (average fitness: 0), Michalewicz (average fitness: -7.6) which are all also better than the results obtained with GA for the same functions respectively except for Stretch V Sine Wave where a similar value was obtained.
- Strategy #5 produced the best results for Ackley One (average fitness: -7.45) which is also better than what was obtained with the GA for the same function.
- Strategy #6 produced better results for pathological (average fitness: 5.8) and Levy (average fitness: 1.65) which are both also better that the results obtained with the GA for the same functions respectively.
- Strategy #7 produced the best results for Schwefel (average fitness: 7387.95) which is worse than what was obtained with the GA for the same function (average fitness: 2058.25).
- Strategy #8 produced the best results for Rastrigin (average fitness: -25371.11) which is also better than what was obtained with the GA for the same function.

3 Conclusion

In conclusion, among all 10 strategies, strategies #1 #2 #3 and #6 appear to be the most efficient as each of them produced the best results for at least 2 of the 18 functions. The DE is better in general than the iterative local search even though the latter sometimes produces better results for some of the functions such as for Griewangk and Sine Envelope Sine Wave. The DE is better in general than the GA even though the latter sometimes produces better results for some of the functions such as for Griewangk. The GA is better in general than the iterative local search even though the latter sometimes produces better results for some of the functions such as for De Jong 1, Sine Envelope Sine Wave, Egg holder, Rana, Michalewicz and Step.