

Abstract

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1 Theoretical background

This experiments serves to examine the position fluctuations of polystyrene particles with a diameter of

$$d = 4,28 \mu\text{m}$$

in an aqueous dispersion that is highly dilute. Therefore, the form of the potential surrounding the single particles is to be determined. Moreover, the determination of the dependency of the diffusion coefficient on the distance between particle and the walls of the cuvette containing the dispersion along with the total distance for a given particle are main goals of the experiment. Regarding the used optical tweezers, the light force on the particles is measured. Last, through measurements of silica particles' movement in aqueous dispersions with different concentrations of salt, the variation of the screening length is examined.

1.1 Brownian motion

The motion of particles suspended in a fluid resulting from their collision with fast moving molecules (or atoms) of the fluid is called the *Brownian motion*.

Within a fluid at thermal equilibrium at a given temperature, no preferential direction of flow exists. Therefore, the movement the fluid's molecules is random yielding no linear or angular momenta over time. Sufficently small particles suspended in this fluid move in random patterns, too, changing its velocity \vec{v} upon colliding with one of the fluid's molecules. As a matter of fact, the observation has been used as evidence for the existence of individual water (fluid) molecules.

Because of the sheer number of involved fluid molecules, the many-body interactions resulting in the *Brownian motion* cannot be solved relying only on classical mechanics. To put a number to it, the number of collisions of a single particle suspended in the fluid (a so called *Brownian particles*) with the fluid molecules is roughly of the order 10^{14} . Therefore, among other, Albert Einstein produced a probabilistic model using statistical mechanics. Einstein started by formulating a diffusion equation for the *Brownian particles*. To this end, he regarded a one dimensional x -space with the origin at the initial position of the modelled *Brownian particles*. Assuming the conservation of the number of fluid molecules and introducing the density function $\varphi(\Delta)$, with the random variable Δ , he expanded the *Brownian particle* density ρ at a time $t + \tau$ in a Taylor series

$$\begin{aligned} \rho(x, t) + \tau \frac{\partial \rho(x)}{\partial t} + \dots &= \rho(x, t + \tau) = \rho(x, t) \cdot \int_{-\infty}^{\infty} \varphi(\Delta) d\Delta \\ &= \rho(x, t) \cdot \int_{-\infty}^{\infty} \varphi(\Delta) d\Delta - \frac{\partial \rho}{\partial x} \cdot \int_{-\infty}^{\infty} \Delta \varphi(\Delta) d\Delta \\ &\quad + \frac{\partial^2 \rho}{\partial x^2} \cdot \int_{-\infty}^{\infty} \frac{\Delta^2}{2} \varphi(\Delta) d\Delta + \dots \\ &= \rho(x, t) \cdot 1 + 0 + \frac{\partial^2 \rho}{\partial x^2} \cdot \int_{-\infty}^{\infty} \frac{\Delta^2}{2} \varphi(\Delta) d\Delta + \dots \end{aligned}$$

While the integral in the second line equals one by definition of the probability, terms with even

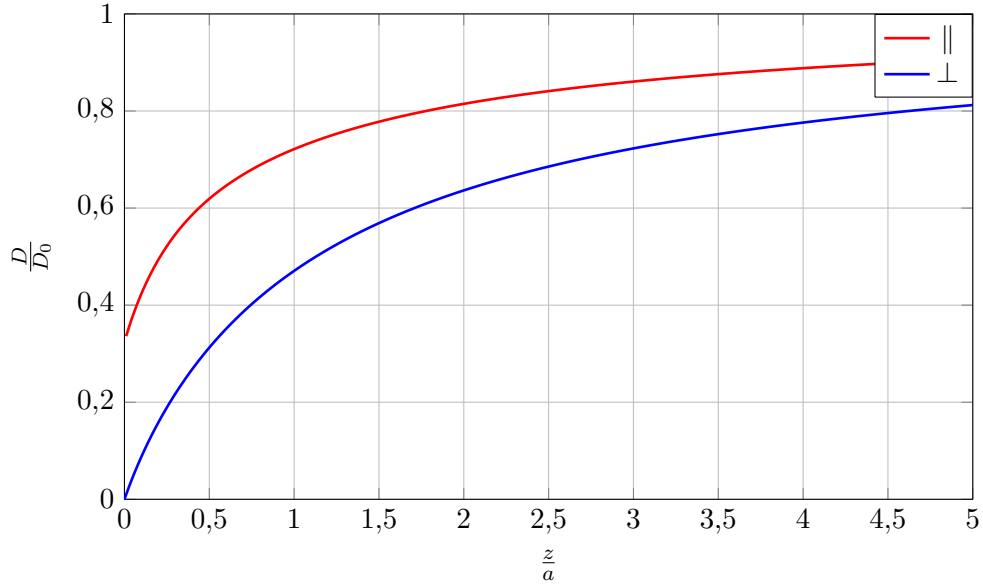


Figure 1.1

partials vanish due to symmetry of the 1D space. The above equation is equivalent to

$$\begin{aligned}\frac{\partial \rho}{\partial t} &\approx \frac{\partial^2 \rho}{\partial x^2} \cdot \int_{-\infty}^{\infty} \frac{\Delta^2}{2\tau} \varphi(\Delta) d\Delta \\ &= D \cdot \frac{\partial^2 \rho}{\partial x^2}\end{aligned}\quad (1.1)$$

if only terms with orders smaller than 2 of Δ are regarded and using the *mass diffusivity* or *diffusion coefficient*

$$D = \int_{-\infty}^{\infty} \frac{\Delta^2}{2\tau} \varphi(\Delta) d\Delta. \quad (1.2)$$

1.1.1 Brownian motion in proximity of the cavity walls

In Abschnitt 1.1, only the fluid's molecules surrounding the *Brownian particles* have been taken into account for the diffusion equation Gleichung (1.1). When the fluid is confined in a cavity though, the cavity's walls affects the flow of the fluid. This statement can be proven by simply regarding a fluid molecule or *Brownian particle* next the cavity wall. It's motion's component orthogonal to the wall is limited to one direction which is away from the wall. The interaction between the confining walls and the particles is called *hydrodynamic interaction*.

The mathematical description of this phenomenon makes use of distance z dependent diffusion coefficients

$$D_{||}(z) = D_0 \left[1 - \frac{9}{16} \left(\frac{a}{z+a} \right) + \frac{1}{8} \left(\frac{a}{z+a} \right)^3 - \frac{45}{256} \left(\frac{a}{z+a} \right)^4 - \frac{1}{16} \left(\frac{a}{z+a} \right)^5 + \dots \right] \quad (1.3)$$

$$\begin{aligned}D_{\perp}(z) &= D_0 \left[\frac{4}{3} \sinh(\alpha) \sum_{n=0}^{\infty} \frac{n(n+1)}{(2n-1)(2n+3)} \left(\frac{2 \sinh((2n+1)\alpha) + (2n+1) \sinh(2\alpha)}{4 \sinh^2((n+\frac{1}{2})\alpha) - (2n+1)^2 \sinh^2(\alpha)} - 1 \right) \right]^{-1} \\ &\approx D_0 \cdot \frac{6 \left(\frac{z}{a} \right)^2 + 2 \frac{z}{a}}{6 \left(\frac{z}{a} \right)^2 + 9 \frac{z}{a} + 2}\end{aligned}\quad (1.4)$$

for the motion orthogonal (\perp) and parallel ($||$) to the walls, where z is the distance between particle and wall. For the approximation of Gleichung (1.4) please refer to [1].

Bibliography

- [1] M. A. Bevan und D. C. Prieve. "Hindered diffusion of colloidal particles verry near to a wall: revisited". In: *J. Chem. Phys.* 113 (2000).