Table of Contents


```
clear all
close all
clc
format long
start_up

% Propagating the two trajectory through regular integration
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
global mu_Earth ae JD AU Target Chasser index ...
    acc2body time2body accrelative timerelative

index = 1;
ae = 6378.136;
mu_Earth = 3.986004415E5;
JD = 2456296.25;
AU = 149597870.7;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
% Target initial conditions(Orbital Elements)
a1      = 1.5E+4; % Semi-major axis in Km
e1      = 0.1;    % Eccentricity
inc1    = 10;     % Inclination in deg0
BigOmg1 = 50;     % RAAN in deg
LitOmg1 = 10;     % AOP in deg
f1     = -80;     % True anomaly in deg

Target = Spacecraft([10 6 2 40 10 0.2 0.5 6 12]); % Describing the
 Chief sailcraft characteristic (refer Spacecraft class definition)
qr_target = [1 0 0 0]'; % No attitude because we are assuming the
 target is a cannon ball
qr_target = qr_target/norm(qr_target);
Omega_target = zeros(3,1); % No Angular velocity because we are
 assuming the target is a cannonball

%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
% Chaser initial conditions(Orbital Elements)
a2      = 1.75E+4; % Semi-major axis in Km
e2      = 0.52;    % Eccentricity
inc2    = 60.1 ;   % Inclination in deg
BigOmg2 = 10;      % RAAN in deg
LitOmg2 = 0;       % AOP in deg
f2     = 124;      % True anomaly in deg
```

```
Chasser = Spacecraft([30 30 4 116 40 0.9 0.9 3000 6000]); % Describing
 the Deputy sailcraft characteristic (refer Spacecraft class
 definition)
qr_chasser = [1 1 1 9]'; % [1 0 0 0]'; %
qr_chasser =  qr_chasser/norm(qr_chasser); %
Omega_chaser = 0.2*[deg2rad(-.5) deg2rad(.4) deg2rad(.1)]'; % Angular
 velocity in inertial frame
```

Converting angles in radians

```
inc1 = deg2rad(inc1); BigOmg1 = deg2rad(BigOmg1); LitOmg1 =
 deg2rad(LitOmg1); f1 = deg2rad(f1);
COE1 = [a1,e1,inc1,BigOmg1,LitOmg1,f1];
[Position_target,Velocity_target]  = COEstoRV(COE1,mu_Earth);

inc2 = deg2rad(inc2); BigOmg2 = deg2rad(BigOmg2); LitOmg2 =
 deg2rad(LitOmg2); f2 = deg2rad(f2);
COE2 = [a2,e2,inc2,BigOmg2,LitOmg2,f2];
[Position_chaser,Velocity_chaser]  = COEstoRV(COE2,mu_Earth);
```

Initial condtions

```
Period = 2*pi*sqrt(a2^3/mu_Earth);
int_time = 5*Period;
tspan  = linspace(0,int_time,10000);
options = odeset('RelTol',2.22045e-14,'AbsTol',2.22045e-40);
```

Integrate the trajectory in cartesian coordinates (Validation purposes only)

```
for k = 1
    CN = Quaternion_to_DCM(qr_chasser); Omega_chaser_B =
 CN*Omega_chaser;
    Aug_chasser_Xo = [qr_chasser; Omega_chaser_B; Position_chaser;
 Velocity_chaser];
    TN = Quaternion_to_DCM(qr_target); Omega_target_B =
 TN*Omega_target;
    Aug_target_Xo = [qr_target; Omega_target_B; Position_target;
 Velocity_target];
    Aug_Xo = [Aug_chasser_Xo; Aug_target_Xo];
    tic
    [time1, Aug_X] =
 ode113(@(t,Aug_X)my2Bodyfunc(t,Aug_X,Target,Chasser),tspan,Aug_Xo,options);
    toc

    % Computing the Sun's position
    JulianDay = JD+tspan(1)/86400;
    [XS, Vs, ~] = Ephem(JulianDay,3,'EME2000');
    XSUN_ECI = XS/norm(XS);
    TN = DCM(Position_target,Velocity_target);
```

```
    XSUN_LVLH = TN*XSUN_ECI;

    quaternion_chasser1 = Aug_X(:,1:4);
    quaternion_target1  = Aug_X(:,14:17);
    omega_chasser1 = nan(length(time1),3);
    omega_target1 = nan(length(time1),3);
    R_target2 = nan(length(time1),3);
    V_target2 = nan(length(time1),3);
    R_chasser2 = nan(length(time1),3);
    V_chasser2 = nan(length(time1),3);
    TR_rel = nan(length(time1),3);
    TV_rel = nan(length(time1),3);

    for j = 1:length(time1)

        CN = Quaternion_to_DCM(quaternion_chasser1(j,:)');
        omega_chasser1(j,:) = (CN'*Aug_X(j,5:7)')';
        Chasser_r = Aug_X(j,8:10)'; Chasser_v = Aug_X(j,11:13)';
 Target_r = Aug_X(j,21:23)'; Target_v = Aug_X(j,24:26)';
        h_vec = cross(Target_r,Target_v); h_norm = norm(h_vec);
        eh = h_vec/h_norm; rt_norm = norm(Target_r);
        U_eci_Target  = F_CanonBall(time1(j),Target_r,Target); % SRP
 force on the Target
        N_nudot = h_vec/rt_norm^2 + dot(U_eci_Target,eh)*Target_r/
h_norm;

        TN = DCM(Target_r,Target_v); NOmega_target =
 cross(Target_r,Target_v)/norm(Target_r)^2;
        NR_rel = Chasser_r - Target_r; NV_rel = Chasser_v - Target_v;
        TR_rel(j,:) = TN*NR_rel; TV_rel(j,:) = TN*(NV_rel -
 cross(N_nudot,NR_rel));

    end
    R_chasser1 = [Aug_X(:,8) Aug_X(:,9) Aug_X(:,10)]; V_chasser1 =
 [Aug_X(:,11) Aug_X(:,12) Aug_X(:,13)];
    R_target1 = [Aug_X(:,21) Aug_X(:,22) Aug_X(:,23)]; V_target1 =
 [Aug_X(:,24) Aug_X(:,25) Aug_X(:,26)];


end
```

Elapsed time is 7.938048 seconds.

Integrating the relative motion in the chief's LVLH frame

```
TN = DCM(Position_target,Velocity_target); rt_norm =
 norm(Position_target);
h_vec = cross(Position_target,Velocity_target); h_norm = norm(h_vec);
eh = h_vec/h_norm;
U_eci_Target  = F_CanonBall(time1(1),Position_target,Target); % SRP
 force on the Target
```

```
N_nudot = h_vec/rt_norm^2 + dot(U_eci_Target,eh)*Position_target/
h_norm;
NR_rel = Position_chaser - Position_target; NV_rel2 = Velocity_chaser
 - Velocity_target;
TR_rel0 = TN*NR_rel; TV_rel0 = TN*(NV_rel2 - cross(N_nudot,NR_rel));
X_aug0 = [Position_target; Velocity_target; qr_chasser;
 Omega_chaser_B; TR_rel0; TV_rel0];
index = 1;
tic
[time, X_rel] = ode113(@(t,X_aug)RelativeMotionODE(t, X_aug, Target,
 Chasser),time1,X_aug0,options);
toc
```

Elapsed time is 6.649005 seconds.

Plots

```
for k = 1

    %~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Plotting the 3D trajectories
    for ii = 1
        c1 = rgb('DeepPink'); c2 = rgb('Cyan'); c3 = rgb('Goldenrod');

        % Attitude Description of the Spacecraft
        B_axis1 = [1 0 0]';
        B_axis2 = [0 1 0]';
        B_axis3 = [0 0 1]';
        Len = 5000;
        CN = Quaternion_to_DCM(qr_chasser);
        P = R_chasser1(1,:) ; % you center point
        L = 2*[1000,1000,100];   % your cube dimensions
        shift = 1/2*CN'*L';
        O = P-shift';            % Get the origin of cube so that P is
 at center

        %=============================================================
%
        % Ploting the time evolution of the referece frame
        Reference_N =
 ReferenceFrame(R_chasser1(1,:)',R_target1(1,:)');
        NTarget1 = Reference_N'*B_axis1;
        NTarget2 = Reference_N'*B_axis2;
        NTarget3 = Reference_N'*B_axis3;
        v2a = P+Len*NTarget1';
        va = [v2a;P];
        v2b = P+Len*NTarget2';
        vb = [v2b;P];
        v2c = P+Len*NTarget3';
        vc = [v2c;P];

        for jj=1
```

Main Script Part 2

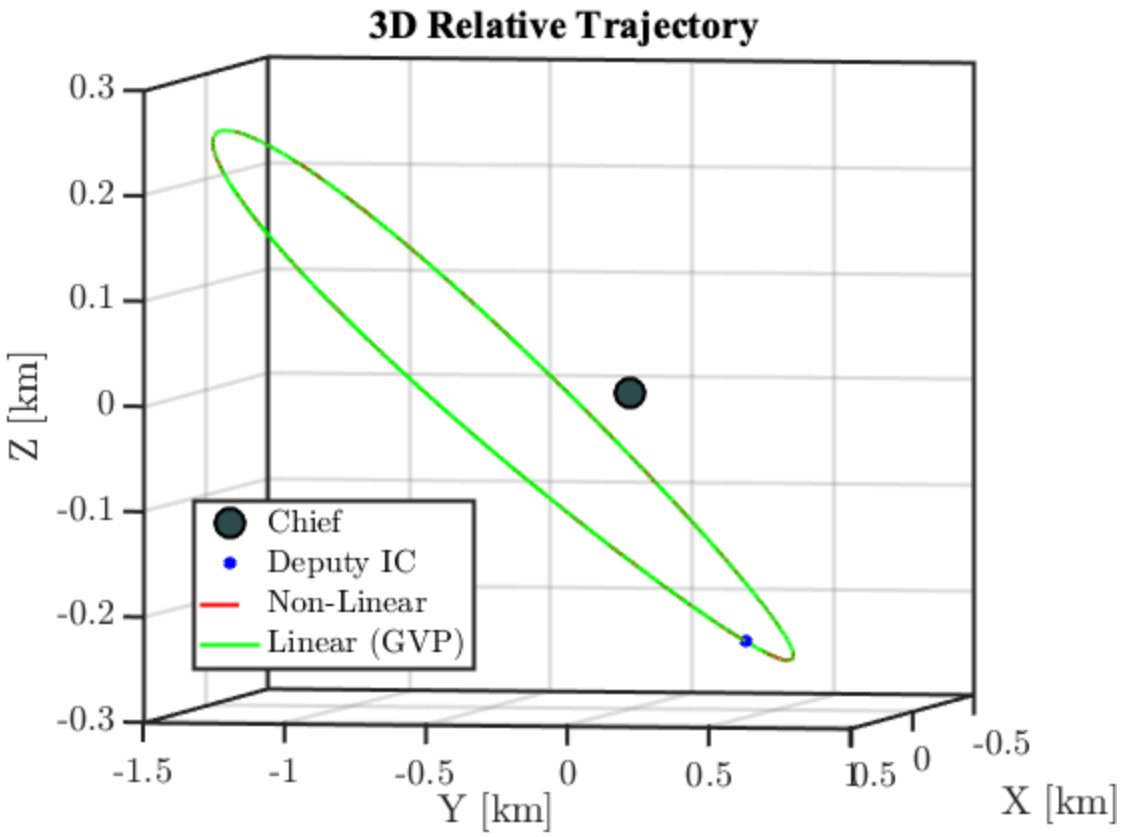
```
clear all
close all
clc
format long e
global mu_Earth ae JD AU Target Chasser
mu_Earth = 3.986004415E5;
ae = 6378.136;
JD = 2456296.25;
AU = 149597870.7;

% Setting the initial conditon
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for k = 1
    % Target initial conditions(Orbital Elements)
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Target initial conditions(Orbital Elements)
    a1      = 1.5E+4; % Semi-major axis in Km
    e1      = 0.1;    % Eccentricity
    inc1    = 10;     % Inclination in deg0
    BigOmg1 = 50;     % RAAN in deg
    LitOmg1 = 10;     % AOP in deg
    f1     = -80;       % True anomaly in deg
    Target = Spacecraft([10 6 2 40 10 0.2 0.5 6 12]); % Describing the
 Chief sailcraft characteristic (refer Spacecraft class definition)

    % Relative orbital elements of the deputy
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    dela = 0;                    % Relative semi-major axis in Km
    deli = 1/(4*a1);             % Relative inclination in deg0
    dele = -1/(2*a1);           % Relative eccentricity, in rad
    delLitOmg = 1/(2*a1);        % Relative RAAN in rad
    delBigOmg = 0;               % Relative AOP in rad
    delf = 0;                    % Relative true anomaly in rad


    % Chaser initial conditions(Orbital Elements)
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    a2      = a1;          % Semi-major axis in Km
    e2      = e1 + dele;   % Eccentricity
    inc2    = inc1;        % Inclination in deg
    BigOmg2 = BigOmg1;     % RAAN in deg
    LitOmg2 = LitOmg1;     % AOP in deg
    f2      = f1;          % True anomaly in deg
    Chasser = Spacecraft([30 30 4 116 40 0.9 0.9 3000 6000]); %
 Describing the Deputy sailcraft characteristic (refer Spacecraft
 class definition)

    % Converting angles in radians
    inc1 = deg2rad(inc1); BigOmg1 = deg2rad(BigOmg1); LitOmg1 =
 deg2rad(LitOmg1); f1 = deg2rad(f1);
```

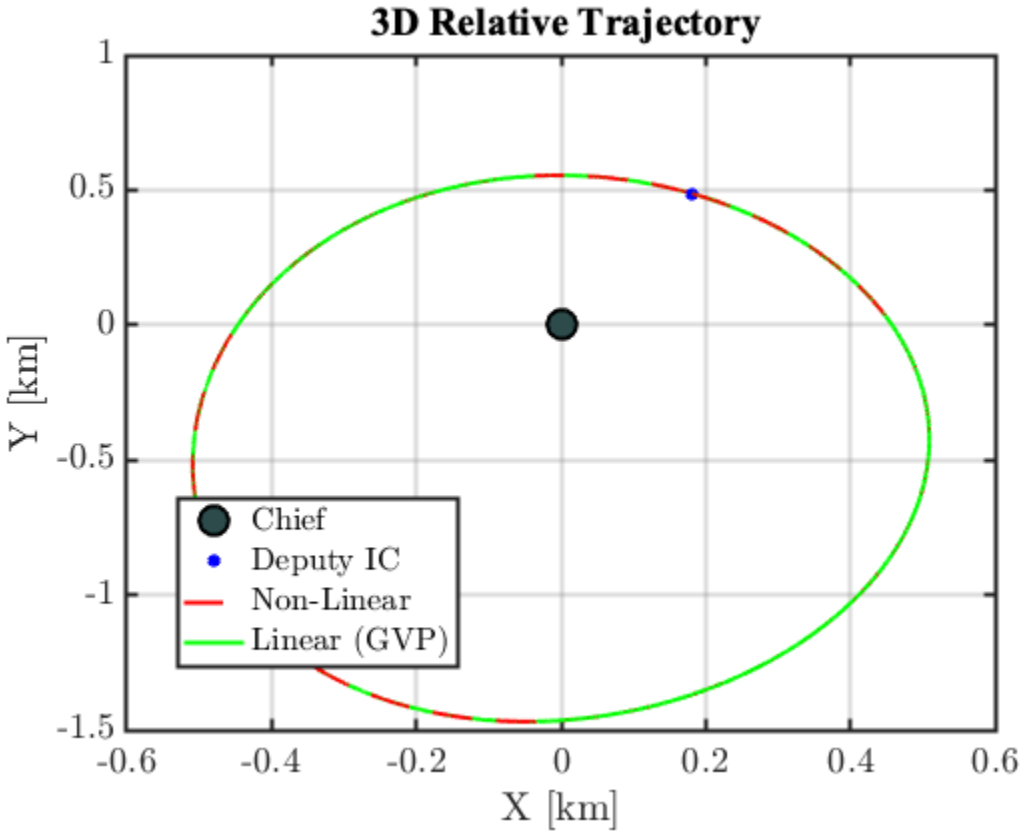
```
    COE1 = [a1,e1,inc1,BigOmg1,LitOmg1,f1];
    [Position_target,Velocity_target]  = COEstoRV(COE1,mu_Earth);

    inc2 = deg2rad(inc2) + deli; BigOmg2 = deg2rad(BigOmg2); LitOmg2 =
 deg2rad(LitOmg2)+delLitOmg; f2 = deg2rad(f2);
    COE2 = [a2,e2,inc2,BigOmg2,LitOmg2,f2];
    [Position_chaser,Velocity_chaser]  = COEstoRV(COE2,mu_Earth);

    % Setting the integrator parameters
    Period = 2*pi*sqrt(a1^3/mu_Earth);
    IntegrationTime = Period;
    tspan  = linspace(0,IntegrationTime,10000);
    options = odeset('RelTol',2.22045e-14,'AbsTol',2.22045e-30);
end

for k=1
    tic
    [~, Chief_OE] =
 ode113(@(t,COE,nu,u,f)GVE(t,COE,Target),tspan,COE1,options);
    toc

    tic
    [~, Deputy_OE] =
 ode113(@(t,COE,nu,u,f)GVE(t,COE,Chasser),tspan,COE2,options);
    toc
    X_rel_linear1 = nan(length(tspan),6);
    R_target2 = nan(length(tspan),3);
    V_target2 = nan(length(tspan),3);

    for j = 1:length(tspan)

        [R_target2(j,:),V_target2(j,:)]  =
 COEstoRV(Chief_OE(j,:),mu_Earth);

       % Computing the relative using the linear maping COE2 =
 [a2,e2,inc2,BigOmg2,LitOmg2,f2];
       % Nonsingular Chief orbital elements
       a_c = Chief_OE(j,1); e_c = Chief_OE(j,2); LitOmg_c =
 Chief_OE(j,5);  f_c = Chief_OE(j,6);
       theta_c = LitOmg_c + f_c;
       inc_c = Chief_OE(j,3);
       q1_c = e_c * cos(LitOmg_c);
       q2_c = e_c * sin(LitOmg_c);
       BigOmg_c = Chief_OE(j,4);

       % Nonsingular Deputy orbital elements
       a_d = Deputy_OE(j,1); e_d = Deputy_OE(j,2); LitOmg_d =
 Deputy_OE(j,5);  f_d = Deputy_OE(j,6);
       theta_d = LitOmg_d + f_d;
       inc_d = Deputy_OE(j,3);
       q1_d = e_d * cos(LitOmg_d);
       q2_d = e_d * sin(LitOmg_d);
       BigOmg_d = Deputy_OE(j,4);
```

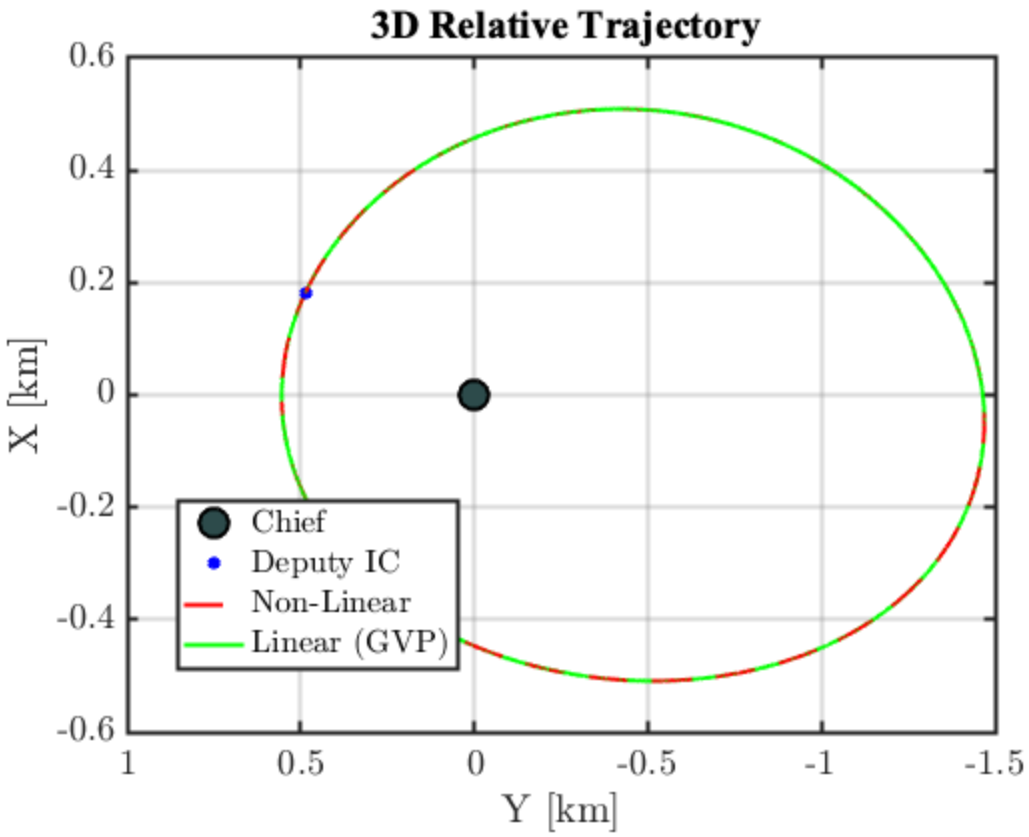
```
        % Relative orbital elements
        del_a = a_d - a_c; del_e = e_d - e_c;  delLitOmg = LitOmg_d -
 LitOmg_c;
        del_theta = theta_d - theta_c;
        del_inc = inc_d - inc_c;
        del_q1 = del_e*cos(LitOmg_c) - e_c*sin(LitOmg_c)*delLitOmg;
        del_q2 = del_e*sin(LitOmg_c) + e_c*cos(LitOmg_c)*delLitOmg;
        del_BigOmg = BigOmg_d - BigOmg_c;

        COE = [a_c, theta_c, inc_c, q1_c, q2_c, BigOmg_c]';
        delCOE = [del_a, del_theta, del_inc, del_q1, del_q2,
 del_BigOmg]';
        [A] = ForwardMapping(COE, mu_Earth);
        Rho_aug = A*delCOE;
        X_rel_linear1(j,:) = Rho_aug';


    end
end

% Integrating the relative motion in the chief's LVLH frame
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for i = 1

    TN = DCM(Position_target,Velocity_target); rt_norm =
 norm(Position_target);
    h_vec = cross(Position_target,Velocity_target); h_norm =
 norm(h_vec);
    eh = h_vec/h_norm;
    U_eci_Target  = 0*F_CanonBall(tspan(1),Position_target,Target); %
 SRP force on the Target
    N_nudot = h_vec/rt_norm^2 + dot(U_eci_Target,eh)*Position_target/
h_norm;
    NR_rel = Position_chaser - Position_target; NV_rel2 =
 Velocity_chaser - Velocity_target;
    TR_rel0 = TN*NR_rel; TV_rel0 = TN*(NV_rel2 -
 cross(N_nudot,NR_rel));
    X_aug0 = [Position_target; Velocity_target; TR_rel0; TV_rel0];
    index = 1;
    tic
    [~, X_rel] = ode113(@(t,Xaug)RelativeMotionODE_cannonball(t, Xaug,
 Target, Chasser),tspan,X_aug0,options);
    toc

end


% Plotting the 3D realtive trajectory
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for k = 1:3
    % Plotting the 3D trajectories
    c1 = rgb('Crimson'); c2 = rgb('DarkSlateGray');
    figure
    for jj=1
```

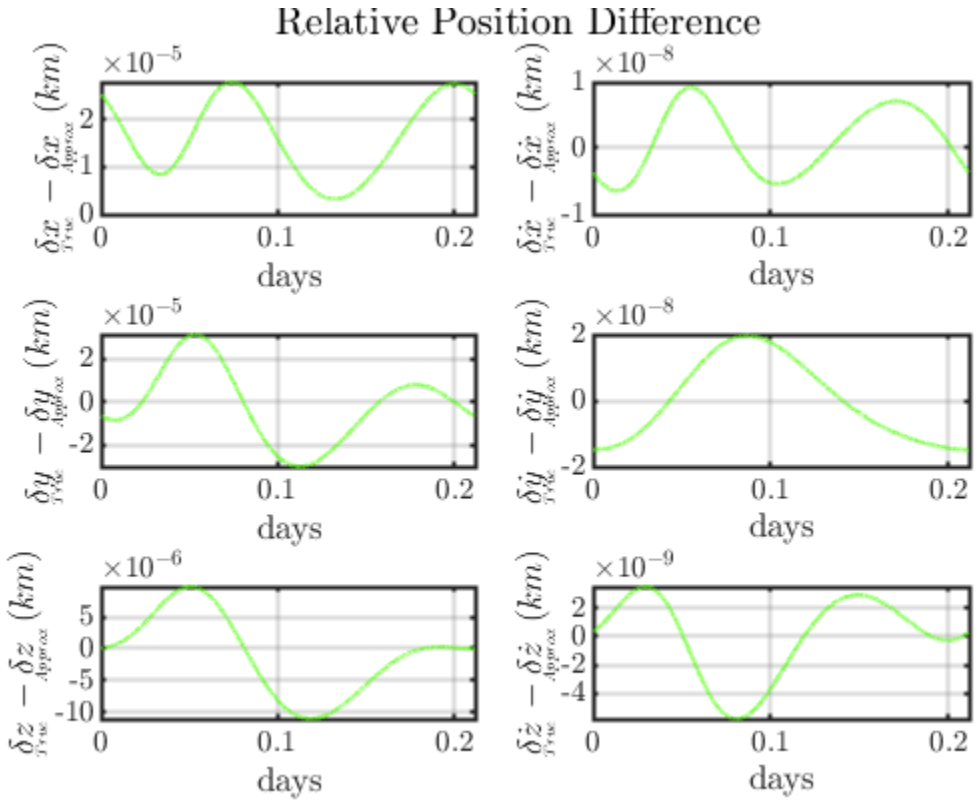
```
        h1 =
 plot3(X_rel_linear1(:,1),X_rel_linear1(:,2),X_rel_linear1(:,3),'g');
        hold on
        h3 = plot3(X_rel(:,7),X_rel(:,8),X_rel(:,9),'r--');

        h4 = plot3(X_rel(1,7),X_rel(1,8),X_rel(1,9),'go',...
            'LineWidth',2,...
            'MarkerEdgeColor','b',...
            'MarkerFaceColor','b',...
            'MarkerSize',5);

        h5 = plot3(0,0,0,'bo',...
            'LineWidth',2,...
            'MarkerEdgeColor','k',...
            'MarkerFaceColor',c2,...
            'MarkerSize',15);
        grid on
        xlabel('X [km]')
        ylabel('Y [km]')
        zlabel('Z [km]')
        title('{\color{black} 3D Relative
 Trajectory}','interpreter','tex')
        h = legend([h5, h4, h3, h1],{'Chief','Deputy IC','Non-
Linear','Linear (GVP)'});
        rect = [0.25, 0.25, 0.1, 0.1];
        set(h, 'Position', rect)
        box on
    end

    if k == 1
        view(100,3)
    elseif k==2
        view(0,90)
    else
        view(-90,90)
    end

end


for jj=1

    DQ1 = [X_rel_linear1(:,1) X_rel_linear1(:,4) X_rel_linear1(:,2)
 X_rel_linear1(:,5) X_rel_linear1(:,3) X_rel_linear1(:,6)]; %
    DQ2 = [X_rel(:,7) X_rel(:,10) X_rel(:,8) X_rel(:,11) X_rel(:,9)
 X_rel(:,12)]; %
    figure
    YLabel={'$\delta x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
```

```
        '$\delta \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$'};
    for i=1:6
        subplot(3,2,i)
        plot1 = plot(tspan/86400,DQ1(:,i)-DQ2(:,i));
        ylabel(YLabel(i))
        xlabel('days')
        grid on

        plot1.Color(4) = 0.6;
        %             ax = gca;
        %             ax.GridColor = [0.9, 0.9, 0.9];  % [R, G, B]
        %             ax.XColor = 'k';
        %             ax.YColor = 'k';
        %             set(gca,'color','k')
        %             set(gcf,'color',[0.3, 0.3, 0.3])
    end
    ha = axes('Position',[0 0 2.2 1],'Xlim',[0 1],'Ylim',[0
 1],'Box','off','Visible','off','Units','normalized', 'clipping'
 , 'off');
    text(0.13, 0.98,sprintf('Relative Position Difference'))
end

Elapsed time is 0.487324 seconds.
Elapsed time is 0.296204 seconds.
Elapsed time is 8.264004 seconds.
```


3D Relative Trajectory
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Z [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)
-1.5
-1
-0.5
0
0.5
1
0.5
0
-0.5
Y [km]
X [km]


3D Relative Trajectory
1
0.5
0
-0.5
-1
-1.5
Y [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X [km]


3D Relative Trajectory
X [km]
Y [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)


Relative Position Difference
days

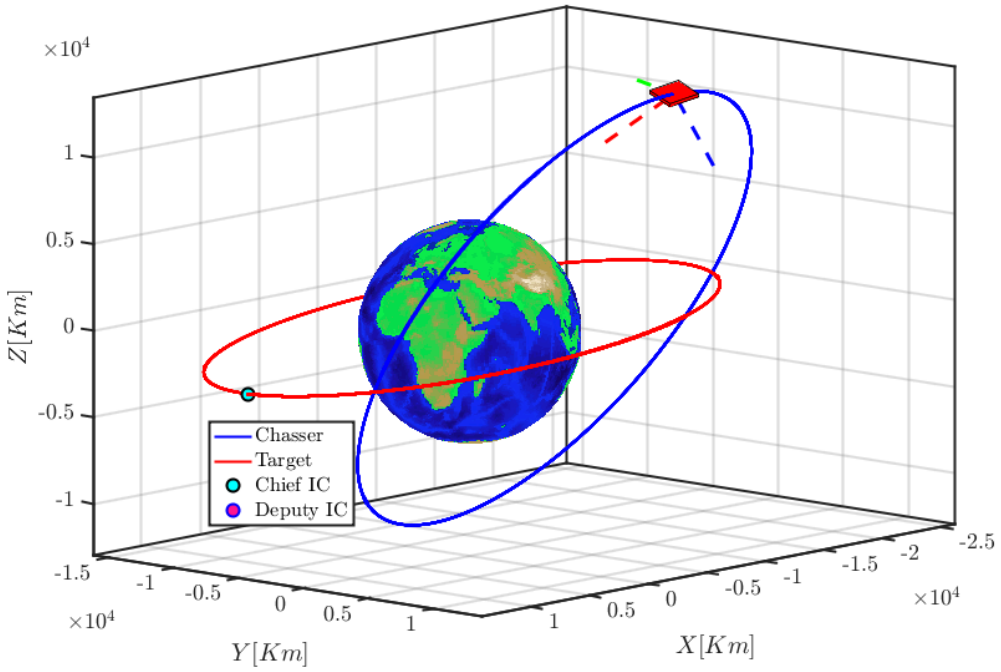
```
            figure('Renderer', 'painters', 'Position', [15 15 900
600])
            h1 =
plot3(R_chasser1(:,1),R_chasser1(:,2),R_chasser1(:,3),'b');
            hold on
            cube = plotcube(L,O,1,[1 0 0],CN');  % use function
plotcube
            Taxis_head1 =
plot3(va(:,1),va(:,2),va(:,3),'g--','LineWidth',3);
            Taxis_head2 =
plot3(vb(:,1),vb(:,2),vb(:,3),'b--','LineWidth',3);
            Taxis_head3 =
plot3(vc(:,1),vc(:,2),vc(:,3),'r--','LineWidth',3);

            h2 =
plot3(R_target1(:,1),R_target1(:,2),R_target1(:,3),'r');
            earth_sphere('km');
            h3 =
plot3(R_chasser1(1,1),R_chasser1(1,2),R_chasser1(1,3),'ro',...
                'LineWidth',2,...
                'MarkerEdgeColor','b',...
                'MarkerFaceColor',c1,...
                'MarkerSize',10);
            h4 =
plot3(R_target1(1,1),R_target1(1,2),R_target1(1,3),'bo',...
                'LineWidth',2,...
                'MarkerEdgeColor','k',...
                'MarkerFaceColor',c2,...
                'MarkerSize',10);
            grid on
            hold off
            axis equal
            xlabel('$X [ Km ]$')
            ylabel('$Y [ Km ]$')
            zlabel('$Z [ Km ]$')
            h = legend([h1 h2 h4,h3],{'Chasser', 'Target','Chief
IC','Deputy IC'});
            rect = [0.25, 0.25, 0.1, 0.1];
            set(h, 'Position', rect)
            box on
            view(132,10)
        end

    end

    %~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Plotting the 3D trajectories
    for ii =1
        c1 = rgb('Crimson'); c2 = rgb('DarkSlateGray');
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        for kk = 1:4
            subplot(2,2,kk)
            for jj=1
                h1 = plot3(TR_rel(:,1),TR_rel(:,2),TR_rel(:,3),'b-');
```

```
                hold on
                h4 = plot3(X_rel(:,14),X_rel(:,15),X_rel(:,16),'r--');
                h2 =
 plot3(TR_rel(1,1),TR_rel(1,2),TR_rel(1,3),'-','color', c3,...
                    'LineWidth',2,...
                    'MarkerEdgeColor','b',...
                    'MarkerFaceColor',c3,...
                    'MarkerSize',5);
                h3 =
 plot3(TR_rel(end,1),TR_rel(end,2),TR_rel(end,3),'bo',...
                    'LineWidth',2,...
                    'MarkerEdgeColor','k',...
                    'MarkerFaceColor',c1,...
                    'MarkerSize',5);
                h5 = plot3(0,0,0,'bo',...
                    'LineWidth',2,...
                    'MarkerEdgeColor','k',...
                    'MarkerFaceColor',c2,...
                    'MarkerSize',15);
                arrow3D([.5e4,.5e4,.5e4] ,8e3*XSUN_LVLH,c3)
                h = legend([h1, h4, h5, h2],
{'Truth','Relative','chief','Sun Direction'});
                rect = [0.43, 0.435, 0.125, 0.12];
                set(h, 'Position', rect)
                box on
                grid on
                xlabel('X [km]')
                ylabel('Y [km]')
                zlabel('Z [km]')
            end

            if kk == 1
                view(70,10)
            elseif kk==2
                view(0,90)  % XY
            elseif kk==3
                view(0,0)   % XZ
            else
                view(90,0)  % YZ
            end

        end
        sgt = sgtitle('Relative Trajectory','interpreter','tex');
        sgt.FontSize = 30;
    end


    %~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Plotting the difference between ECI Cartesian coordinates of the
 deputy
    for jj=1

        DQ1 = [X_rel(:,14) X_rel(:,17) X_rel(:,15) X_rel(:,18)
 X_rel(:,16) X_rel(:,19)];
```

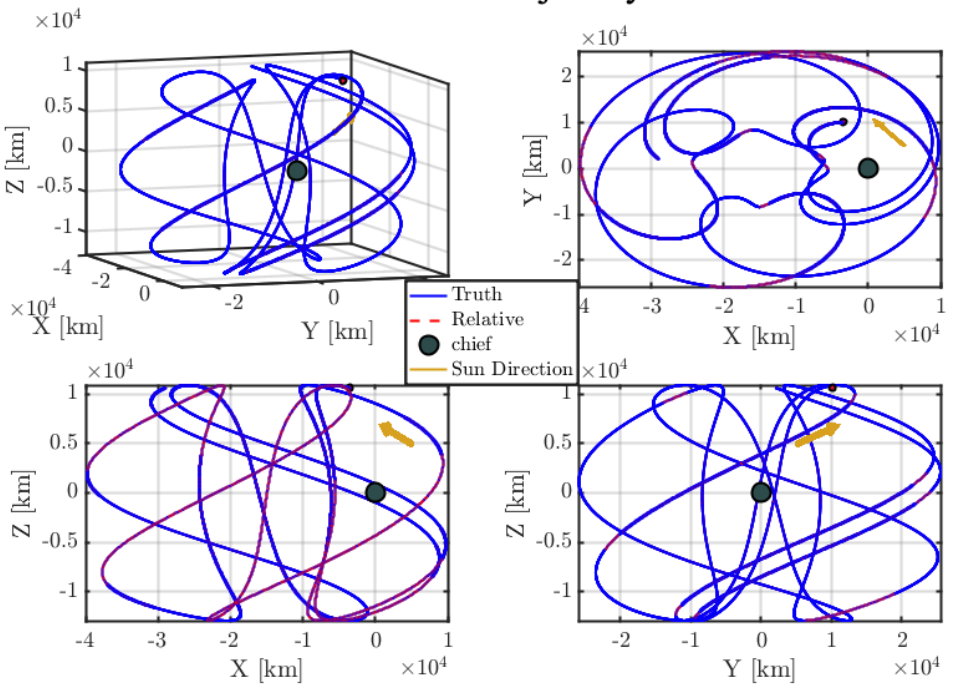
```
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        YLabel={'$\delta x [km]$',...
            '$\delta \dot{x} [km/s]$',...
            '$\delta y [km]$',...
            '$\delta \dot{y} [km/s]$',...
            '$\delta z [km]$',...
            '$\delta \dot{z} [km/s]$'};
        for i=1:6
            subplot(3,2,i)
            plot1 = plot(time/86400,DQ1(:,i));
            ylabel(YLabel(i))
            xlabel('days')
            grid on
        end
        sgt = sgtitle('Relative Position','interpreter','tex');
        sgt.FontSize = 30;
    end

    %~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Plotting the difference between ECI Cartesian coordinates of the
deputy
    for jj=1

        DQ1 = [X_rel(:,14) X_rel(:,17) X_rel(:,15) X_rel(:,18)
X_rel(:,16) X_rel(:,19)];
        DQ2 = [TR_rel(:,1) TV_rel(:,1) TR_rel(:,2) TV_rel(:,2)
TR_rel(:,3) TV_rel(:,3)];
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        YLabel={'$\delta x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} -
\delta x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}[km]$',...
            '$\delta \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} -
\delta \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}[km/s]$',...
            '$\delta y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}[km]$',...
            '$\delta \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} -
\delta \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}[km/s]$',...
            '$\delta z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
            '$\delta \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} -
\delta \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}[km/s]$'};
        for i=1:6
            subplot(3,2,i)
            plot1 = plot(time/86400,DQ1(:,i)-DQ2(:,i));
            ylabel(YLabel(i))
            xlabel('days')
            grid on
        end
        sgt = sgtitle('Relative Position
Difference','interpreter','tex');
        sgt.FontSize = 30;
    end

    for jj=1
```

```
        acc2body = [acc2body(:,1), acc2body(:,4), acc2body(:,2),
 acc2body(:,5), acc2body(:,3), acc2body(:,6)];
        accrelative = [accrelative(:,1), accrelative(:,4),
 accrelative(:,2), accrelative(:,5), accrelative(:,3),
 accrelative(:,6)];
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        YLabel={'$u_{_{\!1}}$', '$\tau_{_{\!1}}$', '$u_{_{\!2}}$',...
            '$\tau_{_{\!2}}$', '$u_{_{\!3}}$', '$\tau_{_{\!3}}$'};
        for i=1:6
            subplot(3,2,i)
            plot(time2body/86400,acc2body(:,i),'b') % -Q2(:,i)
            hold on
            plot(timerelative/86400,accrelative(:,i),'r--') % -Q2(:,i)
            ylabel(YLabel(i))
            xlabel('days')
            grid on
            % legend('Classical Dynamics', 'Dual Quaternions')
        end
        sgt = sgtitle('Acceleration and Torque on the Flat-
Plate','interpreter','tex');
        sgt.FontSize = 30;
    end


    for jj=1
        Q1 = Aug_X(:,1:4);
        Q2 = X_rel(:,7:10);
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        YLabel={'$q_{_{\!1}}$','$q_{_{\!2}}$','$q_{_{\!3}}$','$q_{_{\!
4}}$'};
        for i=1:4
            subplot(2,2,i)
            plot(time/86400,Q1(:,i),'g') % -Q2(:,i)
            ylabel(YLabel(i))
            xlabel('days')
            grid on
            % legend('Classical Dynamics', 'Dual Quaternions')
        end
        sgt = sgtitle('Deputy Attitude
 (Quaternions)','interpreter','tex');
        sgt.FontSize = 30;
    end

    %~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Plotting the angular velocy
    for jj=1
        figure('Renderer', 'painters', 'Position', [15 15 900 600])
        YLabel={'$\omega_{_{\!1}}$','$\omega_{_{\!2}}$','$\omega_{_{\!
3}}$'};
        for i=1:3
            subplot(3,1,i)
            % plot(time/86400,omega_chasser1(:,i)-omega_target1(:,i))
            plot(time/86400,omega_chasser1(:,i),'b')
            ylabel(YLabel(i))
```

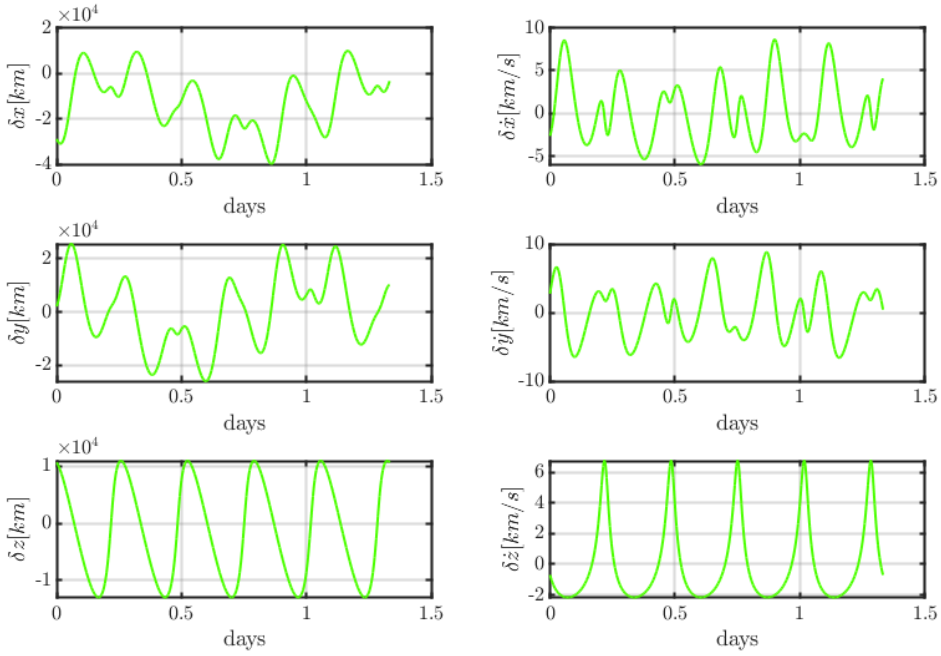
```
            xlabel('days')
            grid on
            % legend('Classical Dynamics', 'Dual Quaternions')
        end
        sgt = sgtitle('Deputy Angular Velocity','interpreter','tex');
        sgt.FontSize = 30;
    end

end % Plots for relative motions (Chaser and Target) with classical
 dynamics
```



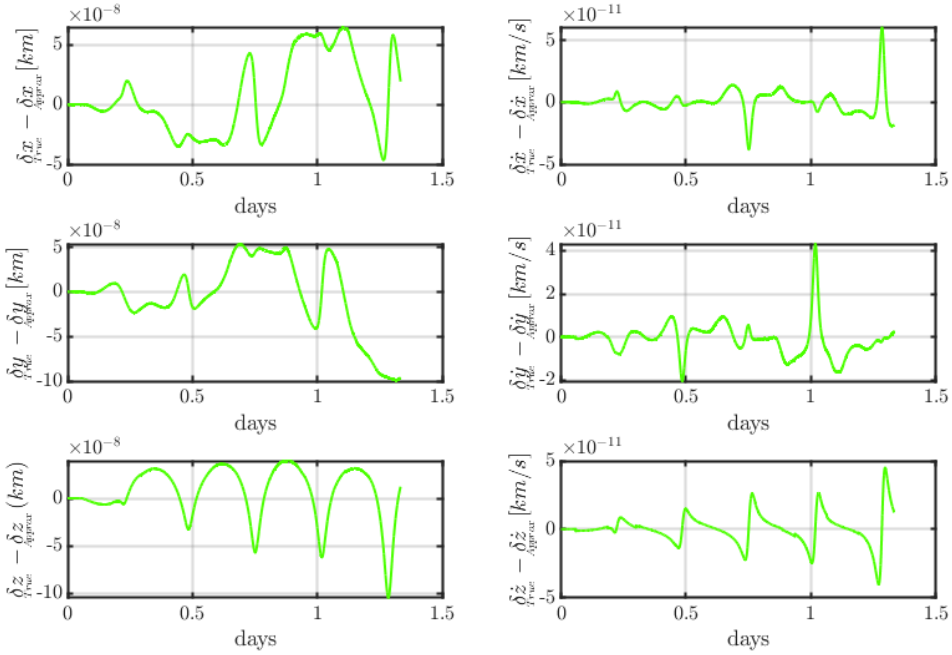
Relative Trajectory



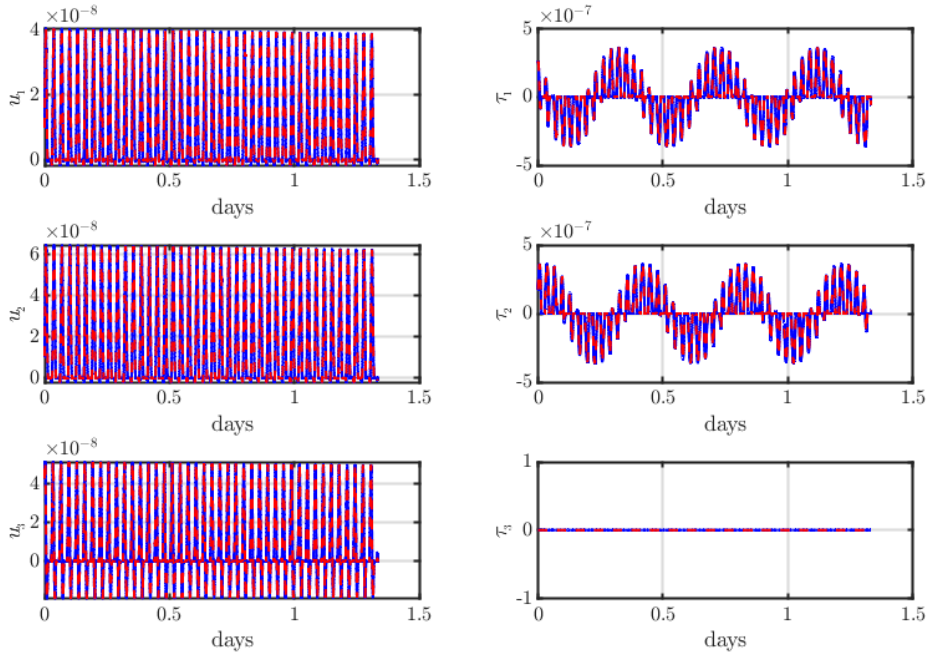
Relative Position

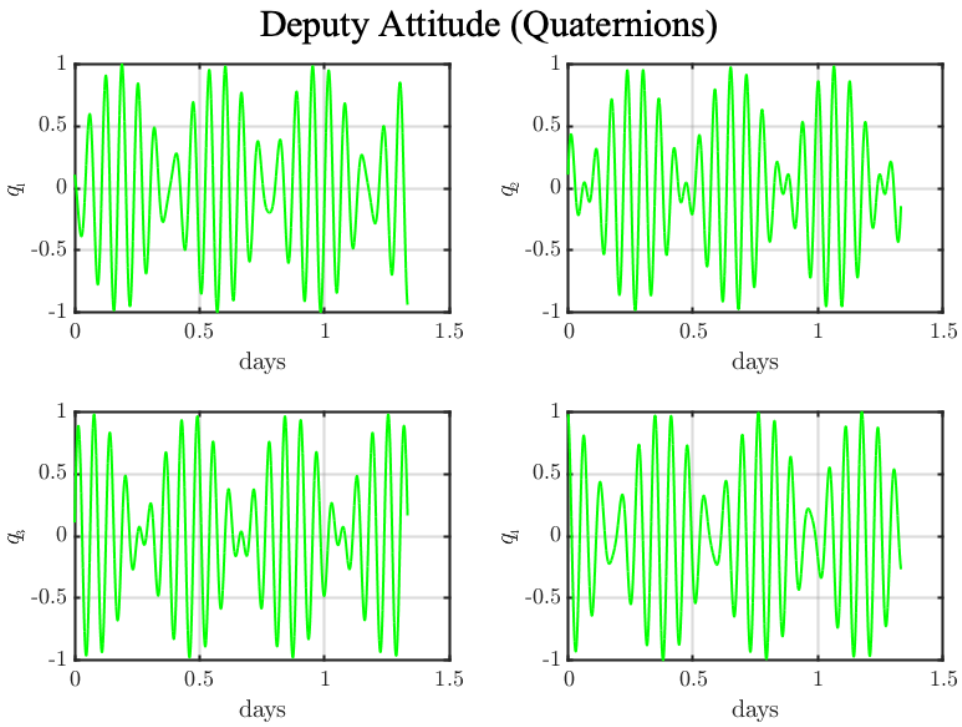


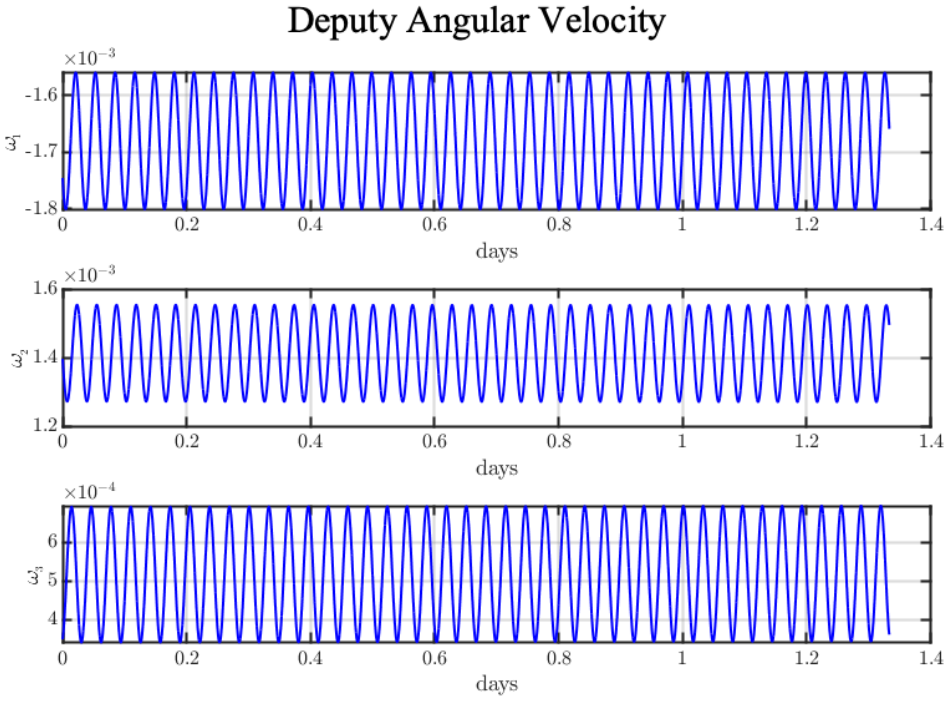
Relative Position Difference



Acceleration and Torque on the Flat-Plate







Published with MATLAB® R2018b

Main Script Part 2

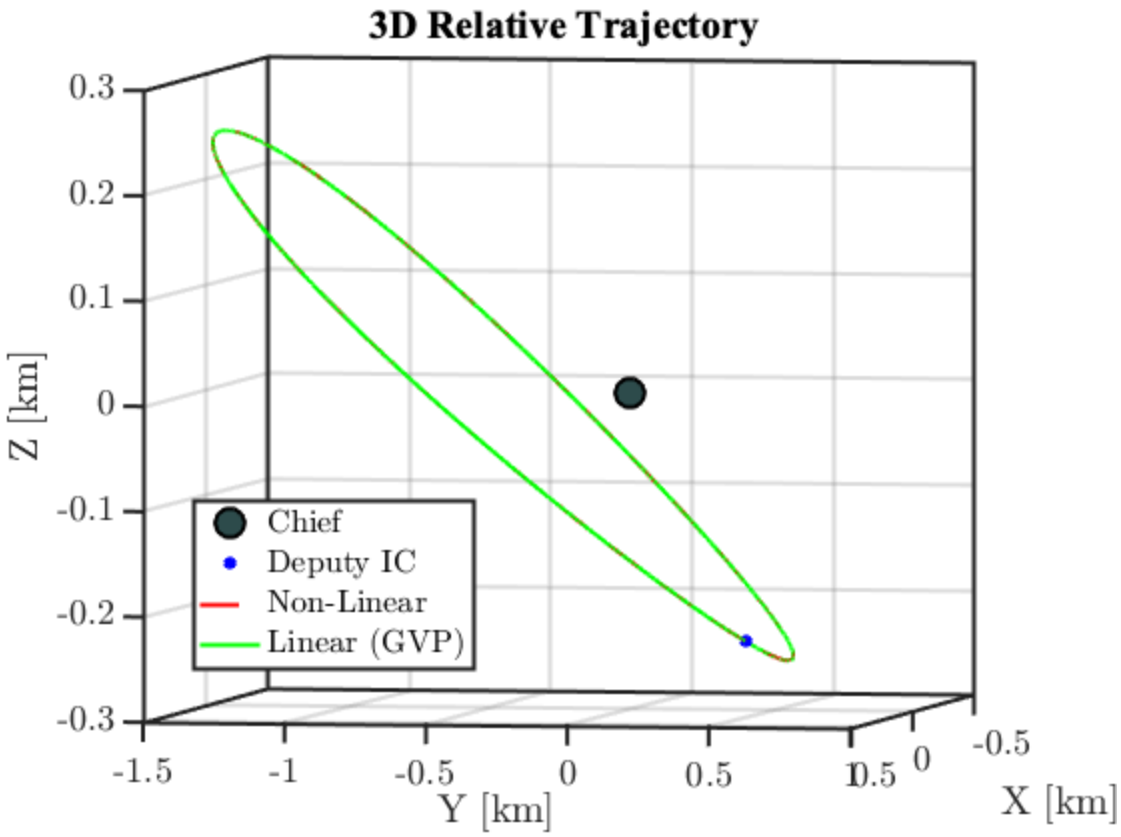
```
clear all
close all
clc
format long e
global mu_Earth ae JD AU Target Chasser
mu_Earth = 3.986004415E5;
ae = 6378.136;
JD = 2456296.25;
AU = 149597870.7;

% Setting the initial conditon
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for k = 1
    % Target initial conditions(Orbital Elements)
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    % Target initial conditions(Orbital Elements)
    a1      = 1.5E+4; % Semi-major axis in Km
    e1      = 0.1;    % Eccentricity
    inc1    = 10;     % Inclination in deg0
    BigOmg1 = 50;     % RAAN in deg
    LitOmg1 = 10;     % AOP in deg
    f1     = -80;       % True anomaly in deg
    Target = Spacecraft([10 6 2 40 10 0.2 0.5 6 12]); % Describing the
 Chief sailcraft characteristic (refer Spacecraft class definition)

    % Relative orbital elements of the deputy
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    dela = 0;                    % Relative semi-major axis in Km
    deli = 1/(4*a1);             % Relative inclination in deg0
    dele = -1/(2*a1);           % Relative eccentricity, in rad
    delLitOmg = 1/(2*a1);        % Relative RAAN in rad
    delBigOmg = 0;               % Relative AOP in rad
    delf = 0;                    % Relative true anomaly in rad


    % Chaser initial conditions(Orbital Elements)
    %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
    a2      = a1;          % Semi-major axis in Km
    e2      = e1 + dele;   % Eccentricity
    inc2    = inc1;        % Inclination in deg
    BigOmg2 = BigOmg1;     % RAAN in deg
    LitOmg2 = LitOmg1;     % AOP in deg
    f2      = f1;          % True anomaly in deg
    Chasser = Spacecraft([30 30 4 116 40 0.9 0.9 3000 6000]); %
 Describing the Deputy sailcraft characteristic (refer Spacecraft
 class definition)

    % Converting angles in radians
    inc1 = deg2rad(inc1); BigOmg1 = deg2rad(BigOmg1); LitOmg1 =
 deg2rad(LitOmg1); f1 = deg2rad(f1);
```

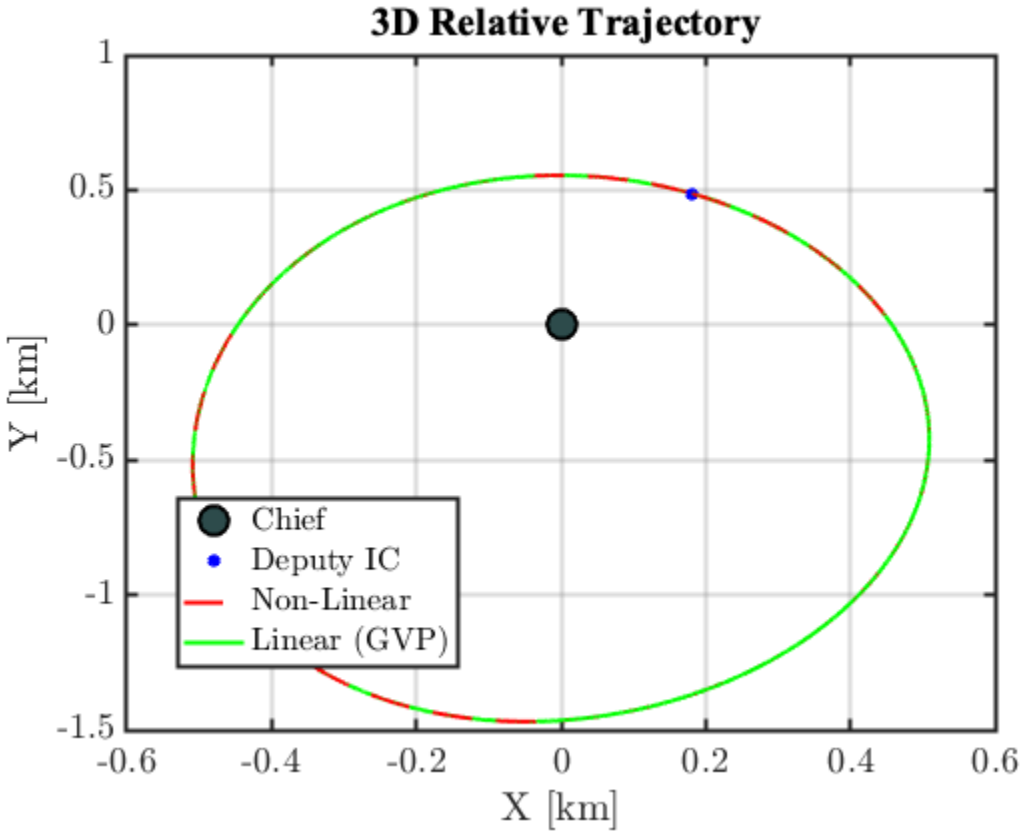
```
    COE1 = [a1,e1,inc1,BigOmg1,LitOmg1,f1];
    [Position_target,Velocity_target]  = COEstoRV(COE1,mu_Earth);

    inc2 = deg2rad(inc2) + deli; BigOmg2 = deg2rad(BigOmg2); LitOmg2 =
 deg2rad(LitOmg2)+delLitOmg; f2 = deg2rad(f2);
    COE2 = [a2,e2,inc2,BigOmg2,LitOmg2,f2];
    [Position_chaser,Velocity_chaser]  = COEstoRV(COE2,mu_Earth);

    % Setting the integrator parameters
    Period = 2*pi*sqrt(a1^3/mu_Earth);
    IntegrationTime = Period;
    tspan  = linspace(0,IntegrationTime,10000);
    options = odeset('RelTol',2.22045e-14,'AbsTol',2.22045e-30);
end

for k=1
    tic
    [~, Chief_OE] =
 ode113(@(t,COE,nu,u,f)GVE(t,COE,Target),tspan,COE1,options);
    toc

    tic
    [~, Deputy_OE] =
 ode113(@(t,COE,nu,u,f)GVE(t,COE,Chasser),tspan,COE2,options);
    toc
    X_rel_linear1 = nan(length(tspan),6);
    R_target2 = nan(length(tspan),3);
    V_target2 = nan(length(tspan),3);

    for j = 1:length(tspan)

        [R_target2(j,:),V_target2(j,:)]  =
 COEstoRV(Chief_OE(j,:),mu_Earth);

       % Computing the relative using the linear maping COE2 =
 [a2,e2,inc2,BigOmg2,LitOmg2,f2];
       % Nonsingular Chief orbital elements
       a_c = Chief_OE(j,1); e_c = Chief_OE(j,2); LitOmg_c =
 Chief_OE(j,5);  f_c = Chief_OE(j,6);
       theta_c = LitOmg_c + f_c;
       inc_c = Chief_OE(j,3);
       q1_c = e_c * cos(LitOmg_c);
       q2_c = e_c * sin(LitOmg_c);
       BigOmg_c = Chief_OE(j,4);

       % Nonsingular Deputy orbital elements
       a_d = Deputy_OE(j,1); e_d = Deputy_OE(j,2); LitOmg_d =
 Deputy_OE(j,5);  f_d = Deputy_OE(j,6);
       theta_d = LitOmg_d + f_d;
       inc_d = Deputy_OE(j,3);
       q1_d = e_d * cos(LitOmg_d);
       q2_d = e_d * sin(LitOmg_d);
       BigOmg_d = Deputy_OE(j,4);
```

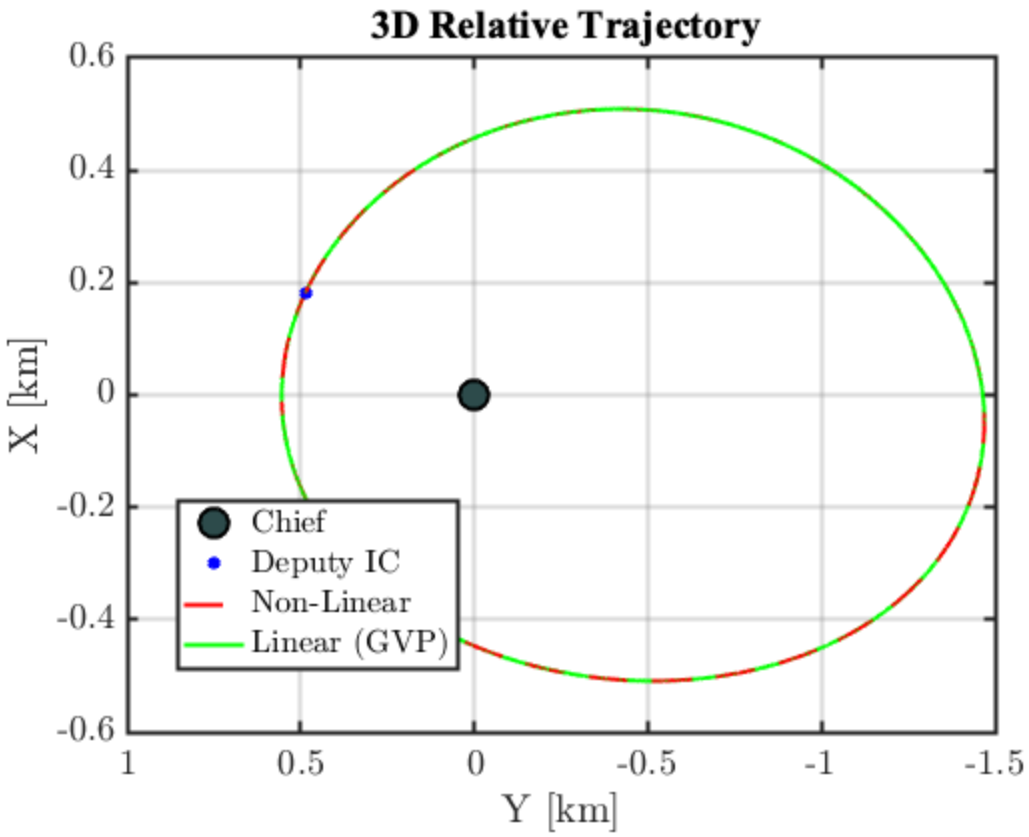
```
        % Relative orbital elements
        del_a = a_d - a_c; del_e = e_d - e_c;  delLitOmg = LitOmg_d -
 LitOmg_c;
        del_theta = theta_d - theta_c;
        del_inc = inc_d - inc_c;
        del_q1 = del_e*cos(LitOmg_c) - e_c*sin(LitOmg_c)*delLitOmg;
        del_q2 = del_e*sin(LitOmg_c) + e_c*cos(LitOmg_c)*delLitOmg;
        del_BigOmg = BigOmg_d - BigOmg_c;

        COE = [a_c, theta_c, inc_c, q1_c, q2_c, BigOmg_c]';
        delCOE = [del_a, del_theta, del_inc, del_q1, del_q2,
 del_BigOmg]';
        [A] = ForwardMapping(COE, mu_Earth);
        Rho_aug = A*delCOE;
        X_rel_linear1(j,:) = Rho_aug';


    end
end

% Integrating the relative motion in the chief's LVLH frame
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for i = 1

    TN = DCM(Position_target,Velocity_target); rt_norm =
 norm(Position_target);
    h_vec = cross(Position_target,Velocity_target); h_norm =
 norm(h_vec);
    eh = h_vec/h_norm;
    U_eci_Target  = 0*F_CanonBall(tspan(1),Position_target,Target); %
 SRP force on the Target
    N_nudot = h_vec/rt_norm^2 + dot(U_eci_Target,eh)*Position_target/
h_norm;
    NR_rel = Position_chaser - Position_target; NV_rel2 =
 Velocity_chaser - Velocity_target;
    TR_rel0 = TN*NR_rel; TV_rel0 = TN*(NV_rel2 -
 cross(N_nudot,NR_rel));
    X_aug0 = [Position_target; Velocity_target; TR_rel0; TV_rel0];
    index = 1;
    tic
    [~, X_rel] = ode113(@(t,Xaug)RelativeMotionODE_cannonball(t, Xaug,
 Target, Chasser),tspan,X_aug0,options);
    toc

end


% Plotting the 3D realtive trajectory
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
for k = 1:3
    % Plotting the 3D trajectories
    c1 = rgb('Crimson'); c2 = rgb('DarkSlateGray');
    figure
    for jj=1
```

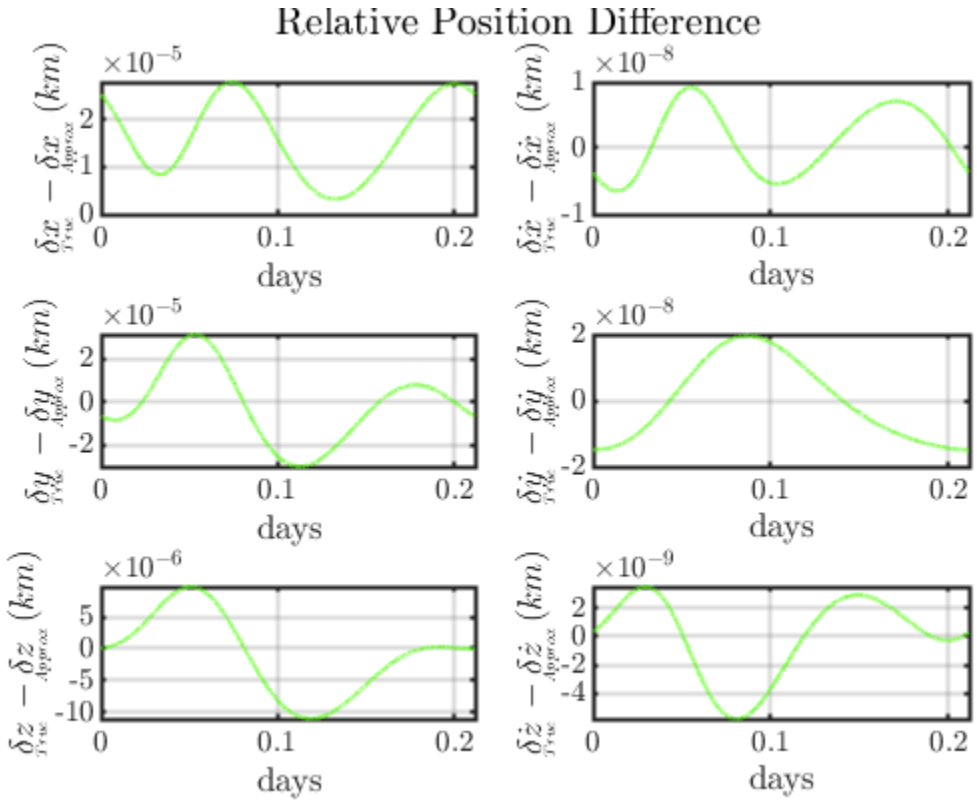
```
        h1 =
 plot3(X_rel_linear1(:,1),X_rel_linear1(:,2),X_rel_linear1(:,3),'g');
        hold on
        h3 = plot3(X_rel(:,7),X_rel(:,8),X_rel(:,9),'r--');

        h4 = plot3(X_rel(1,7),X_rel(1,8),X_rel(1,9),'go',...
            'LineWidth',2,...
            'MarkerEdgeColor','b',...
            'MarkerFaceColor','b',...
            'MarkerSize',5);

        h5 = plot3(0,0,0,'bo',...
            'LineWidth',2,...
            'MarkerEdgeColor','k',...
            'MarkerFaceColor',c2,...
            'MarkerSize',15);
        grid on
        xlabel('X [km]')
        ylabel('Y [km]')
        zlabel('Z [km]')
        title('{\color{black} 3D Relative
 Trajectory}','interpreter','tex')
        h = legend([h5, h4, h3, h1],{'Chief','Deputy IC','Non-
Linear','Linear (GVP)'});
        rect = [0.25, 0.25, 0.1, 0.1];
        set(h, 'Position', rect)
        box on
    end

    if k == 1
        view(100,3)
    elseif k==2
        view(0,90)
    else
        view(-90,90)
    end

end


for jj=1

    DQ1 = [X_rel_linear1(:,1) X_rel_linear1(:,4) X_rel_linear1(:,2)
 X_rel_linear1(:,5) X_rel_linear1(:,3) X_rel_linear1(:,6)]; %
    DQ2 = [X_rel(:,7) X_rel(:,10) X_rel(:,8) X_rel(:,11) X_rel(:,9)
 X_rel(:,12)]; %
    figure
    YLabel={'$\delta x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 x_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{x}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 y_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
```

```
        '$\delta \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{y}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 z_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$',...
        '$\delta \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!True}}} - \delta
 \dot{z}_{_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!Approx}}}(km)$'};
    for i=1:6
        subplot(3,2,i)
        plot1 = plot(tspan/86400,DQ1(:,i)-DQ2(:,i));
        ylabel(YLabel(i))
        xlabel('days')
        grid on

        plot1.Color(4) = 0.6;
        %             ax = gca;
        %             ax.GridColor = [0.9, 0.9, 0.9];  % [R, G, B]
        %             ax.XColor = 'k';
        %             ax.YColor = 'k';
        %             set(gca,'color','k')
        %             set(gcf,'color',[0.3, 0.3, 0.3])
    end
    ha = axes('Position',[0 0 2.2 1],'Xlim',[0 1],'Ylim',[0
 1],'Box','off','Visible','off','Units','normalized', 'clipping'
 , 'off');
    text(0.13, 0.98,sprintf('Relative Position Difference'))
end

Elapsed time is 0.487324 seconds.
Elapsed time is 0.296204 seconds.
Elapsed time is 8.264004 seconds.
```


3D Relative Trajectory
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Z [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)
-1.5
-1
-0.5
0
0.5
1
0.5
0
-0.5
Y [km]
X [km]


3D Relative Trajectory
1
0.5
0
-0.5
-1
-1.5
Y [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X [km]


3D Relative Trajectory
X [km]
Y [km]
Chief
Deputy IC
Non-Linear
Linear (GVP)


Relative Position Difference
days