Final Assignment
DNS course 2015

Compressible 1D flow, Sod shock tube
Compressible 2D flow, Lid driven cavity
Finite Volume Method

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Sod Shock tube

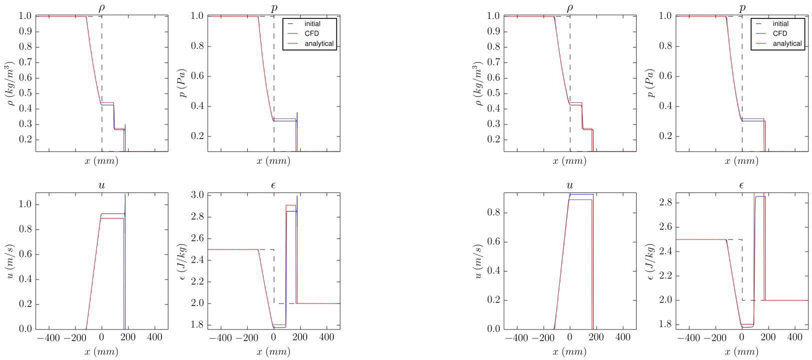


Figure : N=10000, $t = 0.1s$, left $\nu_H = 0.1$,right$\nu_H = 0.6$

Lid driven cavity

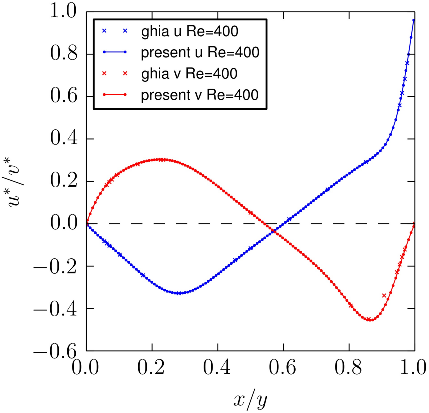
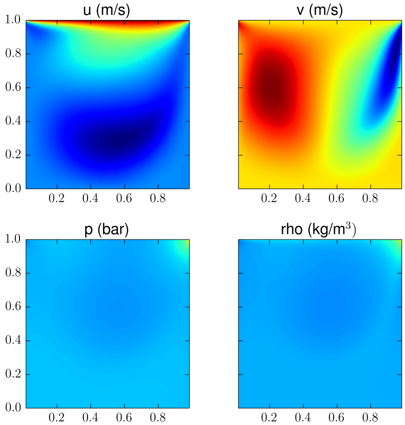


Figure : Test data

Results




```
Co        0.01152
clf diff  0.0036864
clf soni  0.0384

u lid     90.0
Re        400
nu        0.225
mu        0.271125

max u     0.963856162576
min u     -0.337051956293
max v     0.320720990696
min v     -0.637082135204
max rho   1.40450644535
min rho   1.12269824544
max p     116642.464675
min p     92493.3107295
```

Figure : Overview

Algorithm

- Initialize with smoothed fields
- Integrate time steps (Euler or Runge-Kutta 4)
  1. Solve continuity equation $\rightarrow d\rho$
  2. Solve momentum equation $\rightarrow d\rho U$
  3. Solve energy equation $\rightarrow de$
  4. Update primitives $\rightarrow du, dp$
  5. Filter ρ, e, p after every 100th time step with Laplace filter and $0.07\left(\frac{\Delta x}{\pi}\right)^2$ as the cutter
- Post-process

Governing equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \overline{u}) \\
\frac{\partial (\rho \overline{u})}{\partial t} &= -\nabla \cdot (\rho \overline{u}\overline{u}) - \frac{\partial p}{\partial x} + \mu \Delta \overline{u} \\
\frac{\partial e}{\partial t} &= -\nabla \cdot (\overline{u}(\rho e + p)) \\
e &= \frac{p}{(\gamma - 1)} + \tfrac{1}{2}\rho |\overline{u}|^2
\end{aligned}
\tag{1}
$$

Python implementation snippet, N-S

```
drho  = self.dt * ( - self.dx * rhoU
                    - self.dy * rhoV
                    )

drhoU = self.dt * ( - self.dx * (rhoU * u)
                    - self.dy * (rhoU * v)
                    - self.dxZeroGrad * p
                    + self.mu * self.laplacian * u
                    + self.bRhoU # Source term
                    )

drhoV = self.dt * ( - self.dx * (rhoV * u)
                    - self.dy * (rhoV * v)
                    - self.dyZeroGrad * p
                    + self.mu * self.laplacian * v
                    )

de    = self.dt * ( - self.dx * ( u * (e + p))
                    - self.dy * ( v * (e + p))
                  )
```

Figure : Governing equation in Python

Discretization, continuity

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \overline{u}) \\
&= -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} \\
\Rightarrow \left(\frac{\partial \rho}{\partial t}\right)_{i,j} &\approx -\frac{1}{2\Delta x}\Big(\underbrace{[(\rho u)_{i+1} + (\rho u)_i}_{right} \underbrace{-[(\rho u)_{i-1} + (\rho u)_i]}_{left} \\
&\quad + \underbrace{[(\rho u)_{j+1} + (\rho u)_j}_{up} \underbrace{-[(\rho u)_{j-1} + (\rho u)_j]}_{down} \Big)
\end{aligned} \tag{2}
$$

At boundaries the corresponding term is set to zero. Operators dx and dy, no sources

Python implementation snippet, $\frac{\partial f}{\partial x}$

```
C = (2 * self.h)**-1
for row in range(self.nx):
    for col in range(self.nx):
        matrix_index = row*self.nx + col

        # left
        if col > 0:
            # middle
            xrows.append(matrix_index)
            xcols.append(matrix_index)
            xvals.append(-C)
            # left
            xrows.append(matrix_index)
            xcols.append(matrix_index-1)
            xvals.append(-C)
        # right
        if col < self.nx -1 :
            # middle
            xrows.append(matrix_index)
            xcols.append(matrix_index)
            xvals.append(C)
            # right
            xrows.append(matrix_index)
            xcols.append(matrix_index+1)
            xvals.append(C)

# No boundary effects
self.dx = sparse.csr_matrix(sparse.coo_matrix((xvals, (xrows, xcols))))
```

Figure : $\frac{\partial f}{\partial x}$ operator