1 Compactness

1. $\mathcal{T}$ is satisfiable iff every finite subset of it is satisfiable
2. $\mathcal{T} \models \varphi$ iff there is a finite $\mathcal{T}' \subseteq \mathcal{T}$ such that $\mathcal{T} \models \varphi$

Proof:

(2) $\Rightarrow$ (1): Fact: $\mathcal{T}$ is not satisfiable iff $\mathcal{T} \models (P \wedge (\neg P))$.
For a sentence symbol P, as a truth assignment cannot satisfy $(P \wedge (\neg P))$.
By (2), $\mathcal{T} \models (P \wedge (\neg P))$ iff there is finite $\mathcal{T}' \models (P \wedge (\neg P))$.
So $\mathcal{T}$ is not satisfiable iff there is a finite $\mathcal{T}'$ which is not satisfiable.

(1) $\Rightarrow$ (2): $\mathcal{T} \models \varphi$ iff $\mathcal{T} \cup \{(\neg\varphi)\}$ is not satisfiable.
Suppose (1) holds $\mathcal{T} \models \varphi$ iff $\mathcal{T} \cup \{(\neg\varphi)\}$.
By (1), $\mathcal{T} \cup \{(\neg\varphi)\}$ is not satisfiable iff $\mathcal{T}' \cup \{(\neg\varphi)\}$ is not satisfiable for a finite $\mathcal{T}' \subseteq \mathcal{T}$.
By the fact, this is the case iff $\mathcal{T}' \models \varphi$.

Why Compactness? The set of all truth assignment can be equiped with the product/Tychonoff/Positive-Convergence topology.
Compactness $\Leftrightarrow$ This space is compact.

2 Relevance Lemma

If Σ, Σ' are two assignments that agree on the sentence symbols arising in φ, then Σ satisfies φ iff Σ' satisfies φ.

Definition 2.0.1

- $\zeta(\varphi) = \{\varphi\}$
- $\zeta((\neg\varphi)) = \zeta(\varphi)$ *for any wff* φ
- $\zeta((\varphi \circ \phi)) = \zeta(\varphi) \cup \zeta(\psi)$ *for any wff's* φ, ψ *and any binary connective* $\circ$.