- Since the ODE is $y' = -5y$, in order to make the numerical solution stable, we need

  $$|1 + \lambda h| \leq 1 \Rightarrow |1 - 5h| \leq 1 \Rightarrow h \leq 2/5$$

  for Euler forward method,

  $$\left|\frac{1}{1 - h\lambda}\right| \leq 1 \Rightarrow \left|\frac{1}{1 + 5h}\right| \leq 1 \Rightarrow h > 0$$

  for Euler backward method, and

  $$\left|\frac{1 + h\lambda/2}{1 + h\lambda/2}\right| < 1 \Rightarrow h > 0$$

  for Trapezoid Method.

  As a result, $h = 0.5 > 0.4 = 2/5$ will make the solutions unstable.

- From the equations above, we know that the step is not stable for this ODE using this step size at $t = 0.5$ given by Euler's method.

- Since $y' = f(t, y) = -5y$ and $y_0 = 1$, so we have

  $$y_{0.5} = (1 + 0.5 \cdot (-5))y_0 = -1.5$$

- From the equations above, we know that the step is stable for this ODE using this step size at $t = 0.5$ given by Euler's backward method.

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- Since $y' = f(t, y) = -5y$ and $y_0 = 1$, so we have

  $$y_{0.5} = \left(\frac{1}{1 - 0.5 \cdot (-5)}\right) y_0 = 0.28$$