Homework 3

Due Friday Feb 9

Suppose that $\{P_1, \ldots, P_n\}$ is the set of sentence symbols. Recall that we identify the set of truth assignments $\Sigma : \{P_1, \ldots, P_n\} \to \{T, F\}$ with $\{T, F\}^n$ by identifying the truth assignment Σ with $(\Sigma(P_1), \ldots, \Sigma(P_n))$ and identifying $(x_1, \ldots, x_n) \in \{T, F\}^n$ with the truth assignment Σ given by

$$\Sigma(P_1) = x_1, \Sigma(P_2) = x_2, \ldots, \Sigma(P_n) = x_n.$$

We now associate a function $f_\varphi : \{T, F\}^n \to \{T, F\}$ to each wff φ. Given, $(x_{,1}, \ldots, x_n) \in \{T, F\}$ we let $f_\varphi(x_1, \ldots, x_n) = T$ if and only if the truth assigment associated to $(x_1, \ldots, x_n)$ satisfies φ. If the truth assignment associated to $(x_1, \ldots, x_n)$ does not satisfy φ then $f_\varphi(x_1, \ldots, x_n) = F$.

Problem 1: Show that for every function $g : \{T, F\}^n \to \{T, F\}$ there is a wff ϕ such that $g = f_\varphi$.

Problem 2: Suppose φ, ψ are wffs. Show that $f_\varphi = f_\psi$ if and only if $(\varphi \leftrightarrow \psi)$ is tautology.

Problem 3: We say that wffs φ, ψ are logically equivalent if $(\varphi \leftrightarrow \psi)$ is a tautology. Show that logical equivalence is an equivalence relation with 2^{2^n} classes.