6.38 Solution:

(a)

$$P\{X = i, Y = j\} = \frac{1}{5i}$$

(b)

$$P\{X = j, Y = i\} = \frac{\sum_{k=i}^{5} 1/5k}{5j} = \frac{\sum_{k=i}^{5} 1/k}{j}$$

(c) No, $P\{Y\}$ dependent on $P\{X\}$.

6.40 Solution:

(a)

$$P\{X = 1\} = p(1,1) + p(1,2) = 3/8$$
$$P\{X = 2\} = p(2,1) + p(2,2) = 5/8$$

(b)

$$P\{X = 1|Y = 1\} = 1/3 \neq 1/4 = P\{Y = 1\}$$

X and Y are not independent.

(c)

$$P\{XY \leq 3\} = 1 - 1/2 = 1/2$$
$$P\{X + Y > 2\} = 1$$
$$P\{X/Y > 1\} = 1/8$$

6.41 Solution:

(a)

$$P\{X = x|Y = y\} = \frac{P\{X = x, Y = y\}}{P\{Y = y\}} \frac{xe^{-x(y+1)}}{\int_0^\infty xe^{-x(y+1)}dx} = (y+1)^2 xe^{-x(y+1)}$$

$$P\{Y = y|X = x\} = \frac{xe^{-x(y+1)}}{\int_0^\infty xe^{-x(y+1)}dy} = xe^{-xy}$$

(b)

$$P\{Z = XY < a\} = \int_0^{\infty} \int_0^{a/x} xe^{-x(y+1)} dydx = 1 - e^{-a}$$

6.42 Solution:

$$P\{Y = y|X = x\} = \frac{c(x^2 - y^2)e^{-x}}{\int_{-x}^{x} c(x^2 - y^2)e^{-x} dy} = \frac{3}{4x^3}(x^2 - y^2), -y < x < x$$

6.48 Solution:

(a)

$$P\{\min(X_1, \cdots, X_5) \leq a\} = 1 - \prod_{i=0}^{5} P\{X_i > a\} = 1 - e^{-5\lambda a}$$

(b)

$$P\{\max(X_1, \cdots, X_5) \leq a\} = \prod_{i=0}^{5} P\{X_i > a\} = (1 - e^{-\lambda a})^5$$