5.6 Solution:

(a)

$$E[X] = \int_{-\infty}^{+\infty} xf(x)dx = \int_{0}^{+\infty} \frac{1}{4}x^2 e^{-x/2}dx = 4$$

(b)

$$1 = \int_{-\infty}^{+\infty} f(x)dx \Rightarrow \int_{-1}^{1} c(1-x^2)dx = 1 \Rightarrow c = \frac{1}{2}$$

So

$$E[X] = \int_{-\infty}^{+\infty} xf(x)dx = \int_{-1}^{1} (x - x^3)dx = 0$$

(c)

$$E[X] = \int_{-\infty}^{+\infty} xf(x)dx = \int_{5}^{+\infty} \frac{5}{x}dx = \infty$$

5.10 Solution:

(a)

$$p(A) = 4 \cdot 10/60 = 2/3$$

(b)

$$p(A) = 4 \cdot 10/60 = 2/3$$

5.12 Solution: In this problem, what we want is to minimize the expected value of the distance a bus has to take to a service station when breakdown happens.

In the original case, we have

$$d(x) = \begin{cases} x & \text{if } 0 \le x \le 25 \\ 50 - x & \text{if } 25 \le x \le 50 \\ x - 50 & \text{if } 50 \le x \le 75 \\ 100 - x & \text{if } 75 \le x \le 100 \end{cases}$$

Thus,

$$\begin{aligned}E[d(x)] &= \int_{-\infty}^{+\infty} d(x)f(x)dx \\ &= \frac{1}{100}(\int_0^{25} xdx + \int_{25}^{50}(50-x)dx \\ &+ \int_{50}^{75}(x-50)dx + \int_{75}^{100}(100-x)dx) \\ &= 12.5\end{aligned}$$

In another occasion,

$$d(x) = \begin{cases} 25-x & \text{if } 0 \leq x \leq 25 \\ x-25 & \text{if } 25 \leq x \leq 37.5 \\ 50-x & \text{if } 37.5 \leq x \leq 50 \\ x-50 & \text{if } 50 \leq x \leq 62.5 \\ 75-x & \text{if } 62.5 \leq x \leq 75 \\ x-75 & \text{if } 75 \leq x \leq 100 \end{cases}$$

So,

$$\begin{aligned}E[d(x)] &= \int_{-\infty}^{+\infty} d(x)f(x)dx \\ &= \frac{1}{100}(\int_0^{25}(25-x)dx + \int_{25}^{37.5}(x-25)dx + \int_{37.5}^{50}(50-x)dx \\ &+ \int_{50}^{62.5}(x-50)dx + \int_{62.5}^{75}(75-x)dx + \int_{75}^{100}(x-75)dx) \\ &= 9.375\end{aligned}$$

So in conclusion, it is beneficial to take the suggestion.

5.13 Solution:

(a)

$$P(>10) = 20/30 = 2/3$$

(b)

$$P(+10) = (15-10)/15 = 1/3$$

5.15 Solution: $\mu = 10, \sigma^2 = 36 \Rightarrow$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} = \frac{1}{6\sqrt{2\pi}} e^{-(x-10)^2/72}, -\infty < x < \infty$$

(a)

$$P\{X > 5\} = \int_5^\infty f(x)dx = 0.798$$

(b)

$$P\{4 < X < 16\} = \int_4^{16} f(x)dx = 0.683$$

(c)

$$P\{X < 8\} = \int_{-\infty}^{8} f(x)dx = 0.369$$

(d)

$$P\{X < 20\} = \int_5^\infty f(x)dx = 0.952$$

(e)

$$P\{X > 16\} = \int_5^\infty f(x)dx = 0.159$$

5.18 Solution: $\mu = 5, P\{X > 9\} = 0.2 \Rightarrow$

$$P\{Z > 4/\sigma\} = 0.9 \Rightarrow \sigma = 4.76 \Rightarrow \text{Var}(X) = (4.76)^2 = 22.66$$

5.21 Solution: $\mu = 71, \sigma^2 = 6.25 \Rightarrow$

$$f(x) = \frac{2}{5\sqrt{2\pi}} e^{-(x-71)^2/2(6.25)^2} \Rightarrow$$

$$P\{X > 6'2''\} = \int_{74}^{\infty} \frac{2}{5\sqrt{2\pi}} e^{-(x-71)^2/2(6.25)^2} dx = 0.789$$

$$P\{X > 6'5''\} = \int_{77}^{84} \frac{2}{5\sqrt{2\pi}} e^{-(x-71)^2/2(6.25)^2} dx = 0.742$$

5.22 Solution:

$$\begin{aligned}P\{X>100\} &= 1-P\{X\le 100\}\\ &= 1-\sum_{i=0}^{50}\binom{50}{i}\binom{50}{50-i}0.4^{100-i}(0.6)^i\\ &= 0.973\end{aligned}$$

5.23 Solution:

$$P\{150\le X\le 200\} = \sum_{i=150}^{200}\binom{1000}{i}(1/6)^i(5/6)^{1000-i} = 0.9258$$

$$P\{X<150\} = P\{Z<-0.93\} = 0.1762$$

5.25 Solution:

$$P\{X\le 10\} = P\{Z\le 1.1239\} = 0.8695$$

5.28 Solution:

$$P\{X>19\} = P\{Z>-0.9792\} = 0.8363$$

5.32 Solution:

(a)

$$P\{X>2\} = 1-\int_0^2 \frac{1}{2}e^{-\frac{1}{2}x}dx = e^{-1}$$

(b)

$$P\{X>10|X>9\} = \frac{1-\int_0^{10}\frac{1}{2}e^{-\frac{1}{2}x}dx}{1-\int_0^{9}\frac{1}{2}e^{-\frac{1}{2}x}dx} = e^{-1/2}$$

5.33 Solution:

$$P\{X>8\} = 1-\int_0^8 \frac{1}{8}e^{-\frac{1}{8}x}dx = e^{-1}$$

5.34 Solution:

(a)

$$P(X \geq 30000|X > 10000) = P(X \geq 20000) = \int_{20}^{\infty} e^{-\frac{1}{20}x}dx = e^{-1}$$

(b)

$$P(X > 30|X > 10) = \frac{\int_{30}^{40} \frac{1}{40}xdx}{\int_{10}^{40} \frac{1}{40}xdx} = 1/3$$