

Consider the implicit trapezoid method

$$y_{k+1} = y_k + h_k(\lambda y_k + \lambda y_{k+1})/2$$

and apply it to

$$y' = \lambda y$$

we get

$$y_k = \left( \frac{1 + h\lambda/2}{1 - h\lambda/2} \right)^k y_0$$

We notice that the growing factor

$$\begin{aligned} \frac{1 + h\lambda/2}{1 - h\lambda/2} &= \left( 1 + \frac{\lambda h}{2} \right) \left( 1 + \frac{\lambda h}{2} + \left( \frac{\lambda h}{2} \right)^2 + \left( \frac{\lambda h}{2} \right)^3 + \cdots \right) \\ &= 1 + h\lambda + (h\lambda)^2/2 + (h\lambda)^3/4 + \cdots \end{aligned}$$

which is consistent to  $e^{h\lambda}$  up until  $h^2$  term, so the method has 2nd order accuracy.