

1.7 Solution: If $x = \frac{2}{3}, y = \frac{1}{2}$, we have $x > y$, but $(-1/x) < (-1/y)$.

Hypothesis: $\forall x, y \in \mathbb{R}$, if $|x|, |y| \geq 1$ and $x > y$, then $(-1/x) > (-1/y)$.

1.13 Proof: $\forall x \in A, x = 2k - 1 = 2(k - 1) + 1 \in B \Rightarrow A \subseteq B, \forall x \in B, x = 2k + 1 = 2(k + 1) - 1 \in A \Rightarrow B \subseteq A$.

As a result, $A = B$.

1.32 Proof: $\forall x \in \{x \in \mathbb{R} | x^2 - 2x - 3 < 0\}, x^2 - 2x - 3 < 0 \Rightarrow (x - 3)(x + 1) < 0 \Rightarrow -1 < x < 3 \Rightarrow \{x \in \mathbb{R} | -1 < x < 3\}$, so $\{x \in \mathbb{R} | x^2 - 2x - 3 < 0\} \subseteq \{x \in \mathbb{R} | -1 < x < 3\}$.

Similarly, $\forall x \in \{x \in \mathbb{R} | -1 < x < 3\}, -1 < x < 3 \Rightarrow -4 < x - 3 < 0$ and $0 < x + 1 < 4 \Rightarrow (x - 3)(x + 1) > 0, x \in \{x \in \mathbb{R} | x^2 - 2x - 3 < 0\}$, so $\{x \in \mathbb{R} | x^2 - 2x - 3 < 0\} \supseteq \{x \in \mathbb{R} | -1 < x < 3\}$.

Thus, $\{x \in \mathbb{R} | x^2 - 2x - 3 < 0\} = \{x \in \mathbb{R} | -1 < x < 3\}$.

1.36 Proof: $\forall x, y \in S, 1 \leq x \leq 3, 1 \leq y \leq 3 \Rightarrow 0 \leq 3x + y - 4 \leq 8 \Rightarrow x \in T$, so $S \subseteq T$.

$\forall x, y \in T, 0 \leq 3x + y - 4 \leq 8 \Rightarrow 4 \leq 3x + y \leq 12, x, y \in \mathbb{Z}, (0, 4) \in T$ but $\notin S$, so $S \neq T$.

1.47 Solution:

(a) Since $a, b \in \mathbb{N}$, if a is odd, $(a + 1)(a + 2b) = (2k + 1 + 1)(a + 2b) = 2(k + 1)(a + 2b)$ is even and $f \in \mathbb{N}$. If a is even, $a + 2b = 2k + 2b = 2(k + b)$ is even and $f \in \mathbb{N}$.

Thus, $\forall a, b \in \mathbb{N}, f(a, b) \in \mathbb{N}$.

(b) If $a = 1, f(a, b) = 2b + 1, b \in \mathbb{N}$. If $a = 2, (a + 2b)/2 = b + 1, b \in \mathbb{N} \Rightarrow (a + 2b)/2 \geq 2 \Rightarrow f(a, b) \geq 6$ and f is even. Similarly, if $a = 3, f(a, b) = 2(3 + 2b) = 4(b + 1) + 2 \geq 10$. We can conclude that the image of f is $\mathbb{N} - \{1, 2, 4\}$.

1.50

(a) Proof: Given that $C, D \subseteq \text{domain}, C \cap D \subseteq \text{domain}, \forall x \in C \cap D, x \in C$ or $x \in D$ is true. When $x \in C, f(x) \subseteq C$ and when $x \in D, f(x) \subseteq D$, so $f(x) \subseteq C \cap D$.

Thus, $f(C \cap D) \subseteq f(C) \cap f(D)$.

(b) Solution: If $C \cap D = \emptyset$, but $f(C) \cap f(D) \neq \emptyset$, the equality does not hold. For example, if $C = \{2\}$, $D = \{-2\}$, $f(x) = x^2$, $f(C \cap D) = \emptyset$ but $f(C) \cap f(D) = \{4\}$.