1. The characteristic polynomial of A is

$$\begin{aligned} & P(\lambda) = det(A - \lambda I) \\ \Leftrightarrow\ & P(\lambda) = (1-\lambda)^2 - 4 \\ \Leftrightarrow\ & P(\lambda) = \lambda^2 - 2\lambda - 3 \end{aligned} \tag{1}$$

2. By solving characteristic polynomial (1), we get

$$\lambda^2 - 2\lambda - 3 = (\lambda - 3)(\lambda + 1) = 0$$

so

$$\lambda_1 = 3, \lambda_2 = -1$$

3. As mentioned above, the eigenvalues is

$$\lambda_1 = 3, \lambda_2 = -1$$

4. For $\lambda_1 = 3$,

$$\begin{aligned} & \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} x = 3x \\ \Leftrightarrow\ & \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 3x_2 \end{bmatrix} \\ \Leftrightarrow\ & \begin{bmatrix} x_1 + 4x_2 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 3x_2 \end{bmatrix} \\ \Leftrightarrow\ & \begin{cases} x_1 + 4x_2 = 3x_1 \\ x_1 + x_2 = 3x_2 \end{cases} \\ \Leftrightarrow\ & \begin{cases} 4x_2 = 2x_1 \\ x_1 = 2x_2 \end{cases} \\ \Leftrightarrow\ & \mathbf{x}_1 = \begin{bmatrix} 2c \\ c \end{bmatrix} (c \in \mathbb{R}) \end{aligned} \tag{2}$$

For $\lambda_2 = -1$,

$$\begin{aligned}
&\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} x = -x \\
\Leftrightarrow &\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix} \\
\Leftrightarrow &\begin{bmatrix} x_1 + 4x_2 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix} \\
\Leftrightarrow &\begin{cases} 2x_1 = -4x_2 \\ x_1 = -2x_2 \end{cases} \\
\Leftrightarrow &\mathbf{x}_2 = \begin{bmatrix} -2c \\ c \end{bmatrix} (c \in \mathbb{R})
\end{aligned} \tag{3}$$

5. By applying power iteration once, we get

$$x' = Ax/||Ax|| = [5/\sqrt{29}, 2/\sqrt{29}]^T$$

6. By the result, of question 4 and 5, we see that eigenvector will converge to $[2/\sqrt{5}, 1/\sqrt{5}]^T$.

7. By using Rayleigh quotient, we get that

$$\lambda = \frac{x^T A x}{x^T x} = 3.5$$

8. The inverse iteration will converge to the least eigenvalue in magnitude, which is -1.

9. Since power iteration converges to the closest eigenvalue to the shift, it will converges to $\lambda = 3$ when $\sigma = 2$.

10. Since A is not symmetric, so it will converges to triangular matrix after QR iteration.