

Let L be the set of all strings in $\{0, 1\}^*$ that contain at most two occurrences of the substring **100**.

1. Describe a DFA that over the alphabet $\Sigma = \{0, 1\}$ that accepts the language L . Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA *means*.

You may either draw the DFA or describe it formally, but the states Q , the start state s , the accepting states A , and the transition function δ must be clearly specified.

2. Give a regular expression for L , and briefly argue why the expression is correct.

Solution: 1. The DFA $M = \{Q, \Sigma, \delta, s, A\}$ accepts L should look like the automata below, with state $q \in Q$ in form of (i, j) that i denotes the number of received **100** and j denotes the constructing **100**.

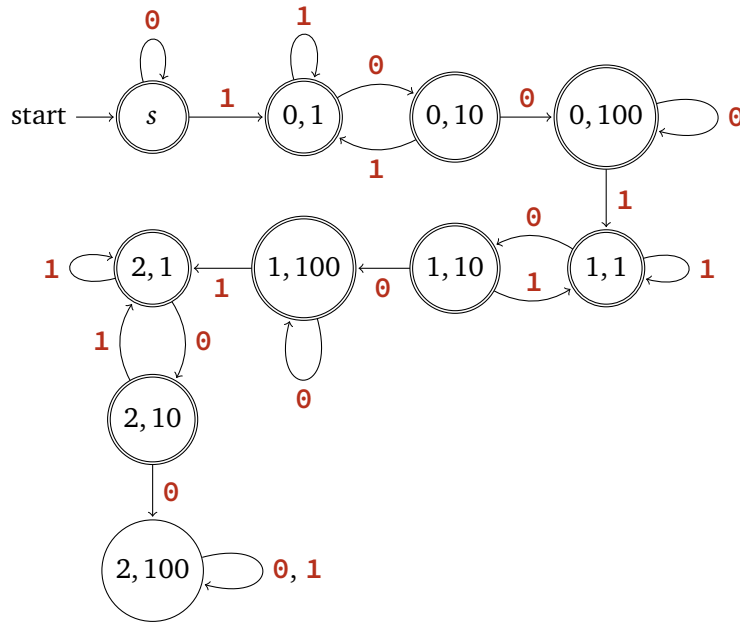


Figure 1. DFA that accepts L

2. $(0^*(\varepsilon + 1)(\varepsilon + 0))^*(100)(0^*(\varepsilon + 1)(\varepsilon + 0))^*(100)(0^*(\varepsilon + 1)(\varepsilon + 0))^*$

Reason: Other than two **100**s, the other parts, if have **1**, should have no more than 2 consecutive **0**s.

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