CS/ECE 374 Spring 2017	Renheng Ruan (rruan2)
Homework 2 Problem 3	Lanxiao Bai (lbai5)

(a) Describe a concrete example of a machine N to show that $L(N_{\text{comp}}) \neq \overline{L(N)}$. You need to explain for your machine N what $\overline{L(N)}$ and $L(N_{\text{comp}})$ are.

(b) Define an NFA that accepts $\overline{L(N)} - L(N_{\text{comp}})$, and explain how it works.

(c) Define an NFA that accepts $L(N_{\text{comp}}) - \overline{L(N)}$, and explain how it works.

Solution:

(a)

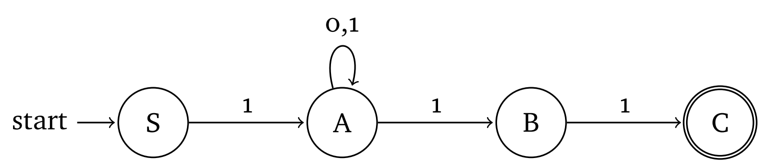

Figure 1.

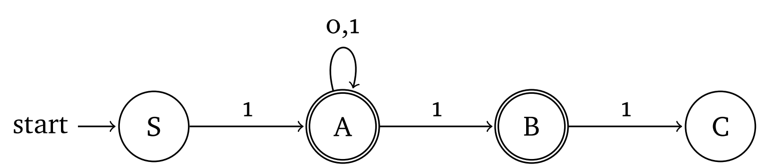

Figure 2.

Let $L(N) = L(1(0+1)^*11)$, as in figure 1, then $\overline{L(N)}$ does not end with 11. By swapping the accept and non-accept states which is shows as figure 2, $L(N_{\text{comp}})$ accepts 011, however, $\overline{L(N)}$ does not end with 11, so that $L(N_{\text{comp}}) \neq \overline{L(N)}$.■

Lemma 1. *Let $A, B \subseteq Q$ and $\delta(S, w_1) = \{A, B\}, \delta(S, w_2) = \{A\}, \delta(S, w_3) = \{B\}, \delta(S, w_4) = \emptyset$. If N accepts both w_1 and w_2, then N' that accepts $\overline{L(N)}$ accepts both w_3 and w_4, and N_{comp} accepts both w_1 and w_3. Denote $L_1 = \{w : \delta(S, w) = \{A, B\}\}, L_2 = \{w : \delta(S, w) = \{A\}\}, L_3 = \{w : \delta(S, w) = \{B\}\}, L_4 = \{w : \delta(S, w) = \emptyset\}$.*

Proof: This lemma is true by the definition of NFA.■

(b) By Lemma 1,

$$\overline{L(N)} - L(N_{\text{comp}}) = L_3 + L_4 - (L_1 + L_3) = L_4 = \emptyset$$

(c) By Lemma 1,

$$L(N_{\text{comp}}) - \overline{L(N)} = L_1 + L_3 - (L_3 + L_4) = L_1 = A + B$$

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