1 Theory

Let Σ be a truth assignment, let $\Delta = \{\text{wff's that } \Sigma \text{ satisfies}\}$.

Δ is trivially finitely satisfiable.

Δ is maximal: $\varphi \in \Delta$ or $(\neg\varphi) \in \Delta$ for any wff.

Δ is the theory of Σ.

The truth assignment

$$\Sigma(P) = \begin{cases} T & \text{if } P \in \Delta \\ T & \text{otherwise} \end{cases}$$

satisfies Δ, Δ is the theory of Σ.

2 First order logic

Let M be a set, a k-nary on M is a subset R of M^k.

We often write $R(x_1, x_2, \cdots, x_k)$ for $(x_1, x_2, \cdots, x_k) \in R$.

If R is a binary relation, we write xRy when $R(x, y)$.

A k-nary function on M is a function $f : M^k \to M$.

2.1 First order language

A first order language L is a set of formal symbols consisting of:

- Logical symbols:
  - $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \forall, \exists$
  - Parathesis: $($, $)$
  - Equality: $=$
- Variables: $x, y, z, \cdots$
- k-ary relation symbols: $R, S, \cdots$
- k-ary function symbols: $f, g, h, \cdots$
- Constant symbols: c, c'

First order language can be uncountable, but we can usually take L to be countable.

An L-structure $\mathcal{M}$ is a nonempty M together with

- a k-ary relation $R^{\mathcal{M}}$ on M for every k-ary relation symbol

- a k-ary function $f^{\mathcal{M}}$ on M for every k-ary function symbol
- an element $c^{\mathcal{M}}$ for each constant symbol c

$\mathcal{M}$ is the structure.
M is the underlying set (domain) of $\mathcal{M}$.
$\mathcal{M}$ is a symmetric L-structure if $xR^{\mathcal{M}y}$ iff $yR^{\mathcal{M}x}$ for all $x, y \in M$.