1. Since $\partial f/\partial x = 2x - 4y$ and $\partial f/\partial y = 2y - 4x$, let both of them equal to 0 then we get $(x, y) = (0, 0)$ as the critical points, then we test the second derivatives.

   Since
   $$H(x, y) = \begin{bmatrix} 2 & -4 \\ -4 & 2 \end{bmatrix}$$
   so that
   $$D(x, y) = det(H) = 4 - 16 = -12 < 0$$
   As a result, $(0, 0)$ is a saddle point.

2. Since $\partial f/\partial x = 4x^3 - 4y$ and $\partial f/\partial y = 4y^3 - 4x$, let both of them equal to 0 then we get $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (-1, -1)$ as the critical points, then we test the second derivatives.

   Since
   $$H(x, y) = \begin{bmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{bmatrix}$$
   so that
   $$D(x, y) = det(H) = 144x^2y^2 - 16$$
   So we have $D(0, 0) < 0$, so $(0, 0)$ is a saddle point; we have $D(1, 1) > 0$ and $f_{xx}(1, 1) > 0$, so $(1, 1)$ is a local minimum; we have $D(-1, -1) > 0$ and $f_{xx}(-1, -1) > 0$, so $(-1, -1)$ is a local minimum.

3. Since $\partial f/\partial x = 6x^2 - 6x - 6y(x - y - 1) - 6xy$ and $\partial f/\partial y = -6x(x - y - 1) + 6xy$, let both of them equal to 0 then we get $(x_1, y_1) = (-1, -1)$, $(x_2, y_2) = (0, -1)$, $(x_3, y_3) = (0, 0)$, $(x_4, y_4) = (1, 0)$ as the critical points, then we test the second derivatives.

   Since
   $$H(x, y) = \begin{bmatrix} 12x - 12y - 6 & -6(x - y - 1) - 6x + 6y \\ -6(x - y - 1) - 6x + 6y & 12x \end{bmatrix}$$
   so that
   $$D(x, y) = det(H) = 36(-1 + 2x - 2y)(1 + 2y)$$
   o we have $D(-1, 1) = 36 > 0$ and $f_{xx}(-1, -1) = -6 < 0$, so $(-1, -1)$ is a local maximum point; we have $D(0, -1) = -36 < 0$ so $(0, -1)$ is a saddle point; we have $D(0, 0) = -36 < 0$, so $(0, 0)$ is a saddle point; we have $D(1, 0) = 36 > 0$ and $f_{xx}(1, 0) = 6 > 0$ so $(1, 0)$ is a local minimum.

4. Since $\partial f/\partial x = 4(x-y)^3 + 2x - 2$ and $\partial f/\partial y = -4(x-y)^3 - 2y + 2$, let both of them equal to 0 then we get $(x_1, y_1) = (1, 1)$ as the critical points, then we test the second derivatives.

   Since
   $$H(x,y) = \begin{bmatrix} 12(x-y)^2 + 2 & -12(x-y)^2 \\ -12(x-y)^2 & 12(x-y)^2 - 2 \end{bmatrix}$$
   so that
   $$D(x,y) = det(H) = -4$$
   o we have $D(x,y) = -4 < 0$, so $(1,1)$ is a saddle point.