4.38 Solution: Since $\text{Var}(X) = E[X^2] - E[X]^2 = 5$ and $E[X] = 1$ we have $E[X^2] = 6$.

(a)

$$E[(2+X)^2] = E[X^2 + 4X + 4] = E[X^2] + 4E[X] + 4 = 6 + 4 + 4 = 14$$

(b)

$$\text{Var}(4 + 3X) = 9\text{Var}(X) = 45$$

4.40 Solution:

$$P(>4) = \binom{5}{4}(\frac{1}{3})^4(\frac{2}{3}) + \binom{5}{5}(\frac{1}{3})^5 = 11/243$$

4.42 Solution:

(a)

$$E[\text{A and B}] = 10 \cdot 0.7 \cdot 0.4 = 2.8$$

(b)

$$\text{Var}(A \textit{ and } B) = E[X^2] - (E[X])^2 = 14.84$$

4.45 Solution:

$$P(3) = \frac{1}{3}[\binom{3}{2}(0.8)^2(0.2) + (0.8)^3] + \frac{2}{3}[\binom{3}{2}(0.4)^2(0.6) + (0.4)^3] = 0.533$$

$$P(5) = \frac{1}{3}\sum_{i=3}^{5}\binom{5}{i}(0.8)^2(0.2)^{5-i} + \frac{2}{3}\sum_{i=3}^{5}\binom{5}{i}(0.4)^i(0.6)^{5-i} = 0.3038$$

So it would be better to have 3 examiners

4.48 Solution:

$$P(1) = 3\sum_{i=2}^{10}(0.01)^i(0.99)^{10-i}(1 - \sum_{i=2}^{10}(0.01)^i(0.99)^{10-i})^2$$

4.50 Solution:

(a)

$$P(HTT|6H) = \frac{P(HTT)P(6H|HTT)}{P(6H)} = \frac{pq^2\binom{7}{5}p^5q^2}{\binom{10}{6}p^6q^4} = 1/10$$

(b)

$$P(THT|6H) = \frac{P(THT)P(6H|THT)}{P(6H)} = \frac{pq^2\binom{7}{5}p^5q^2}{\binom{10}{6}p^6q^4} = 1/10$$

4.55 Solution:

$$p(X_A = 0) = e^{-3}$$

$$p(X_A = 0) = e^{-4.2}$$

Thus,

$$P = \frac{1}{2}(e^{-3} + e^{-4.2})$$

4.57 Solution:

(a)

$$p(\geq 3) = 1 - \sum_{i=0}^{2} e^{-3}\frac{3^i}{i!} = 1 - \frac{17}{2}e^{-3}$$

(b)

$$p(\geq 3| \geq 1) = \frac{1 - \frac{17}{2}e^{-3}}{1 - e^{-3}}$$

4.59 Solution: Since

$$\lambda = np = \frac{1}{2}$$

(a)

$$p(x \geq 1) = 1 - e^{-\frac{1}{2}}$$

(b)

$$p(x = 1) = \frac{1}{2}e^{-1/2}$$

(c)

$$p(x >= 2) = 1 - p(x < 2) = 1 - \frac{3}{2}e^{-1/2}$$

4.61 Solution: Since $\lambda = np = 1.4$,

$$P = 1 - e^{-1.4} - 1.4e^{-1.4}$$

4.63 Solution: People enter at the rate of 1 every 2 minutes means that $P(\text{enter}) = 0.5$, thus $\lambda = np = 2.5$

(a)

$$P(\text{No one in 5 min}) = e^{-2.5}$$

(b)

$$P(\geq 4) = 1 - e^{-2.5} - 2.5e^{-2.5} - \frac{(2.5)^2}{2}e^{-2.5} - \frac{(2.5)^3}{3!}e^{-2.5}$$

4.72 Solution:

$$P(i) = \binom{i-1}{3}(0.6)^4(0.4)^{i-4}$$

$$\begin{aligned}\Rightarrow P(4) &= 0.1296\\ P(5) &= 0.20736\\ P(6) &= 0.20736\\ P(7) &= 0.16588\end{aligned}$$

So

$$P(\text{win } 4) = 0.7102$$

$$P(2 \text{ in } 3) = \binom{3}{2}(0.6)^2(0.4) = 0.432$$

So in the first game, the possibility of victory is higher.

4.73 Solution:

$$P(i) = \sum_{i=4}^{7} i \cdot 2\binom{i-1}{i-4}(1/2)^4(1/2)^{i-3} = 5.8125$$

4.77 Solution:

$$P(\text{Empty} \cap k) = 2\binom{2n-k}{n}(1/2)^{2n-k}$$

4.78 Solution:

$$P(n) = (1 - \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}})^{n-1}(\frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}})$$

4.79 Solution:

(a)

$$P\{X = 0\} = \frac{\binom{6}{0}\binom{94}{10}}{\binom{100}{10}}$$

(b)

$$P\{X > 2\} = \sum_{i=3}^{6} \frac{\binom{6}{i}\binom{94}{10-i}}{\binom{100}{10}}$$

4.84 Solution:

(a)

$$E[X] = \sum_{i=1}^{5} P(X_i) = \sum_{i=1}^{5}(1 - p_i)^{10}$$

(b)

$$E[X] = \sum_{i=1}^{5} P(X_i) = \sum_{i=1}^{5} 10p_i(1 - p_i)^9$$

4.85 Solution:

$$E[X] = \sum_{i=1}^{k} P(X) = \sum_{i=1}^{k}(1 - (1 - p_i)^n) = k - \sum_{i=1}^{k}(1 - p_i)^n$$