0.1 Solution:

(a)

$$n = (2 \times 3)^{15} = 470184984576$$

(b)

$$n = (2 \times 3)^{14} \times 15 \times 3 = 3526387384320$$

(c)

$$(2 \times 2)^{15} = 1073741824$$

0.2 Solution:

(a)

$$P(A^S \cap B^S) = 1 - 0.3 - 0.5 = 0.2$$

(b)

$$P(A \cap B^S) = P(A) = 0.3$$

(c)

$$P(A \cap B) = P(\emptyset) = 0$$

0.3 Solution:

$$\begin{aligned} P(Accept) &= P(American\ Express) + P(Visa) - P(American\ Express) \cap P(Visa) \\ &= 0.24 + 0.61 - 0.11 \\ &= 0.74 \end{aligned}$$

0.4 Solution:

$$P(Blackjack) = \frac{4 \times 16}{\binom{52}{2}} = \frac{64}{1326} = 0.048$$

0.5 Solution:

$$\begin{aligned} P(No\ blackjack) &= 1 - P(I \cup Dealer) \\ &= 1 - (P(I) + P(Dealer) - P(I) \cap P(Dealer)) \\ &= 1 - (0.048 + 0.048 - \frac{2(4 \times 16)(3 \times 15)}{\binom{52}{4}}) \\ &= 1 - (0.096 - 5760/270725) = 1 - 0.075 = 0.925 \end{aligned}$$

0.6 Solution:

(a)

$$
\begin{aligned}
P(1) &= 4/20 = 0.2 \\
P(2) &= 8/20 = 0.4 \\
P(3) &= 5/20 = 0.25 \\
P(4) &= 2/20 = 0.1 \\
P(5) &= 1/20 = 0.05
\end{aligned}
$$

(b)

$$
\begin{aligned}
&n = 4 \times 1 + 8 \times 2 + 5 \times 3 + 2 \times 4 + 5 = 48 \\
&P(1) = 4/48 = 1/12 \\
&P(2) = 16/48 = 1/3 \\
&P(3) = 15/48 = 5/16 \\
&P(4) = 8/48 = 16 \\
&P(5) = 5/48
\end{aligned}
$$

0.7 Solution: In a single roll,

$$
\begin{aligned}
&P(5) = 4/36 = 1/9 \\
&P(7) = 6/36 = 1/6 \\
&P(5 \cup 7) = 10/36 = 5/18 \\
&P(\mathit{not}\ 5\ \mathit{and}\ \mathit{not}\ 7) = 1 - 5/18 = 13/15
\end{aligned}
$$

Thus,

$$P(E_n) = (13/15)^{n-1}/9$$

and

$$P(5\ \mathit{comes\ first}) = \sum_{n=1}^{\infty} P(E_n) = \lim_{n \to \infty} \frac{5}{6}[1 - (\frac{13}{15})^n] = \frac{5}{6}$$

0.8 Solution: In a single draw,

$$P(\mathit{red}) = 3/10$$

$$P(\mathit{black}) = 7/10$$

If denote the event that A will draw the first red ball in the nth cycle as E_n,

$$
\begin{aligned}
P(E_n) &= \frac{3}{10 - 2(n-1)} \prod_{i=1}^{n-1} (\frac{9 - 2n}{12 - 2n} \frac{8 - 2n}{11 - 2n} \\
&= \frac{3}{12 - 2n} \prod_{i=1}^{n-1} (\frac{9 - 2n}{12 - 2n} \frac{8 - 2n}{11 - 2n})
\end{aligned}
$$

Since there are only 7 black balls, so A can only draw red ball in the first 4 cycles. Thus,

$$P(A) = \sum_{i=1}^{4} P(E_n) = 3/10 + 7/40 + 1/12 + 1/40 = 7/12$$

0.9 Solution:

(a)

$$P(same) = P(red) + P(green) + P(blue) = (5/19)^3 + (6/19)^3 + (8/19)^3 = 0.124$$

(b)

$$P(diff) = (5/19)(6/19)(8/19) = 0.035$$

0.10 Solution: Having a girl on position i means to randomly choose a girl to interlope her into the rest $b+g-1$ people to divide any permutation of them into 2 part. Thus,

$$P(i) = \frac{(b+g-1)!g}{(b+g)!} = \frac{g}{b+g}$$

0.11 Proof: $EF^C = (E^C)^C F^C = (E^C \cup F)^C$, then $P(EF^C) = P((E^C \cup F)^C) = 1 - P(E^C \cup F) = 1 - (P(E^C) + P(F) - P(E^C F)) = 1 - ((1 - P(E)) + P(F) - P(E^C F)) = 1 - 1 + P(E) - P(F) + P(E^C F) = P(E) - (P(F) - P(E^C F)) = P(E) - P(EF)$■

0.12 Proof: $P(EF) = P((E^C)^C (F^C)^C) = P((E^C \cup F^C)^C) = 1 - P((E^C \cup F^C)) = 1 - (P(E^C) + P(F^C) - P(E^C F^C)) = 1 - ((1 - P(E)) + (1 - P(F) - P(E^C F^C))) = 1 - (2 - (P(E) + P(F) - P(E^C F^C))) = (P(E) + P(F)) - 1 + P(E^C F^C) \geq P(E) + P(F) - 1$■

Base case is proved above, we can suppose that when $n = k$

$$P(\bigcap_{i=1}^{k} E_i) \geq \sum_{i=1}^{k} P(E_i) - (k-1)$$

is true.

When $n = k + 1$,

$$\begin{aligned}
P(\bigcap_{i=1}^{k+1} E_i) &= P(\bigcap_{i=1}^{k} E_i \cap E_{k+1}) \\
&= P(((\bigcap_{i=1}^{k} E_i)^C)^C \cap (E_{k+1}^C)^C) \\
&= P(((\bigcap_{i=1}^{k} E_i)^C) \cup E_{k+1}^C)^C) \\
&= 1 - P((\bigcap_{i=1}^{k} E_i)^C \cup E_{k+1}^C) \\
&= 1 - (P((\bigcap_{i=1}^{k} E_i)^C) + P(E_{k+1}^C) - P((\bigcap_{i=1}^{k} E_i)^C E_{k+1}^C) \\
&= 1 + P((\bigcap_{i=1}^{k} E_i)^C E_{k+1}^C) - ((1 - P(\bigcap_{i=1}^{k} E_i)) + (1 - P(E_{k+1}))) \\
&= P((\bigcap_{i=1}^{k} E_i)^C E_{k+1}^C) + P(\bigcap_{i=1}^{k} E_i) + P(E_{k+1}) - 1
\end{aligned}$$

According to our hypothesis

$$P(\bigcap_{i=1}^{k} E_i) \geq \sum_{i=1}^{k} P(E_i) - (n-1)$$

Then

$$\begin{aligned}
P(\bigcap_{i=1}^{k+1} E_i) &\geq \sum_{i=1}^{k} P(E_i) - (n-1) + P(E_{k+1}) - 1 \\
&\geq \sum_{i=1}^{k} P(E_i) - (n-1) + P(E_{k+1}) - 1 \\
&\geq \sum_{i=1}^{k+1} P(E_i) - n
\end{aligned}$$

We can conclude that $\forall n \in \mathbb{N}$

$$P(\bigcap_{i=1}^{n} E_i) \geq \sum_{i=1}^{n} P(E_i) - (n-1)$$

■

0.13 Solution:

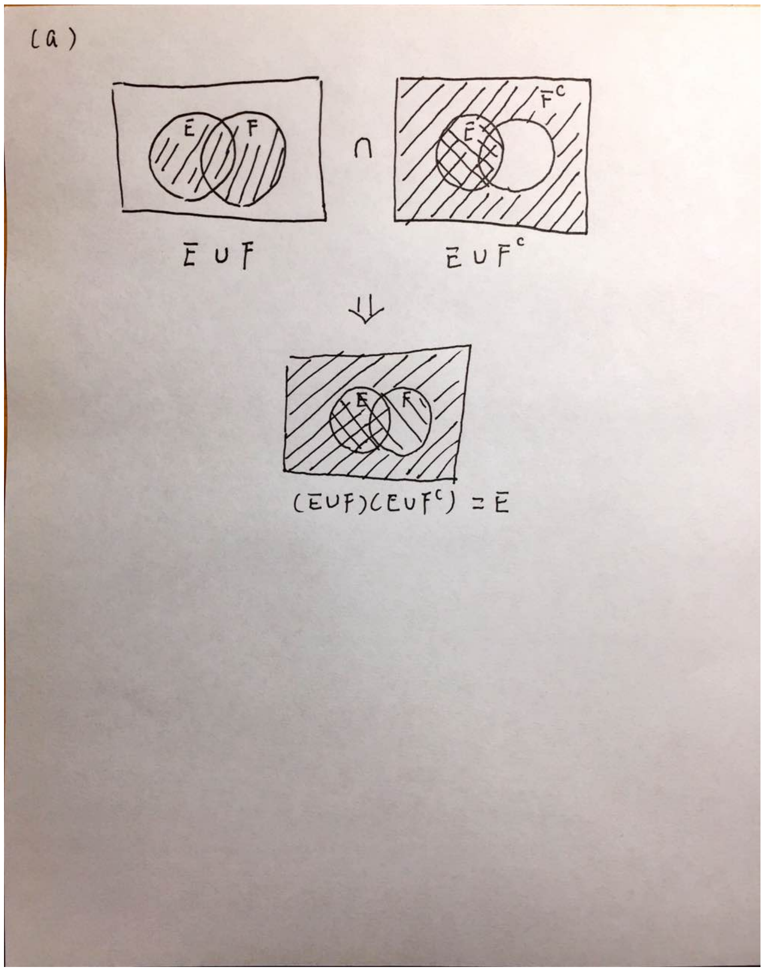

Figure 1: 0.13.a

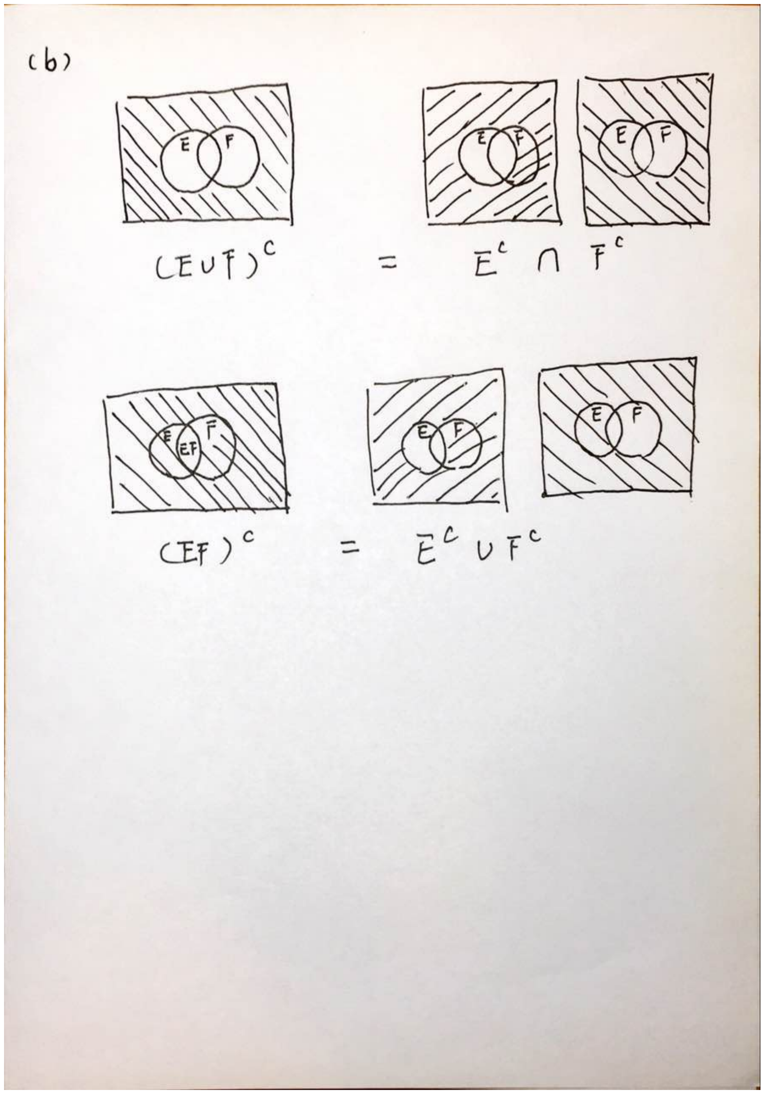

Figure 2: 0.13.b