0.1 Solution: If all of them can play all 4 kinds of instruments, then the number of possible outcomes

$$n_1 = P(4) = 4! = 24$$

If Jay and John can only play piano and drums, then there's

$$n_2 = 2 \times 2 = 4$$

possible outcomes.

0.2 Solution: There are

$$n = 8 \times 2 \times 10 = 160$$

possible outcomes. And there're

$$n_{Start\ with\ 4} = 1 \times 2 \times 10 = 20$$

possible outcomes if the codes start with 4.

0.3 Solution:

(a)

$$n = \binom{6}{3} \times 3! \times 3! = 6! = 720$$

(b) We can seperate the 6 seats into 3 pair of seats for each pair of boy and girl, so the only problem left is just how they match with each other. Then there are

$$n = 3! \times 3! \times 2^3 = 288$$

possible outcomes.

(c) If the boys take consecutive 3 seats, it can be regarded as 1 seat choose from 4 seat, then

$$n = \binom{4}{1} \times 3! = 24$$

(d) Under presented rule, only two possible permutation is allowed - MFMFMF or FMFMFM, so

$$n = 2 \times 3! \times 3! = 72$$

0.4 Solution:

(a)

$$n = 5! = 120$$

(b)

$$n = \frac{7!}{2!2!} = 1260$$

(c)

$$n = \frac{11!}{4!4!2!} = 34650$$

(d)

$$n = \frac{7!}{2!2!} = 1260$$

0.5 Solution:

(a)

$$n = \binom{30}{1}^5 = 24300000$$

(b)

$$n = 30 \times \cdots \times 26 = 17100720$$

0.6 Solution:

$$n = \binom{5}{2}\binom{6}{2}\binom{4}{3} = 600$$

0.7 Solution:

(a)

$$n = \binom{8}{3}(\binom{6}{3} - \binom{6-2}{1}) = 896$$

(b)

$$n = (\binom{8}{3} - \binom{8-2}{1})\binom{6}{3} = 1000$$

(c)

$$n = \binom{8}{3}\binom{6}{3} - \binom{8+6-2}{6-2} = 625$$

0.8 Solution:

(a)

$$n = \binom{8}{5} - \binom{8-2}{5-2} = 36$$

(b)

$$n = \binom{6}{5} + \binom{2}{2}\binom{6}{3} = 26$$

0.9 Solution: You have to take 4 steps right and 3 steps up to reach B from A, no matter which way, so the possible path is to choose 3 up from all seven, thus

$$n = \binom{7}{3} = 35$$

0.10 Solution: According to Binomial Theorem,

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Thus,

$$(3x^2+y)^5 = \sum_{k=0}^{5} \binom{5}{k} (3x)^k y^{5-k} = 243x^5+405x^4y+270x^3y^2+90x^2y^3+15xy^4+y^5$$

0.11 Solution:

$$n = \binom{12}{3}\binom{12-3}{4}\binom{12-3-4}{5} = 27720$$

0.12 Proof: $\binom{n+m}{r}$ means choosing r from $n+m$, which means you can choose k from n and choose $r-k$ from m. Suppose $k=0$, the number of outcomes is $\binom{n}{0}\binom{m}{r}$, if $k=1$, $\binom{n}{1}\binom{m}{r-1}$... ect.

Thus,

$$\binom{n+m}{r} = \sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k} \tag{1}$$

is proved to be true.

0.13 Proof: According to equation (1), let $m = n, r = n$, then

$$\binom{n+n}{r} = \binom{2n}{r} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{k} = \sum_{k=0}^{n} \binom{n}{k}^2$$

0.14 Solution: Choosing r from n, is like pick the rest $n-r$ out of n.

0.15 Solution: To choose k objects form a set, we need at least k elements in it. Suppose it has the size of $n = k$, then the possible outcomes in this subset is $\binom{n-1}{k-1}$. Thus from k to n, we can seperately have the possible outcomes under which circumstances. Thus, to get the total number of possible outcomes, we sum them together into

$$\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1}$$