Base on the three data points $(-1, 1), (0, 0), (1, 1)$, we can build out array $x = [-1, 0, 1]$ and $y = [1, 0, 1]$.

- Using monomial basis we build the Vandermonde Matrix

$$V = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

  And solve quation

$$Va = y$$

  and get

$$a = [0, 0, 1]$$

  which means that the function we get by interpolation is

$$p(t) = t^2$$

- Base on the data we have, we get the following Lagrange basis functions

  1.
$$l_1(t) = \frac{t(t-1)}{-1 \cdot -2} = \frac{t^2 - t}{2}$$

  2.
$$l_2(t) = \frac{(t-1)(t+1)}{1 \cdot -1} = t^2 - 1$$

  3.
$$l_3(t) = \frac{t(t+1)}{2 \cdot 1} = \frac{t^2 + t}{2}$$

  So we have

$$p(t) = l_1(t) + l_3(t) = t^2$$

- Base on the data we have, we get the lower triangular matrix formed by Newton basis functions

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

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So we have

$$a = [1, -1, 1]$$

so that

$$p(t) = 1 - (t + 1) + (t + 1)t = 1 - t - 1 + t^2 + t = t^2$$