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Section: AYV

1. 406, 408
2. (a). $\frac{n+1}{2n}$

   $$\prod_{k=2}^{n}\left(1-\frac{1}{k^2}\right)=\prod_{k=2}^{n}\left(1-\frac{1}{k}\right)(1+\frac{1}{k})=\prod_{k-1}^{n}\frac{k-1}{k}\cdot\prod_{k=2}^{n}\left(\frac{1+k}{k}\right)=\frac{n+1}{2n}$$

   (b). 4

   Since that 36 mod 7=1, then we have 3^{1000}=$3^{6\times166+4}$=3^4=4 mod 7

   (c). 1

   $$\sum_{r=1}^{\infty}\left(\frac{1}{2}\right)^r=1-\lim_{r\to\infty}\frac{1}{2^r}=1$$

   (d). 2

   $$\frac{\log_7 81}{\log_7 97}=\log_9 81=2$$

   (e). 4n

   $$\log_2 4^{2n}=2n\log_2 4=4n$$

   (f). 1

   $$\log_{17}211-\log_{17}13=\log_{17}\left(\frac{211}{13}\right)=1$$

3. (n+1)!

   Proof :

   Base case: k = 1, Solution = 2

   Induction Step: Suppose k = n, $\sum_{j=0}^{k}j\cdot j!=(\mathrm{k}+1)!$ Is true

   When k = n+1, $\sum_{j=0}^{k}j\cdot j!\ =\ \sum_{j=0}^{k}j\cdot j!+(n+1)\cdot n$

   $=(\mathrm{n}+1)!\ +(\mathrm{n}+1)\cdot\mathrm{n}$

   $=(\mathrm{n}+1)\cdot\mathrm{n}+(\mathrm{n}+1)\cdot\mathrm{n}$

   $=(\mathrm{n}+1)!$

   Q.E.D

4. (a).

   $\mathrm{f}(n)=4^{\log_4 n}=n, g(n)=2n+1$

   Obviously, c1 = 1, c2 = 3, n $\in\mathbb{R}^+$ exist to make $0\le c_1g(n)\le f(n)\le c_2g(n)$

   $so\ f(n)=\ \Theta(\mathrm{g(n)})$

   (b).

   $\mathrm{g(n)}=(\sqrt{2})^{\log_2 n}$=$n^{\frac{1}{2}}=\sqrt{n}$, while $\mathrm{f(x)}=n^2$

   Then, we have $c_1=1, n\ge1$, to make $c_1g(n)\le f(n)$

   Therefore, $f(n)=\ \mathrm{O}(\mathrm{g(n)})$

   (c).

   $f(n)=\ \mathrm{O}\big(\mathrm{g(n)}\big)$

(d).

$f(n) = O(g(n))$

5.

(a). We can just expand the recursive tree and get that T(n) = $5\log_2 n$+1

(b). Expand the recursive tree, we have T(n) = 0 + $\sum_{k=1}^{n} \frac{1}{k}$, which is a harmonic series. So this recurrence relation does not have a result.

(c).Proof:

Base case: T(1) = $5\log_2 1$+1 = 1

Inductive step:

Suppose $\forall k \leq n$, T(k) = $5\log_2 k$+1

Then, T(k+1) = 5+T(k+1/2) = 5 + $5\log_2 \frac{k+1}{2}$+1

= 5 + $5(\log_2(k+1) - 1)$ + 1

= $\log_2(k+1)$

Q.E.D

6.(a).

Recurrence	$T(n) = T\left(\frac{n}{2}\right) + c$
Base case	$T(1) = c$
Recurrence Solution	$T(n) = c\log_2 n$

(b).

Recurrence	$T(n) = 2T\left(\frac{n}{2}\right) + cn$
Base case	$T(1) = c$
Recurrence Solution	$T(n) = cn\log_2 n + cn$

7.

(a).

x	n	return
2	12	derp(4, 6)
4	6	derp(16,3)
16	3	16*derp(256, 1)
256	1	16*256*derp(256^2,0)
		16*256=4096

(b).Calculate the x^n

(c).$T(n) = T\left(\frac{n}{2}\right) + c$

(d).$T(n) = \mathrm{c}\log_2 n$