[L]Math 417 [C]HW2 [R]Lanxiao Bai(lbai5)

1.5.3 Solution:

(a)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 6 & 3 & 7 & 4 & 1 \end{pmatrix} = (1\ 2\ 5\ 7)(3\ 6\ 4)$$

(b)

$$(1\ 2)(1\ 2\ 3\ 4\ 5) = (2\ 3\ 4\ 5)$$

(g)

$$(1\ 2)(1\ 3)(1\ 4) = (4\ 3\ 2\ 1)$$

(h)

$$(1\ 3)(1\ 2\ 3\ 4)(1\ 3) = (1\ 4)(3\ 2)$$

1.5.6 Claim:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 6 & 3 & 7 & 4 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 1 & 4 & 6 & 2 & 3 & 5 \end{pmatrix}$$

Solution: To find the inverse of a permutation is to find the permutation that, when producted with the original one, get e. As a result, there's only need to check how we can put the disordered element back. Thus,

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 6 & 3 & 7 & 4 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 1 & 4 & 6 & 2 & 3 & 5 \end{pmatrix}$$

1.5.11 Solution:

- When $size = 2$, $\pi = (1\ 2)$, $order = 2$
- When $size = 4$, $\pi = (1\ 2\ 4\ 3)$, $order = 4$
- When $size = 6$, $\pi = (1\ 2\ 4)(3\ 6\ 5)$, $order = 3$
- When $size = 8$, $\pi = (1\ 2\ 4\ 8\ 7\ 5)(3\ 6)$, $order = 6$
- When $size = 10$, $\pi = (1\ 2\ 4\ 8\ 5\ 10\ 9\ 7\ 3\ 6)$, $order = 10$
- When $size = 12$, $\pi = (1\ 2\ 4\ 8\ 3\ 6\ 12\ 11\ 9\ 5\ 10\ 7)$, $order = 12$
- When $size = 14$, $\pi = (1\ 2\ 4\ 8)(3\ 6\ 12\ 9)(5\ 10)(7\ 14\ 13\ 11)$, $order = 4$
- When $size = 16$, $\pi = (1\ 2\ 4\ 8\ 16\ 15\ 13\ 9)(3\ 6\ 12\ 7\ 14\ 11\ 5\ 10)$, $order = 8$
- When $size = 52$, $\pi = (1\ 2\ 4\ 8\ 16\ 32\ 11\ 22\ 44\ 35\ 17\ 34\ 15\ 30\ 7\ 14\ 28\ 3\ 6\ 12\ 24\ 48\ 43\ 33\ 13\ 26\ 52\ 51\ 49\ 45\ 37\ 21\ 42\ 31\ 9\ 18\ 36\ 19\ 38\ 23\ 46\ 39\ 25\ 50\ 47\ 41\ 29\ 5\ 10\ 20\ 40\ 27)$, $order = 52$

1.6.3 Claim: If $p \in N$, $(p|ab \Rightarrow (p|a) \wedge (p|b)) \Rightarrow$ p is a prime.