

## 1 Syntax vs. Semantics

### Syntax

- Symbols
- Recursive rules to form formulas out of symbols
- Recursive nature of formulas allows proofs by induction
- Complexity of formula is the number of times you need to apply rules to build the formula
- Inductive arguments are by induction on complexity

### Semantics

- A collection of mathematical objects & ways to interpreting out wff's as statements about these objects
- If we fix one such object every formula becomes a true or false statement about the object
- Roughly: Every sentence symbol is a statement that is true or false.

## 2 Uniqueness

Interpretation must be unique.

### Example of Ambiguity:

$$P \wedge Q \rightarrow R$$

could be explained as

$$(P \wedge Q) \rightarrow (R)$$

or

$$(P) \wedge (Q \rightarrow R)$$

**Unique Readability** Every wff is either a sentence symbol or is composite.

Every composite wff is of the form

- $(\neg\psi)$
- $(\phi \circ \psi)$

for wff's  $\phi, \psi$  and  $\circ$  is one of  $\vee, \text{wedge}, \rightarrow, \leftrightarrow$ <sup>1</sup>.

**Theorem 2.0.1** *Every composite wff has a unique primary connective & immediate subformulas.*

### 3 Truth Assignment

**Definition 3.0.1** *A truth assignment is function*

$$\Sigma : W_0 \rightarrow \{False, True\}$$

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<sup>1</sup>In  $(\neg\psi)$ ,  $\neg$  is the primary connective and  $\psi$  is the immediate subformula.  
In  $(\phi \circ \psi)$ ,  $\circ$  is the primary connective and  $\phi, \psi$  are the immediate subformulas.