1 Recall

A first order language L is a set of formal symbols consisting of:

- Logical symbols:
  - $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \forall, \exists$
  - Parathesis: (,)
  - Equality: $=$
- Variables: $x, y, z, \cdots$
- k-ary relation symbols: $R, S, \cdots$
- k-ary function symbols: $f, g, h, \cdots$
- Constant symbols: c, c'

First order language can be uncountable, but we can usually take L to be countable.

An L-structure $\mathcal{M}$ is a nonempty M together with

- a k-ary relation $R^{\mathcal{M}}$ on M for every k-ary relation symbol
- a k-ary function $f^{\mathcal{M}}$ on M for every k-ary function symbol
- an element $c^{\mathcal{M}}$ for each constant symbol c

$\mathcal{M}$ is the structure.
M is the underlying set (domain) of $\mathcal{M}$.
We also write

$$\mathcal{M} = (M; R^{\mathcal{M}})$$

Ex. If $M = \mathbb{R}$ and $R^{\mathcal{M}} = \leq$, then we write $(\mathbb{R}; \leq)$

Definition 1.0.1 *$\mathcal{M}$ is a symmetric L-structure if $xR^{\mathcal{M}}y$ iff $yR^{\mathcal{M}}x$ for all $x, y \in M$.*

Definition 1.0.2 *A partial order is a L-structure satisfying*

- $$\forall x(xRx)$$
- $$\forall x \forall y[(xRy) \wedge (yRx)] \rightarrow (x = y)$$

- $$\forall x \forall y \forall z[(xRy) \wedge (yRz)] \rightarrow (xRz)$$

Definition 1.0.3 *$R^{\mathcal{M}}$ is total order if*

$$\forall x \forall y (x \leq y) \vee (y \leq x)$$

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Definition 1.0.4 (homomorphism) *Fix a first order language L. Let A and B be L-structure with underlying sets A, B. A map $h : A \rightarrow B$ is a homomorphism.*

Definition 1.0.5 (automorphism) *Let $\mathcal{M}$ be an L-structure, an automorphism of $\mathcal{M}$ is an isomorphism from M to M.*