1. **Proof:**

   Suppose there's a function g that there is no wff φ such that $g = f_\varphi$. Then for any wff φ, whenever $g = T$, $\Sigma(\varphi) = F$. But if $g = x_1$, $\varphi = P_1$ and $\Sigma(P_1) = x_1 = T$, this truth assignment will lead to $g = T = \Sigma_{P_1}$. This fact contradicts with our assumption.

   As a result, for all function g there is a wff φ that $g = f_\varphi$.$\blacksquare$

2. **Proof:**

   $\Rightarrow$:

   Since $f_\varphi = f_\psi$, any truth assignment Σ that either satisfies $(\varphi \wedge \psi)$ or does not satisfy $(\varphi \vee \psi)$. So by definition, Σ either satisfies both φ and satisfies ψ, or does not satisfies φ and does not satisfies ψ.

   This means that for all truth assignment, there is $\Sigma(\varphi) = \Sigma(\psi)$, so $(\varphi \leftrightarrow \psi)$ is always true.

   By definition, $(\varphi \leftrightarrow \psi)$ is a tautology.

   $\Leftarrow$:

   Since $(\varphi \leftrightarrow \psi)$ is a tautology, then for any truth assignment Σ, there is $\Sigma(\varphi) = \Sigma(\psi)$, so that Σ that either satisfies $(\varphi \wedge \psi)$ or does not satisfy $(\varphi \vee \psi)$.

   As a result, $f_\varphi = f_\psi$.

   $\blacksquare$

3. By counting the number of classes, we can calculate how many different mappings from $\{T, F\}^n$ to $\{T, F\}$.

   From the knowledge from Analysis, we know the number is

   $$|\{T, F\}|^{|\{T,F\}|^n} = 2^{2^n}$$