Part1

(a)

$$\begin{aligned}
\text{cond}_{\text{abs}}(f) &= \max \frac{||\Delta y||}{||\Delta x||} \\
&= \max \frac{||A(x+\Delta x) - Ax||}{||\Delta x||} \\
&= \max \frac{||A\Delta x||}{||\Delta x||} \\
&= \frac{||A||||\Delta x||}{||\Delta x||} = ||A||
\end{aligned}$$

(b)

$$\begin{aligned}
\text{cond}_{\text{rel}}(f) &= \max \frac{||\Delta y||/||y||}{||\Delta x||/||x||} \\
&= \max \frac{||\Delta y||||x||}{||\Delta x||||y||} \\
&= \max \frac{||A\Delta x||||A^{-1}y||}{||\Delta x||||y||} \\
&= ||A||||A^{-1}||\frac{||\Delta x||||y||}{||\Delta x||||y||} = ||A||||A^{-1}||
\end{aligned}$$

Part2

(a) **Upper bound:**

$$\begin{aligned}
||y||_2 &= ||A^k x||_2 \\
&\leq ||A^k||_2||x||_2 && \text{by submultiplicity} \\
&\leq ||A||_2^k||x||_2 && \text{by submultiplicity}
\end{aligned}$$

Lower bound:

Since A is invertible, so is A^k, as a result,

$$x = (A^k)^{-1}y$$

So that

$$||x||_2 = ||(A^k)^{-1}y||_2$$

then

$$\begin{aligned}
&||x||_2 = ||(A^k)^{-1}y||_2 \leq ||(A^k)^{-1}||_2||y||_2 \\
&\Rightarrow ||y||_2 \geq \frac{||x||_2}{||(A^k)^{-1}||}
\end{aligned}$$

(b) **Condition:**

$$\begin{aligned}
&\frac{||\Delta y||/||y||}{||\Delta x||/||x||} \leq 1 \\
&\Rightarrow \frac{||\Delta y||||x||}{||\Delta x||||y||} \leq 1 \\
&\Rightarrow \frac{||A^k \Delta x||||(A^k)^{-1}y||}{||\Delta x||||y||} \leq 1 \\
&\Rightarrow \text{cond}(A^k) \leq 1
\end{aligned} \tag{1}$$

And

$$k \geq -\log_{10}\varepsilon - \log_{10}\text{cond}(A^k) = -\log_{10}\varepsilon\text{cond}(A^k)$$