3.64

Solution: Denote the probability of correct for strategy a as $P(a)$ and $P(b)$ for the probability of correct for strategy b.

Then

$$P(a) = p$$

and

$$P(b) = p^2 + 2p(1-p) \cdot 1/2 = p$$

So we can conclude that those two strategies have the same possibility to give the correct answer.

3.66

Solution:

$$P(a) = p_1p_2p_5 + p_3p_4p_5 - p_1p_2p_3p_4p_5$$

$$P(b) = p_1p_4 + p_1p_3p_5 + p_2p_5 + p_2p_3p_4 - p_1p_3p_4p_5 - p_1p_2p_4p_5 - p_1p_2p_3p_4$$
$$-p_1p_2p_3p_5 - p_2p_3p_4p_5 + 2p_1p_2p_3p_4p_5$$

3.78 Solution:

(a)

$$P(4) = P(\text{A win}) + P(\text{B win}) = 2p^3(1-p) + 2(1-p)^3p$$

(b)

$$\begin{aligned} P(\text{A win}) &= \sum_{i=1}^{\infty} P(\text{A win 2i}) \\ &= \sum_{i=1}^{\infty} 2^{i-1}p^{i+1}(1-p)^{i-1} = \frac{p^2}{1-2p(1-p)} \end{aligned} \tag{1}$$

3.83

(a)

$$P(\text{red}) = 1/2 \cdot (2/3 + 1/3) = 1/2$$

(b)

$$P(\text{red}|\text{first 2 red}) = \frac{\frac{1}{2}(1/3)^3 + \frac{1}{2}(2/3)^3}{\frac{1}{2}(1/3)^2 + \frac{1}{2}(2/3)^2} = 3/5$$

(c)

$$P(\text{H}|\text{first 2 red}) = \frac{\frac{4}{9}\frac{1}{2}}{\frac{4}{9}\frac{1}{2} + \frac{1}{9}\frac{1}{2}} = 4/5$$

3.84

(a)

$$\begin{aligned} P(A) &= (\sum_{i=1}^{\infty}((2/3)^3)^{i-1})(1/3) \\ &= \frac{1}{3}\sum_{i=1}^{\infty}(\frac{8}{27})^{i-1} \\ &= \frac{1}{3}\frac{27}{19}(1 - \lim_{n\to\infty}(\frac{8}{27})^n) \\ &= 9/19 \end{aligned}$$

$$\begin{aligned} P(B) &= (\sum_{i=1}^{\infty}((2/3)^3)^{i-1})(2/3)(1/3) \\ &= 6/19 \end{aligned}$$

$$\begin{aligned} P(C) &= (\sum_{i=1}^{\infty}((2/3)^3)^{i-1})(2/3)^2(1/3) \\ &= 4/19 \end{aligned}$$

(b)

$$
\begin{aligned}
P(A) &= 1/3 + (2/3)(7/11)(6/10)(4/9) + (2/3)(7/11)(6/10)(5/9)(4/8)(3/7)(4/6) \\
&= 7/15 \\
P(B) &= (8/12)(4/11) + (2/3)(7/11)(6/10)(5/9)(4/8) \\
&+ (2/3)(7/11)(6/10)(5/9)(4/8)(3/7)(2/6)(4/5) \\
&= 53/165 \\
P(C) &= (8/12)(7/11)(4/10) + (2/3)(7/11)(6/10)(5/9)(4/8)(4/7) \\
&+ (2/3)(7/11)(6/10)(5/9)(4/8)(3/7)(2/6)(1/5)(4/4) \\
&= 7/33
\end{aligned}
$$

3.13 (Theoretical Exercises)

Solution: If the initial flip lands on head, then A will win with $P_{n-1,m}$, if it lands on tail, then it's B's turn to flip the coin with the possibility $P_{m,n}$ with the possibility $(1 - P_{m,n})$.

As a result,

$$P_{n,m} = pP_{n-1,m} + (1-p)(1 - Pm, n)$$

4.1 Solution: It is possible that $X = -2, -1, 0, 1, 2, 4$.

$$
\begin{aligned}
P(-2) &= \frac{8}{14}\frac{7}{13} = 4/13 \\
P(-1) &= \frac{8}{14}\frac{2}{13} + \frac{2}{14}\frac{8}{13} = 16/91 \\
P(0) &= \frac{2}{14}\frac{1}{13} = 1/91 \\
P(1) &= \frac{4}{14}\frac{8}{13} + \frac{8}{14}\frac{4}{13} = 32/91 \\
P(2) &= \frac{4}{14}\frac{2}{13} + \frac{2}{14}\frac{4}{13} = 8/91 \\
P(4) &= \frac{4}{14}\frac{3}{13} = 6/91
\end{aligned}
$$

4.4 Solution: Since there are 5 men and 5 women, $\max(X) = 6$.

$$P\{X = 1\} = 5\frac{9!}{10!} = 1/2$$
$$P\{X = 2\} = 5 \cdot 5\frac{8!}{10!} = 5/18$$
$$P\{X = 3\} = 5 \cdot 4 \cdot 5\frac{7!}{10!} = 5/36$$
$$P\{X = 4\} = 5 \cdot 4 \cdot 3 \cdot 5\frac{6!}{10!} = 5/84$$
$$P\{X = 5\} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 5\frac{5!}{10!} = 5/252$$
$$P\{X = 6\} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5\frac{4!}{10!} = 1/252$$

4.5

Solution: If n is even, we have possibilities of $(0, n), (1, n-1), (2, n-2), \cdots, (n/2, n/2), \cdots, (n-2, 2), (n-1, 1), (n, 0)$.

If n is odd we have possibilities of $(0, n), (1, n-1), (2, n-2), \cdots, (m-1, m), (m, m-1), \cdots, (n-2, 2), (n-1, 1), (n, 0)$.

Thus, $X \in \{0, 1, 2, 3, ..., n\}$.

4.13

$$P\{X = 0\} = 0.7 \cdot 0.4 = 0.28$$
$$P\{X = 500\} = 0.3 \cdot 0.5 \cdot 0.4 + 0.7 \cdot 0.5 \cdot 0.6 = 0.27$$
$$P\{X = 1000\} = 0.3 \cdot 0.5 \cdot 0.4 + 0.7 \cdot 0.5 \cdot 0.6 + 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 = 0.315$$
$$P\{X = 1500\} = 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 \cdot 2 = 0.09$$
$$P\{X = 2000\} = 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 = 0.045$$

4.14

Solution:

$$P\{X = 0\} = (1/5)((1/4) + (2/4) + (3/4) + (4/4)) = 1/2$$
$$P\{X = 1\} = (1/5)((3/4)(1/3) + (2/4)(2/3) + (1/4)(3/3)) = 1/6$$
$$P\{X = 2\} = (1/5)((3/4)(2/3)(1/2) + (2/4)(1/3)(2/2)) = 1/12$$
$$P\{X = 3\} = (1/5)(3/4)(2/3)(1/2) = 1/20$$
$$P\{X = 4\} = 1/5$$

4.17 Solution:

(a)

$$P\{X=1\} = 1/2 - 1/4 = 1/4$$
$$P\{X=2\} = 11/12 - 3/4 = 1/6$$
$$PX = 3 = 1 - 11/12 = 1/12$$

(b)

$$P(1/2 < X < 3/2) = 5/8 - 1/8 = 1/2$$