1 Relation $\models$

We defined the relation

$$\Sigma \models \phi$$

, as the truth assignment Σ satsfies ϕ a wff.

Let $\mathcal{T}$ be a set of wff's, we say that $\mathcal{T} \models \phi$ if whenever a truth assignment Σ satisfies every $\psi \in \mathcal{T}$ then Σ also satisfies ϕ.

2 Tautology

- φ is a tautology if every truth assignment satisfies φ.φ is tautology iff $\emptyset \models \varphi$, as every truth assignment vacuously satisfies every wff in $\emptyset$.
- Contradiction: if no truth assignment satisfies φ
- Satisfiable: if there is at least one truth assignment satisfying φ.
- Note: φ is tautology iff $(\neg\varphi)$ is a contradiction.

3 Compactness

1. $\mathcal{T}$ is satisfiable iff every finite subset of it is satisfiable
2. $\mathcal{T} \models \varphi$ iff there is a finite $\mathcal{T}' \subseteq \mathcal{T}$ such that $\mathcal{T} \models \varphi$

Proof: (2) $\Rightarrow$ (1): Fact: $\mathcal{T}$ is not satisfiable iff $\mathcal{T} \models (P \wedge (\neg P))$.
For a sentence symbol P, as a truth assignment cannot satisfy $(P \wedge (\neg P))$.
By (2), $\mathcal{T} \models (P \wedge (\neg P))$ iff there is finite $\mathcal{T}' \models (P \wedge (\neg P))$.
So $\mathcal{T}$ is not satisfiable iff there is a finite $\mathcal{T}'$ which is not satisfiable.
(1) $\Rightarrow$ (2):