5.37 Solution: Since $\lambda(t) = t^3, t > 0$

(a)

$$F(t) = 1 - \exp\{-\int_0^t \lambda(t)dt\} = 1 - e^{-\frac{1}{4}t^4}$$
$$\Rightarrow f(t) = t^3 e^{-\frac{1}{4}t^4}$$
$$\Rightarrow P(X > 2) = 1 - F(2) = e^{-4}$$

(b)

$$P(0.4 < X < 1.4) = F(1.4) - F(0.4) = \exp\{-(0.4)^4\} - \exp\{-(1.4)^4/4\}$$

(c)

$$P(X > 2|X > 1) = \exp\{\int_1^2 t^3 dt\} = e^{-15/4}$$

5.38 Solution:

$$4x^2 + 4xY + Y + 2 = 0 \Rightarrow x = \frac{1}{2}(\pm\sqrt{y^2 - y - 2} - y)$$

Two roots are real $\Rightarrow y^2 - y - 2 \geq 0 \Rightarrow y \geq 2$ and $y \leq -1$
So $P(real) = 3/5$

5.40 Solution:

$$f_Y(y) = f_X(ln(y))|\frac{1}{y}| = 1/y, 1 < y < e$$

5.41 Solution:

$$f_Y(y) = f_X(\arcsin(y/A))|\frac{1}{a\sqrt{1 - \frac{x^2}{a^2}}}| = \frac{1}{\pi a\sqrt{1 - \frac{x^2}{a^2}}}$$

6.2 Solution:

(a)

$$p(0,0) = (8/13)(7/12) = 14/39$$
$$p(0,1) = p(1,0) = (8/13)(5/12) = 10/39$$
$$p(1,1) = (5/13)(4/12) = 5/39$$

(b)

$$p(0,0,0) = (8/13)(7/12)(6/11) = 28/143$$
$$p(0,0,1) = p(0,1,0) = p(1,0,0) = (8/13)(7/12)(5/11) = 70/429$$
$$p(0,1,1) = p(1,0,1) = p(1,1,0) = (8/13)(5/12)(4/11) = 40/429$$
$$p(1,1,1) = (5/13)(4/12)(3/11) = 5/143$$

6.7 Solution:

$$p(X_1, X_2) = (1-p)^{X_1+X_2}p^2$$

6.8 Solution:

(a)

$$\int_0^\infty \int_{-y}^{y} c(y^2 - x^2)e^{-y}dxdy = 1 \Rightarrow c = 1/8$$

(b)

$$f_Y(y) = \frac{y^3e^{-y}}{6}, 0 < y < \infty$$
$$f_X(x) = \frac{1}{8}\int_{|x|}^{\infty}(y^2 - x^2)e^{-y}dy = \frac{1}{4}e^{-|x|}(1 + |x|)$$

(c)

$$E[X] = \int xp(x)dx = 0$$

6.9 Solution:

(a) Proof:

$$\iint f(x,y)dxdy = \frac{6}{7}\int_0^2 \int_0^1 (x^2 + \frac{xy}{2})dxdy = 1 \blacksquare$$

(b)

$$f_X(x) = \int_0^2 f(x,y)dy = \frac{6}{7}(2x^2 + x)$$

(c)

$$P(X > Y) = \int_0^1 \int_0^x f(x,y)dxdy = \frac{15}{56}$$

(d)

$$P(Y > \frac{1}{2}|X < \frac{1}{2}) = \frac{\int_{1/2}^2 \int_0^{1/2} (x^2 + \frac{xy}{2})dxdy}{\int_0^{1/2}(2x^2 + x)dx} = 69/80$$

(e)

$$E[X] = \int_0^1 xp(x)dx = 5/7$$

(f)

$$E[Y] = \int_0^2 \int_0^1 f(x,y)dxdy = 8/7$$

6.10 Solution:

(a)

$$P(X < Y) = \int_0^\infty \int_0^y e^{-(x+y)}dxdy = 1/2$$

(b)

$$P(X < a) = \int_0^a \int_0^\infty e^{-(x+y)}dydx = 1 - e^{-a}$$