6.13 Solution:

$$P(<5) = \frac{1}{2} \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} = \frac{1}{4}$$

$$P(man) = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$$

6.14 Solution:

$$P\{|Y - X| < a\} = \frac{2}{L^2} \int_0^L \int_y^{\min(y+a,L)} dxdy = \frac{a}{L}(2 - \frac{a}{L}), 0 < a < L$$

6.20 Solution:

$$f_X(x) = xe^{-x}, f_Y(y) = e^{-y}, f(x,y) = xe^{-(x+y)} = f_X(x)f_Y(y)$$

So X and Y are independent.

$$f_X(x) = 2(1-x)$$

$$f_Y(y) = 2y$$

$$f_X(x)f_Y(y) = 4y(1-x) \neq f(x,y)$$

So X and Y are dependent.

6.21 Solution:

(a)

$$\iint 24xydxdy = 24 \int_0^1 \int_0^{1-y} xydxdy = 1$$

(b)

$$E[X] = \int_0^1 x \int_0^{1-x} 24xydydx = 2/5$$

(c)

$$E[Y] = \int_0^1 y \int_0^{1-y} 24xydxdy = 2/5$$

6.22 Solution:

(a) No, $f(x,y)$ does not factor.

(b)

$$f_X(x) = \int_0^1 (x+y)dy = x + 1/2, 0 < x < 1$$

(c)

$$P\{X+Y<1\} = \int_0^1 \int_0^{1-x} (x+y)dydx = 1/3$$

6.23 Solution:

(a)

$$f_X(x) = \int_0^1 12xy(1-x)dy = 6x(1-x), 0 < x < 1$$
$$f_Y(y) = \int_0^1 12xy(1-x)dx = 2y, 0 < y < 1$$
$$f_X(x)f_Y(y) = 12xy(1-x) = f(x,y)$$

So X and Y are independent.

(b)

$$E[X] = \int_0^1 x \int_0^1 12xy(1-x)dydx = 1/2$$

(c)

$$E[Y] = \int_0^1 y \int_0^1 12xy(1-x)dxdy = 2/3$$

(d)

$$Var(X) = \int_0^1 6x^3(1-x)dx - 1/4 = 1/20$$

(e)

$$Var(Y) = \int_0^1 2y^3dy - 4/9 = 1/18$$

6.27 Solution:

$$P\{a\} = \int_0^\infty \int_0^{ay} \lambda_1 e^{-\lambda_1 x}\lambda_2 e^{-\lambda_2 x}dxdy = \frac{\lambda_1 a}{a\lambda_1 + \lambda_2}$$
$$P\{X_1 < X_2\} = P\{X_1/X_2 < 1\} = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

6.28 Solution:

$$f(t) = e^{-x}$$

(a)

$$P\{MJ\} = \frac{1}{2}e^{-t}$$

(b)

$$P\{MJ\} = 1 - 3e^{-2}$$

6.29 Solution:

(a)

$$P\{X > 5000\} = P\{Z > \frac{5000 - 4400}{\sqrt{2(230)^2}}\} = 0.0326$$

(b)

$$P\{X > 2000\} = P\{Z > (2000 - 2200)/230\} = 0.8078$$

6.33 Solution:

(a)

$$p = 1 - e^{-0.2} - 0.2e^{-0.2} - e^{-0.2}(0.2)^2/2!$$

(b)

$$p = 1 - \sum_{i=0}^{2} e^{-0.2}(0.2)^i/i!$$