4.22 Proof: Since $f(x) = \frac{2x-1}{2x(1-x)} = \frac{1}{1-x} - \frac{1}{2x(1-x)}$. Take $a \neq b \in (0,1)$, then $2a-1 \neq 2b-1$, $2a(1-a) \neq 2b(1-b)$ so that $f(a) \neq f(b)$. As a result, f is injective.

Let $y = f(x) = \frac{2x-1}{2x(1-x)}$, $2xy(1-x) = 2x-1 \Leftrightarrow 2xy - 2x^2y = 2x-1 \Leftrightarrow (2y)x^2 + (2-2y) - 1 = 0$. Then we have $x = \sqrt{1 - \frac{1}{2y}}, 0 < x < 1$. So f is surjective.

So we can conclude that f is bijective.$\blacksquare$

4.31 Proof: Since f is a bijection that is increasing, the if $a, b \in A$ and $a > b$, we have $f(a) > f(b)$. Suppose f^{-1} is not increasing on B, then there's at least one pair of $x, y \in B$ such that $f^{-1}(x) \leq f^{-1}(y) \Rightarrow f(f^{-1}(x)) \leq f(f^{-1}(y))$ since f is increasing on A. Because $f^(f^{-1}(x)) = x$, we have $x \leq y$ which is contradict with the assumption.

Thus, we can conclude that f^{-1} is increasing on B.$\blacksquare$

4.35 Proof:

(a) No, suppose $f(x) = x^2, g(x) = \sqrt{x}$, $f(g(y)) = y$ but $f(x)$ is not even surjective.

(b) No, suppose $f(x) = x^2, g(x) = \sqrt{x}$, we have $g(f(x)) : x|x \geq 0 \to y|y \geq 0$ and $f(g(x)) : \mathbb{R} \to x|x \geq 0$. Then if $y < 0$, $g(f(x))$ is not defined.

4.37 Proof: Let $y = f(f(x))$ denote the map $f \circ f$. Since $f \circ f$ is injective, take $a \neq b \in A$, $f(f(a)) \neq f(f(b))$. Take $B' \subseteq B$ to make $f' : A \to B'$ bijective, we can have a inverse function f^{-1} that $f^{-1}(f(f(a))) = f(a)$, thus $f^{-1}(f(f(a))) \neq f^{-1}(f(f(b))) \Rightarrow f(a) \neq f(b)$.

So we can conclude that if $f \circ f$ is injective, f is also injective.$\blacksquare$

4.43 Proof: Suppose A is an finite set, since $B \subset A$, then B must be a finite set as well and $|B| < |A|$ We know that if A, B are finite sets and bijection $f : A \to B$ was established, $|A| = |B|$ which is conflict to the inequity above.

Thus, A must be infinite set.$\blacksquare$