### 0.1 Solution:

$$P(\text{diff}|\text{one on }6) = \frac{P((\text{diff})\ (\text{one on }6))}{P(\text{one on }6)} = \frac{(1/6)((36-6)/36)}{1/6} = 5/6$$

### 0.2 Solution:

$$P = \frac{6}{15}\frac{5}{14}\frac{9}{13}\frac{8}{12} = 0.066$$

**0.3 Solution:** Let $E$ to be the event that the first and the third are white balls. And let $F$ to denote the event that exactly 3 white balls are drew out.

**With replacement:**

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{(2/3)(1/3)(2/3)^2 + (2/3)^3(1/3)}{(2/3)^3} = 2/3$$

**Without replacement:**

$$P(E|F) = \frac{6+6}{\binom{8}{3}\binom{4}{1}} = 3/56 = 0.054$$

### 0.4 Solution:

$$P = \frac{2\cdot 8\cdot 3 + 2\cdot 4\cdot 1}{2\cdot 8\cdot 3 + 2\cdot 4\cdot 1 + 4\cdot 8\cdot 1} = 7/11$$

### 0.5 Solution:

$$P = \frac{13\cdot 12\cdot 11}{39\cdot 13\cdot 12 + 13\cdot 12\cdot 11} = 0.22$$

### 0.6 Solution:

**(a)**

$$P = \frac{0.02}{0.05} = 2/5 = 0.4$$

**(b)**

$$P = \frac{0.02}{0.52} = 1/26 = 0.038$$

### 0.7 Solution:

**(a)**

$$P = \frac{1 + 1/3}{2 + 1/3 + 2/3} = 4/9$$

**(b)**

$$P = \frac{(1/3) \cdot (2/3)}{4/9} = 1/2$$

### 0.8 Solution:

$$P(\text{black}) = (1/2)(1/2) + (1/2)(2/3) = 1/4 + 1/3 = 7/12$$

$$P(\text{first} \mid \text{white}) = \frac{1/4}{1 - 7/12} = \frac{1/4}{5/12} = 3/5$$

### 0.9 Solution:

$$P(\text{all white}) = \sum_{i=1}^{5} \frac{1}{6} \frac{\binom{5}{i}}{\binom{15}{i}} = 5/66$$

$$P(3|\text{white}) = \frac{(1/6)(1/3)}{5/66} = 11/15$$

### 0.10 Solution:

$$P(E) = \sum_{i=1}^{m} P(E \mid i)P(i) = \sum_{i=1}^{m} (1 - p_i)^{n-1} p_i$$

### 0.11 Solution:

**(a)**

$$P = 2/4 = 1/2$$

**(b)**

$$P = \binom{3}{1}/8 = 3/8$$

**(c)**

$$P = 2/3$$

### 0.12 Solution:

**(a)**

$$P = 0.7 \cdot 0.8 + 0.2 \cdot 0.4 + 0.1 \cdot 0.1 = 0.65$$

**(b)**

- **Strong:**

$$P(\text{Strong}) = 0.56/0.65 = 56/65$$

- **Moderate:**

$$P(\text{Moderate}) = 0.08/0.65 = 8/65$$

- **Weak:**

$$P(\text{Weak}) = 0.01/0.65 = 1/65$$

**(c)**

- **Strong:**

$$P(\text{Strong}) = 0.14/1 - 0.65) = 14/35$$

- **Moderate:**

$$P(\text{Moderate}) = 0.12/(1 - 0.65) = 8/35$$

- **Weak:**

$$P(\text{Weak}) = 0.09/(1 - 0.65) = 9/35$$

### 0.13 Solution:

**Claim:** If $P(A) > 0$

$$P(AB|A) \geq P(AB|A \cup B)$$

**Proof:** Since

$$P(AB|A) = \frac{P(AB \cap A)}{P(A)} = \frac{P(AB)}{P(A)}$$

$$P(AB|A \cup B) = \frac{P(AB \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB)}{P(A \cup B)}$$

So

$$\frac{P((AB|A)}{P(AB|A \cup B}) = \frac{P(A \cup B)}{P(A)} = \frac{P(A) + P(B) - P(AB)}{P(A)} \geq 1 \blacksquare$$

Thus, if $P(A) > 0$

$$P(AB|A) \geq P(AB|A \cup B)$$

## 0.14 Solution:

- $$P(A|B) = \frac{P(A)}{P(B)}$$
- $$P(A|B^C) = 0$$
- $$P(B|A) = 1$$
- $$P(B|A^C) = P(B) - P(A)$$