CS/ECE 374 Spring 2017
Homework 9 Problem 1

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Red street in the city Shampoo-Banana can be modeled as a straight line starting at 0. The street has n houses at locations $x_1, x_2, \ldots, x_n$ on the line. The local cable company wants to install some new fiber optic equipment at several locations such that every house is within distance r from one of the equipment locations. The city has granted permits to install the equipment, but only at some m locations on the street given y locations $y_1, y_2, \ldots, y_m$. For simplicity assume that all the x and y values are distinct. You can also assume that $x_1 < x_2 < \ldots < x_n$ and that $y_1 < y_2 < \ldots < y_m$.

- Describe a greedy algorithm that finds the minimum number of equipment locations that the cable company can build to satisfy the desired constraint that every house is within distance r from one of them. Your algorithm has to detect if a feasible solution does not exist. Prove the correctness of the algorithm. One way to do this by arguing that there is an optimum solution that agrees with the first choice of your greedy algorithm.

Solution: Greedy strategy: Take the largest i that location $y[i]$ that $|y[i] - x[1]| < r$, discard all $x[j] \in [y[i] - r, y[i] + r]$, discard $y[i]$ for all $i \leq y$ and recurse.

Proof: Suppose we take $y[j]$ that $j < i$ and $|y[j] - x[1]| < r$, then if there is some $x \in [y[j] + r, y[i] + r]$ other than $x[1]$, there needs at least one more y to cover these x.

Since i is the largest i that location $y[i]$ that $|y[i] - x[1]| < r$, it is impossible to take $y[j]$ that $j > i$ while still cover $x[1]$.

Hence, we conclude that take $y[i]$ has the optimum substructure, and by induction the greedy strategy is correct.

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