CS/ECE 374 Spring 2017 Lanxiao Bai (lbai5)
Homework 2 Problem 1 Renheng Ruan (rruan2)

1. Prove that the following languages are not regular by providing a fooling set. You need to prove an infinite fooling set and also prove that it is a valid fooling set.
   (a) $L = \{0^k 1^k ww \mid 0 \leq k \leq 3, w \in \{0,1\}^+\}$.
   (b) Recall that a block in a string is a maximal non-empty substring of indentical symbols. Let L be the set of all strings in $\{\mathtt{0}, \mathtt{1}\}^*$ that contain two blocks of $\mathtt{0}$s of equal length. For example, L contains the strings $\mathtt{01101111}$ and $\mathtt{01001011100010}$ but does not contain the strings $\mathtt{000110011011}$ and $\mathtt{00000000111}$.
   (c) $L = \{0^{n^3} \mid n \geq 0\}$.
2. Suppose L is not regular. Show that $L \cup L'$ is not regular for any finite language L'. Give a simple example to show that $L \cup L'$ is regular when L' is infinite.

Solution:

1. (a) Let x, y be arbitrary distinct strings in $\mathtt{011}^*$, then $x = \mathtt{011}^i$, $y = \mathtt{011}^j$, $i \neq j$. Then x, y are distinguishes by suffix $\mathtt{1}^i$ because $xz = \mathtt{011}^i\mathtt{1}^i \in L$ but $yz = \mathtt{011}^j\mathtt{1}^i \notin L$. We conclude that $\mathtt{011}^*$ is a fooling set for L. Since $\mathtt{011}^*$ is infinite, L is not regular.
   (b) Let x, y be arbitrary distinct strings in $\mathtt{00}^*\mathtt{10}$, then $x = \mathtt{00}^i\mathtt{10}$, $y = \mathtt{00}^j\mathtt{10}$, $i \neq j$. Then x, y are distinguishes by suffix $z = \mathtt{0}^i$ because $xz \in L$ and $yz \notin L$. We conclude that $\mathtt{00}^*\mathtt{10}$ is a fooling set of L. Since $\mathtt{00}^*\mathtt{10}$ is infinite, L is not regular.
   (c) Let x, y be arbitrary distinct strings in $\{0^{n^3-n} : n \leq 0\}$, then $x = \mathtt{0}^{i^3-i}$, $y = \mathtt{0}^{j^3-j}$, $i \neq j$. Then x, y are distinguishes by suffix o^i because $xz = \mathtt{0}^{i^3} \in L$ and $yz = o^{j^3-j+i} \notin L$. We conclude that $\{0^{n^3-n} : n \leq 0\}$ is a fooling set for L. Since $\{0^{n^3-n} : n \leq 0\}$ is infinite, L is not regular.

2. Since L is not regular, there is a infinite fooling set F that for all arbitrary $x, y \in F$ there is a z that make $xz \in L$ and $yz \notin L$. Then for a finite language L', if $L \cup L'$ is regular, for all $y \in F$, $yz \in L'$, which is impossible since F is infinite.

   As a result, for all finite language L', $L \cup L'$ cannot be regular.

   An example of infinite language L' can be constructed as $L' = \{yz : y \in F, z \in \{w : \forall x \in F, xw \in L\}\}$.

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Describe a context free grammar for the following languages. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

1. $\{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0 \text{ and } i + \ell = j + k\}$.
2. $L = \{0, 1\}^* \setminus \{0^n 1^n \mid n \geq 0\}$. In other words the complement of the language $\{0^n 1^n \mid n \geq 0\}$.

Solution: 1. The context-free grammar can be designed base on the difference of $i + l$ and $j + k$:

$$
\begin{aligned}
&S \to A|B && \{a^i b^j c^k d^l : i + l \neq j + k\} \\
&A \to aA|Ad|aC|Cd|aD|Dd && \{a^i b^j c^k d^l : i + l > j + k\} \\
&B \to bB|Bc|bC|Cc && \{a^i b^j c^k d^l : i + l < j + k\} \\
&C \to \varepsilon|bCc && \{a^i b^j c^k d^l : i + l = j + k\} \\
&D \to \varepsilon|aDd|aCd && \{a^i b^j c^k d^l : i + l = j + k\}
\end{aligned}
$$

2. For $\{0^i 1^j : i, j \geq 0\}$, the context-free grammar can be designed base on the difference of i and j:

$$
\begin{aligned}
&S \to A|B && \{0^i 1^j : i \neq j\} \\
&A \to 1A|A1|1C|C1 && \{0^i 1^j : i < j\} \\
&B \to 0B|B0|0C|C0 && \{0^i 1^j : i > j\} \\
&C \to \varepsilon|0C1|1C0 && \{0^i 1^j : i = j\}
\end{aligned}
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Homework 2 Problem 3 Renheng Ruan (rruan2)

Let $L = \{0^i 1^j 2^k \mid k = 2(i + j)\}$.

1. Prove that L is context free by describing a grammar for L.
2. Prove that your grammar is correct. You need to prove that if $L \subseteq L(G)$ and $L(G) \subseteq L$ where G is your grammar from the previous part.

Solution: ■