

4.5 Solution: All sets that has more than 2 elements can have bijection that is not identity.

4.6 Solution: No, because $f(\text{monday}) = f(\text{friday}) = 6$

4.11 Solution: Take $a, b \in \mathbb{R}$ and $a \neq b$, we have $f(a) \neq f(b) \Leftrightarrow 2a \neq 2b$, so $f : \mathbb{R} \rightarrow \mathbb{R}$ is injective. $\forall x \in \mathbb{R}$, $f^{-1}(x) = \frac{x}{2} \in \mathbb{R}$. So f is surjective. Thus, $f : \mathbb{R} \rightarrow \mathbb{R}$ is bijective.

However, if we take $3 \in \mathbb{Z}$, $f^{-1}(3) = \frac{3}{2} \notin \mathbb{Z}$, so it's not bijective.

4.21 Proof: For an arbitrary set N with positive number of elements, we can choose an arbitrary element $e \in N$ and all the subgroups with even number of elements. Let the map be like

$$f(N) = \begin{cases} N - \{e\} & \text{if } e \in N \\ N \cup \{e\} & \text{if } e \notin N \end{cases}$$

So we can get the sets of subsets that has even number of elements.

Let N_1, N_2 be 2 subsets, we have either

$$N_1 - \{e\} = f(N_1) = f(N_2) = N_2 - \{e\}$$

or

$$N_1 \cup \{e\} = f(N_1) = f(N_2) = N_2 \cup \{e\}$$

we can have $N_1 = N_2$, so the map is injective.

Let N be an element of the range of f . If N contains n , then $N \setminus \{n\}$ is a subset of even size that maps to N under f . If N does not contain n , then $N \cup \{n\}$ is a subset of even size that maps to N under f . Since everything in the image has something in the domain that maps to it, f is surjective.

As a result, the bijection is established. Thus, $|N_{\text{even}}| = |N_{\text{odd}}|$.

4.24 Proof: Let $f(x) = g(x) = x^2$, then $h(x) = x^4$ and $h^{-1}(x) = x^{1/4}$, let $x = 2 \in \mathbb{Z}$, obviously $h^{-1}(2) = 2^{1/4} \notin \mathbb{Z}$. So h may not be surjective.