Math 241 Midterm Exam #3 November 18, 2014

Your name: ______________________ Your NetID: __________

- **No notes, books, or electronics out or hats or sunglasses on during the exam.**

- **You must show your work on all questions. This means show the work that an average Math 241 student would reasonably require.**

- **Do not guess on multiple choice problems—you will receive one point on any multiple choice problem left blank.**

Mark your discussion Section in the table below:

Discussion Section	Instructor	Time (TuTh)	Discussion Section	Instructor	Time (TuTh)	Discussion Section	Instructor	Time (TuTh)
☐ ADA	Hockensmith	8am	☐ BDA	Field	8am	☐ CDA	Compaan	8am
☐ ADB	Gartland	9am	☐ BDB	Field	9am	☐ CDB	Compaan	9am
☐ ADC	Aramyan	10am	☐ BDC	McDonald	10am	☐ CDC	Tian	10am
☐ ADD	Aramyan	11am	☐ BDD	McDonald	11am	☐ CDD	Weigandt	11am
☐ ADE	Yi	12pm	☐ BDE	Wang	12pm	☐ CDE	Tian	12pm
☐ ADF	Yi	1pm	☐ BDF	Ford	1pm	☐ CDF	Gondolo	1pm
☐ ADG	Song	2pm	☐ BDG	Donepudi	2pm	☐ CDG	Golze	2pm
☐ ADH	Tran	3pm	☐ BDH	Romney	3pm	☐ CDH	Golze	3pm
☐ ADI	Tran	4pm	☐ BDI	Romney	4pm	☐ CDI	Gondolo	4pm
☐ ADJ	Huo	9am	☐ BDJ	Pruitt	8am	☐ CDJ	Taha	8am
☐ ADK	Song	9am	☐ BDK	Pruitt	9am	☐ CDK	Taha	9am
☐ ADL	Gartland	10 am	☐ BDL	Wang	10 am	☐ CDL	Hong	10am
☐ ADM	Huo	11am	☐ BDM	Gao	11am	☐ CDM	Hong	11am
☐ ADN	Lu	12pm	☐ BDN	Ford	2pm	☐ CDN	Toprak	12pm
☐ ADO	Lu	1pm	☐ BDO	Donepudi	3pm	☐ CDO	Toprak	1pm
☐ AD1	Wise	11 am	☐ BDP	Gao	4pm	☐ CDP	Pynn-Coates	2pm
☐ AD2	Loeb	9am				☐ CDQ	Pynn-Coates	3pm
☐ AD3	Michiels	3pm						

Question:	1	2	3	4	5	6	7	8	Total
Points:	7	4	8	4	4	8	7	8	50
Score:									

1. (7 points) Rewrite the triple integral

$$\int_0^2 \int_x^2 \int_1^{3-y} z^2 \, dz \, dy \, dx$$

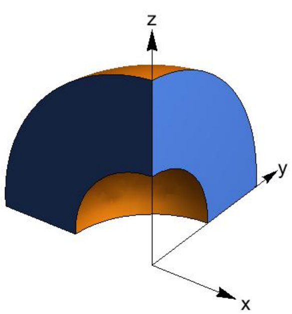
using the different order of integration specified below.

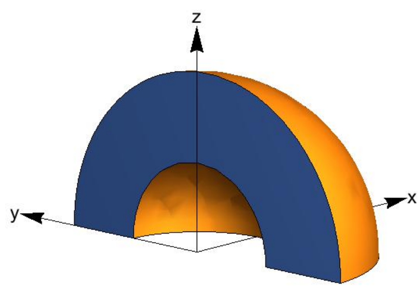
$$\int_0^2 \int_x^2 \int_1^{3-y} z^2 \, dz \, dy \, dx = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \, dx \, dz \, dy$$

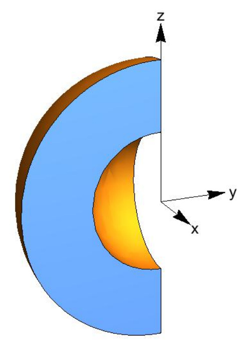
2. (4 points) The volume of a region R is calculated as a triple integral in spherical coordinates as

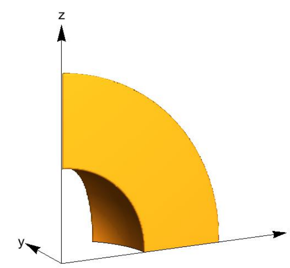
$$\iiint\limits_R dV = \int_1^2 \int_0^{\pi/2} \int_{\pi/2}^{\pi} \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho.$$

Circle the picture of the region R.









3. (8 points) Consider the vector field $\mathbf{F}(x, y, z) = \langle xz, e^z - yz, \cos x \rangle$.

(a) Find $\operatorname{curl} \mathbf{F}$.

$\operatorname{curl} \mathbf{F} = \boxed{\left\langle \quad , \quad , \quad \right\rangle}$

(b) Find $\operatorname{div} \mathbf{F}$.

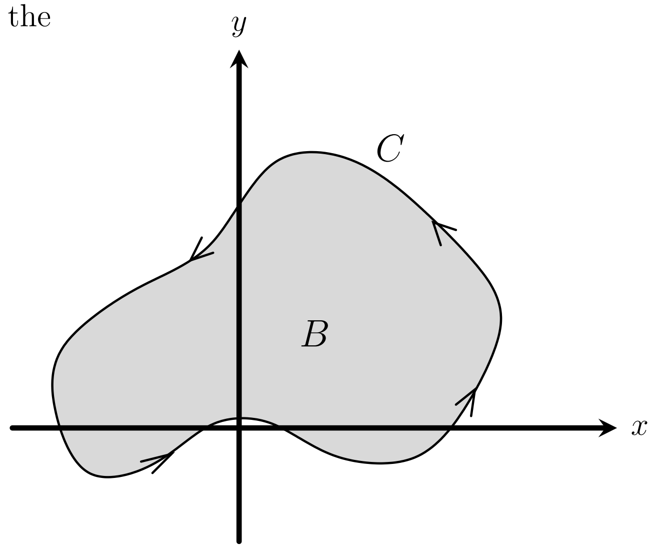
$\operatorname{div} \mathbf{F} = \boxed{\quad}$

(a) Does there exist a function f with $\nabla f = \mathbf{F}$? Circle the correct response.

Yes	No	We do not have enough information

4. (4 points) Let B be the region in the plane bounded by the smooth, simple closed curve C drawn below, where C is oriented counterclockwise.

Which of the integrals below computes the area of B? Circle your response.

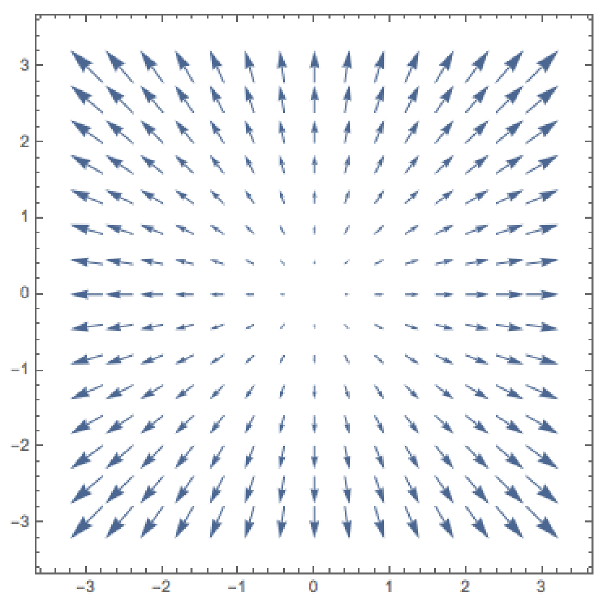


$\int_C 2xe^y dx + x(1+xe^y)dy$	$\frac{1}{2}\int_C ydx + xdy$	$\int_C xdx$	None of these.

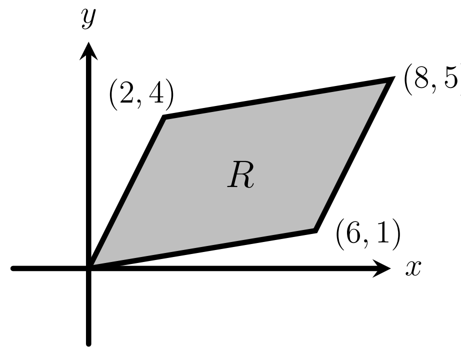
5. (4 points) The vector field $\mathbf{F}$ on $\mathbb{R}^3$ is shown in the xy-plane and looks the same in all other horizontal planes. Circle the best completion of the sentence below.

The divergence of $\mathbf{F}$...

...is positive.	...is negative.
...points up.	...points left.
...is zero.	



6. (8 points) Consider the region R bound by a the parallelogram shown at the right.



(a) Circle the transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ sending the unit square $[0,1] \times [0,1]$ onto the region R.

$T(u,v) =$

$(6u+v, 2u+4v)$	$(6u+2v, u+4v)$	$(6u+v, 4u+2v)$
$(6u+2v, 4u+v)$	$(6u+4v, u+2v)$	

(b) Suppose D is the triangle with vertices $(6,1),(2,4),(8,5)$. Change coordinates *using the transformation T found above* to calculate the integral. Circle the correct answer. **If you left (a) blank, clearly specify one of the choices for T here and calculate using that, assuming it takes the unit square to R.**

$$\iint_D x - y\, dA =$$

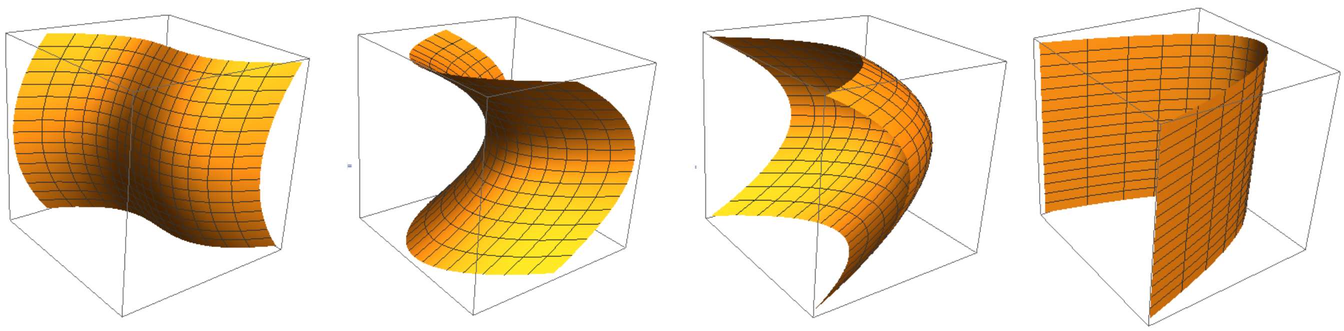
$\displaystyle\int_0^1 \int_0^v F(u,v)\,du\,dv$	$\displaystyle\int_0^1 \int_0^{1-v} F(u,v)\,du\,dv$
$\displaystyle\int_0^1 \int_v^1 F(u,v)\,du\,dv$	$\displaystyle\int_0^1 \int_{1-v}^1 F(u,v)\,du\,dv$

$F(u,v) =$

$88u - 66v$	$110u - 44v$	$16u - 8v$	$4u + 2v$	$40u + 16v$

7. (7 points) Let S be the surface parameterized by $\mathbf{r}(u, v) = \langle v^2 - u^2, u, v \rangle$ with $\{(u, v) \mid -2 \leq u \leq 2 \text{ and } -2 \leq v \leq 2\}$

(a) Circle the picture of S.



(b) The surface area of S is calculated by the integral $A(S) = \int_{-2}^{2} \int_{-2}^{2} F(u, v)\, du\, dv$.
Circle the correct expression for $F(u, v)$.

$F(u, v) =$ $\boxed{u\sqrt{1+u^2+v^2} \qquad \sqrt{1+u^2+v^2} \qquad \sqrt{u^2+v^2} \qquad \sqrt{1+4u^2+4v^2}}$

(c) Circle the correct response:

The integral $\iint\limits_S x^2(y-5)\, dS$ is $\boxed{\text{positive} \qquad \text{negative} \qquad \text{zero}}$

8. (8 points) Find a parameterization $\mathbf{r}(u,v)$ for each of the surfaces described below. Use u, v as your parameters, and specify the domain D of the parameterization.
Important: The domain D **must** be a *rectangle.*

(a) The part of the surface $z = (1-x^2)(4-y^2)$ where $z \geq 0$ and $-2 \leq y \leq 2$.

$$\mathbf{r}(u,v) = \boxed{\left\langle \qquad , \qquad , \qquad \right\rangle}$$

$$D = \left\{ (u,v) \;\middle|\; \boxed{} \leq u \leq \boxed{} \text{ and } \boxed{} \leq v \leq \boxed{} \right\}$$

(b) The part of the cylinder $x^2 + z^2 = 9$ that lies between the planes $y = 0$ and $y = 1$, and for which $z \geq 0$.

$$\mathbf{r}(u,v) = \boxed{\left\langle \qquad , \qquad , \qquad \right\rangle}$$

$$D = \left\{ (u,v) \;\middle|\; \boxed{} \leq u \leq \boxed{} \text{ and } \boxed{} \leq v \leq \boxed{} \right\}$$