[L]Math 347 [C]HW2 [R]Lanxiao Bai(lbai5)

1.21 Solution: $ax^2+bx+c=0, ay^2+by+c=0 \Rightarrow a(x^2-y^2)+b(x-y)=0$, but $a(x^2-y^2)+b(x-y)=0ax^2+bx+c=0, ay^2+by+c=0$. So x that makes $a(x^2-y^2)+b(x-y)=0$ holds does not necessarily make $ax^2+bx+c=0$ hold.

2.4 Solution:

(a) There exists at least one $x \in A$, for all $b \in B$, that $b \leq x$.

(b) For all $x \in A$, there is at least one $b \in B$, $b \leq x$.

(c) There is at least one pair of $x, y \in R$, $(f(x) \neq f(y) \vee x = y$.

(d) There's at least one $b \in R$, for all $x \in R$ such that $f(x) \neq b$.

(e) There's at least one group of $x, y \in R, \epsilon \in P$, for all $\delta \in P$ such that $|x-y| \geq \delta$ or $|f(x)-f(y)| < \epsilon$.

(f) There's at least one $\epsilon \in P$, for all $\delta \in P$ such that there's at least one pair of $x, y \in R$, $|x-y| \geq \delta$ implies $|f(x)-f(y)| < \epsilon$.

2.21 Solution: Negation: There exists at least one $n \in Z, n > 0$, that for all $x \in R, x > 0)$ that $x \geq 1/n$. And the original statement is true.

2.28 Solution:

(a) $x^4y + ay + x = 0 \Leftrightarrow y(x^4 + a) = -x$, that is saying, if we wang to show the equation does not hold for every a and x, we can just construct a pair of (x, a) that $x^4 + a = 0$ and $x \neq 0$. And it can be verified that $x = 1, a = -1$ satisfy this requirement. As a result, the statement is proved to be false.

(b) Since $y(x^4 + a) = -x$, if $x^4 + a = 0$, then the equation holds only when $x = 0$. So it can't be 0. If not, $y = -x/(x^4 + a) \in R$ is guaranteed by the closure of real number. Since $x^4 \geq 0$, just $a > 0$ can make sure that $x^4 + a > 0$.

Thus, the set is $\{a \in R | a > 0\}$.

2.31 Solution:

(a) Believable

(b) Believable

(c) Not believable

(d) Not believable

2.34 Solution:

(a) **Claim:** It is true that if $n \in N$ and $n^2 + (n+1)^2 = (n+2)^2$, then $n = 3$.

Proof: $n^2 + (n+1)^2 = (n+2)^2 \Leftrightarrow 2n^2 + 2n + 1 = n^2 + 4n + 4 \Leftrightarrow n^2 - 2n - 3 = 0$, thus $(n-3)(n+1) = 0$, since $n \in N$, $n \neq -1$. As a result, $n = 3$.

(b) **Claim:** It is true that $\forall n \in N$, it is false that $(n-1)^3 + n^3 = (n+1)^3$.

Proof:$(n-1)^3 + n^3 = (n+1)^3 \Leftrightarrow n^2(n-6) = 2$. Since we can only factorize $2 = 2 \times 1$. It is only possible that $n^2 = 2 \wedge n - 6 = 1$ or $n^2 = 2 \wedge n - 6 = 1$ to make this equation holds. However, neither group of equations has solution.

Thus it is true that $\forall n \in N$, it is false that $(n-1)^3 + n^3 = (n+1)^3$.

2.35 Proof:

Prove sufficiency first:

If $x, y \in R, x \neq y$ and $(x+1)^2 = (y+1)^2$, then $x^2+2x+1 = y^2+2y+1 \Leftrightarrow x^2+2x-y^2-2y = 0$. Thus $(x+y+2)(x-y) = 0$. Since $x \neq y$, $x+y+2 = 0$, and $x + y = -2$

Then we can prove necessity:

If $x+y = -2 \wedge x \neq y$, $x+y+2 = 0$ and $x-y \neq 0$, thus $(x+y+2)(x-y) = 0 \Leftrightarrow x^2 + 2x = y^2 + 2y \Leftrightarrow x^2 + 2x + 1 = y^2 + 2y + 1 \Leftrightarrow (x+1)^2 = (y+1)^2$.

Thus, $x, y \in R, x \neq y$, $(x+1)^2 = (y+1)^2 \Leftrightarrow x + y = -2$.

If $x = y$ is possible, although $x + y = -2 \Rightarrow (x+1)^2 = (y+1)^2$, but $(x+1)^2 = (y+1)^2$ may not imply $x + y = -2$.

2.41 Solution: Since a k-cycle permutation can guarantee that k people get wrong hats and the minimum number of people required to form a cycle is 2, thus the interval is $2 \leq k \leq n$. Also when $k = 0$, no one have wrong hat is obviously true.

Thus $k = 0$ or $2 \leq k \leq n$ if and only if k people get wrong hat.