4.1 Solution: It is possible that $X = -2, -1, 0, 1, 2, 4$.

$$
\begin{aligned}
P(-2) &= \frac{8}{14}\frac{7}{13} = 4/13 \\
P(-1) &= \frac{8}{14}\frac{2}{13} + \frac{2}{14}\frac{8}{13} = 16/91 \\
P(0) &= \frac{2}{14}\frac{1}{13} = 1/91 \\
P(1) &= \frac{4}{14}\frac{8}{13} + \frac{8}{14}\frac{4}{13} = 32/91 \\
P(2) &= \frac{4}{14}\frac{2}{13} + \frac{2}{14}\frac{4}{13} = 8/91 \\
P(4) &= \frac{4}{14}\frac{3}{13} = 6/91
\end{aligned}
$$

4.4 Solution: Since there are 5 men and 5 women, $\max(X) = 6$.

$$
\begin{aligned}
P\{X = 1\} &= 5\frac{9!}{10!} = 1/2 \\
P\{X = 2\} &= 5 \cdot 5\frac{8!}{10!} = 5/18 \\
P\{X = 3\} &= 5 \cdot 4 \cdot 5\frac{7!}{10!} = 5/36 \\
P\{X = 4\} &= 5 \cdot 4 \cdot 3 \cdot 5\frac{6!}{10!} = 5/84 \\
P\{X = 5\} &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 5\frac{5!}{10!} = 5/252 \\
P\{X = 6\} &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5\frac{4!}{10!} = 1/252
\end{aligned}
$$

4.5

Solution: If n is even, we have possibilities of $(0, n), (1, n-1), (2, n-2), \cdots, (n/2, n/2), \cdots, (n-2, 2), (n-1, 1), (n, 0)$.

If n is odd we have possibilities of $(0, n), (1, n-1), (2, n-2), \cdots, (m-1, m), (m, m-1), \cdots, (n-2, 2), (n-1, 1), (n, 0)$.

Thus, $X \in \{0, 1, 2, 3, ..., n\}$.

4.13

Solution:

$$P\{X=0\} = 0.7 \cdot 0.4 = 0.28$$
$$P\{X=500\} = 0.3 \cdot 0.5 \cdot 0.4 + 0.7 \cdot 0.5 \cdot 0.6 = 0.27$$
$$P\{X=1000\} = 0.3 \cdot 0.5 \cdot 0.4 + 0.7 \cdot 0.5 \cdot 0.6 + 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 = 0.315$$
$$P\{X=1500\} = 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 \cdot 2 = 0.09$$
$$P\{X=2000\} = 0.3 \cdot 0.5 \cdot 0.6 \cdot 0.5 = 0.045$$

4.14

Solution:

$$P\{X=0\} = (1/5)((1/4) + (2/4) + (3/4) + (4/4)) = 1/2$$
$$P\{X=1\} = (1/5)((3/4)(1/3) + (2/4)(2/3) + (1/4)(3/3)) = 1/6$$
$$P\{X=2\} = (1/5)((3/4)(2/3)(1/2) + (2/4)(1/3)(2/2)) = 1/12$$
$$P\{X=3\} = (1/5)(3/4)(2/3)(1/2) = 1/20$$
$$P\{X=4\} = 1/5$$

4.17 Solution:

(a)

$$P\{X=1\} = 1/2 - 1/4 = 1/4$$
$$P\{X=2\} = 11/12 - 3/4 = 1/6$$
$$PX = 3 = 1 - 11/12 = 1/12$$

(b)

$$P(1/2 < X < 3/2) = 5/8 - 1/8 = 1/2$$

4.21 Solution:

(a)

$$E[X] \geq E[Y]$$

because the bus with higher weight is more likely to be chosen for X.

(b)

$$E[X] = \sum xp(x) = 40\frac{40}{148} + 33\frac{33}{148} + 25\frac{25}{148} + 50\frac{50}{148} = 39.28$$

$$E[Y] = \sum yp(y) = 148\frac{1}{4} = 37$$

4.23 Solution:

(a) Suppose the best strategy is to buy k ounces at the start of the week with lower price and sell them at last with higher price. Since I have only 1000 dollars, so $0 \leq k \leq 500$.

As a result, we can expect to have

$$E[X] = \frac{1}{2}(1000 - 2k + 4k + 1000 - 2k + k) = 1000 + k/2$$

So when $k = 500$, expectation reaches its maximum value $E[X]_{\max} = 1250$.

Thus, the best strategy is to buy 500 ounces at the beginning of the week and then sell them all out at last.

(b) Similarly, we can calculate the expectation

$$E[Y] = \frac{1}{2}(k + (1000 - 2k)/1 + k + (1000 - 2k)/4) = 625 - k/4$$

So when $k = 0$, $E[Y]_{\min} = 625$.

So the best strategy is to buy all commodity at last.

4.32 Solution: Since any person has 0.1 probability to be ill, then

$$P(\text{has positive}) = 1 - (0.9)^{10}$$

$$E[X] = (1 + 10)(0.9)^{10} + 0(1 - (0.9)^{10}) = 3.835$$

4.35 Solution:

(a) Since $P(\text{same}) = 2 \cdot \frac{1}{2} \cdot \frac{4}{9} = 4/9$, $P(\text{Not same}) = 1 - 4/9 = 5/9$,

$$E[X] = 1.1 \cdot \frac{4}{9} + (-1.0) \cdot \frac{5}{9} = -\frac{0.6}{9} = -\frac{1}{15}$$

(b)

$$\text{Var}(X) = E[X^2] - (E[X])^2 = (1.1^2 \cdot \frac{4}{9} + (-1.0)^2 \cdot \frac{5}{9}) - (-1/15)^2 = 1.089$$

4.37 Solution:

$$\text{Var}[X] = E[X^2] - (E[X])^2 = (40^2 \frac{40}{148} + 33^2 \frac{33}{148} + 25^2 \frac{25}{148} + 50^2 \frac{50}{148}) - (39.28)^2 = 82.2$$

$$\text{Var}[Y] = E[Y^2] - (E[Y])^2 = ((40^2 + 33^2 + 25^2 + 50^2)\frac{1}{4}) - (37)^2 = 84.5$$