1. For projection matrix P,

$$\lambda^2 x = P^2 x = Px = \lambda x$$

leads to $\lambda_1 = 1, \lambda_2 = 0$.

2. Since Householder matrix is symmetric,

$$\begin{aligned}
H^T H &= HH \\
H^T H &= (I - 2\frac{vv^T}{v^T v})(I - 2\frac{vv^T}{v^T v}) \\
&= (I^T - 2\frac{vv^T}{v^T v})(I - 2\frac{vv^T}{v^T v}) \\
&= I - 2\frac{vv^T}{v^T v} - 2\frac{vv^T}{v^T v} + 4(\frac{vv^T}{v^T v})(\frac{vv^T}{v^T v}) \\
&= I
\end{aligned}$$

we see that Householder is an orthogonal matrix, and as a result, the eigenvalues are $\lambda_1 = 1, \lambda_2 = -1$.