

Suppose we need to distribute a message to all the nodes in a rooted tree. Initially, only the root node knows the message. In a single round, any node that knows the message can forward it to at most one of its children. Design an algorithm to compute the minimum number of rounds required for the message to be delivered to all nodes in a given tree. See figure below for an example. Assume that the tree is binary (number of children is at most 2).

Solution: Let T be the input tree. For each node v in T , let $\text{MinRounds}(v)$ denote the minimum number of rounds required, after v learns the message, to inform every descendant of v . And we want to compute $\text{MinRounds}(\text{root}(T))$. The following recursive algorithm evaluates this function recursively, essentially by memoizing $\text{MinRounds}(v)$ at the parent of v , in the temporary variables l and r .

```
MINROUNDS(v):
  if  $v$  is a leaf
    return 0
  else if  $v.\text{right} = \text{NULL}$ 
    return 1 + MINROUNDS( $v.\text{left}$ )
  else if  $v.\text{left} = \text{NULL}$ 
    return 1 + MINROUNDS( $v.\text{right}$ )
  else
     $l \leftarrow \text{MINROUNDS}(v.\text{left})$ 
     $r \leftarrow \text{MINROUNDS}(v.\text{right})$ 
    if  $l < r$ 
      return  $r + 1$ 
    else if  $l > r$ 
      return  $l + 1$ 
    else
      return  $l + 2$ 
```

Run Time: $O(n)$

The solution template is taken from <https://courses.engr.illinois.edu/cs473/homework/hw1-sol.pdf>.

