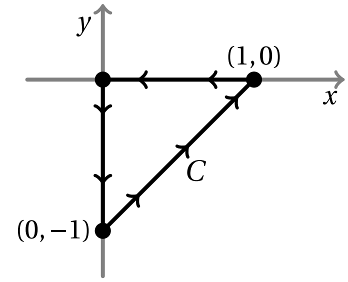
1. Let C denote the curve pictured at right, with the orientation shown.



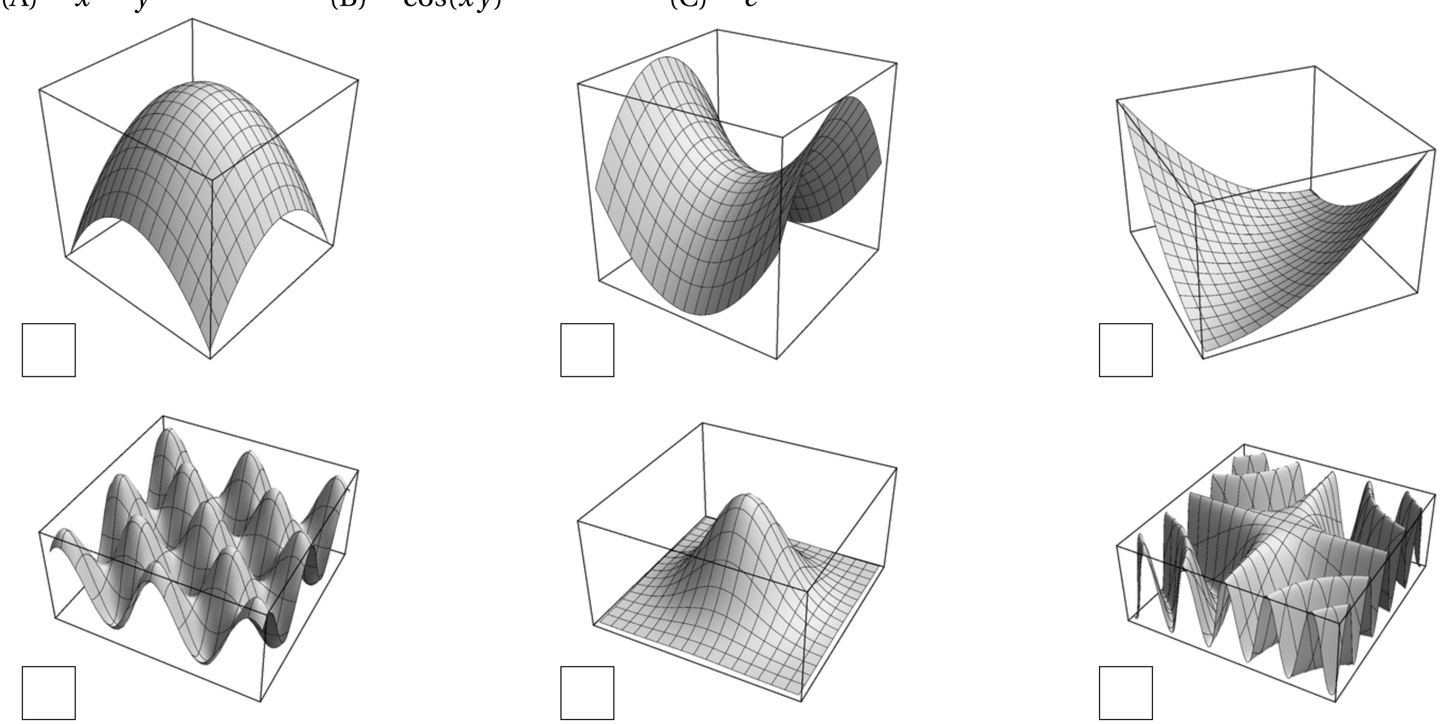
(a) For $\mathbf{F}(x,y) = \langle xy,\, 0\rangle$, compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ directly. **(3 points)**

$\int_C \mathbf{F}\cdot d\mathbf{r} =$

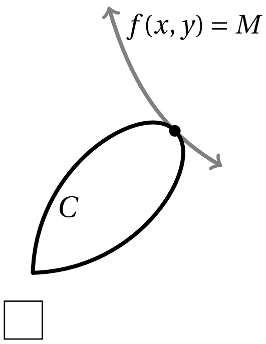
(b) Check your answer to part (a) using Green's Theorem. **(3 points)**

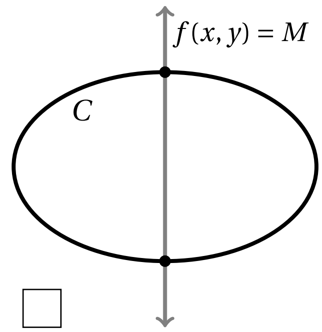
2. For each function label its graph from among the options below: **(2 points each)**

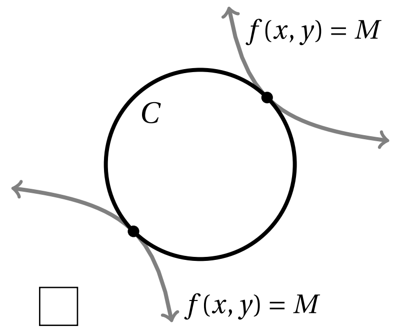
(A) $x^2 - y^2$ (B) $\cos(xy)$ (C) $e^{-(x^2+y^2)}$

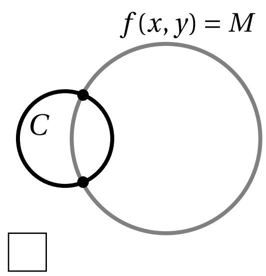


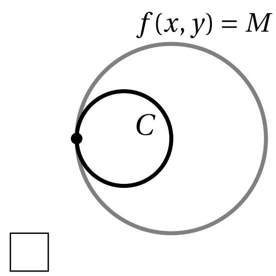
3. (a) Each picture below depicts both (i) a constraint curve C defined by $g(x, y) = 1$ for a function $g(x, y)$, and (ii) a level curve $f(x, y) = M$ of a function $f(x, y)$. Mark the boxes of **all and only those pictures** for which M could be the maximum value of $f(x, y)$ subject to the constraint $g(x, y) = 1$. [In every picture, you should assume that ∇f is always nonzero.] **(2 points)**

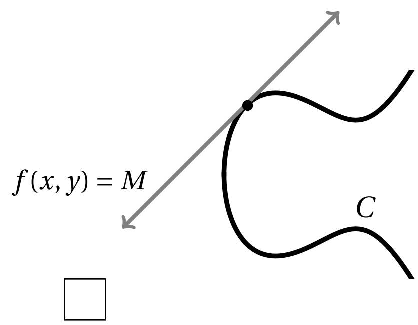












(b) Suppose a function $f(x, y)$ attains its minimum value, subject to the constraint $2x^2 + 2xy^2 + y^3 = 5$, at $(x, y) = (1, 1)$. Assuming that $\nabla f(1, 1) \neq \langle 0, 0 \rangle$, find a nonzero vector $\mathbf{v}$ parallel to $\nabla f(1, 1)$. **(3 points)**

$\mathbf{v} = \Big\langle \qquad , \qquad \Big\rangle$

4. Suppose $f(x, y)\colon \mathbb{R}^2 \to \mathbb{R}$ has the table of values and partial derivatives shown at right. For $x(s, t) = s + 2t$ and $y(s, t) = s^2 - t$, let $F(s, t) = f\big(x(s, t), y(s, t)\big)$ be their composition with f. Compute $\dfrac{\partial F}{\partial t}(2, 1)$. **(3 points)**

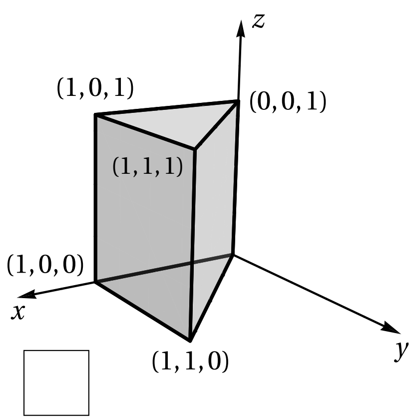
(x, y)	$f(x, y)$	$\dfrac{\partial f}{\partial x}$	$\dfrac{\partial f}{\partial y}$
$(2, 1)$	0	7	6
$(2, -1)$	-12	7	-1
$(4, 3)$	7	3	1
$(5, 3)$	19	-8	5

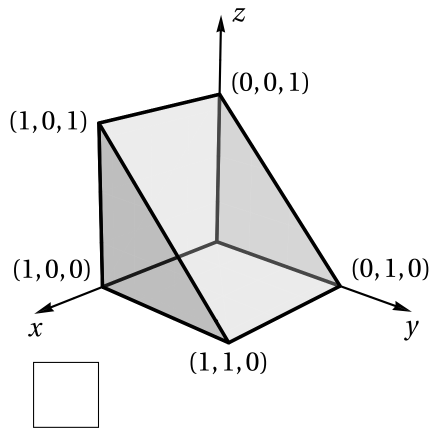
$\dfrac{\partial F}{\partial t}(2, 1) =$

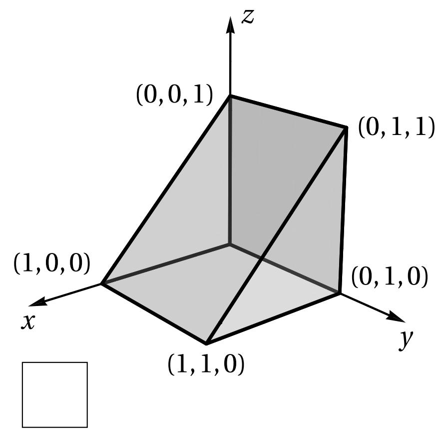
5. For each of the integrals

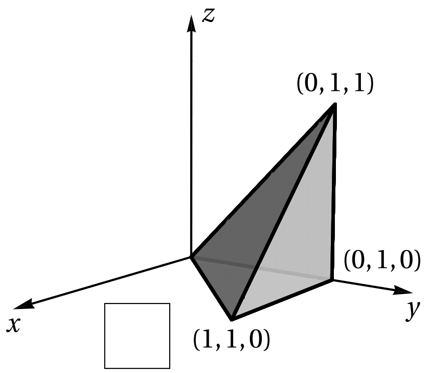
$$(A)\int_0^1\int_0^1\int_0^{1-x} f(x,y,z)\;dz\,dy\,dx \qquad (B)\int_0^1\int_0^1\int_0^{y} f(x,y,z)\;dx\,dy\,dz \qquad (C)\int_0^1\int_x^1\int_0^{y-x} f(x,y,z)\;dz\,dy\,dx$$

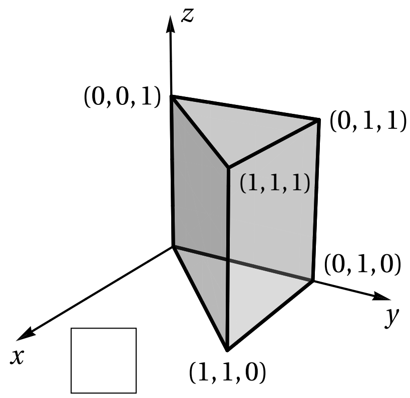
label the solid corresponding to the region of integration below. **(1 point each)**

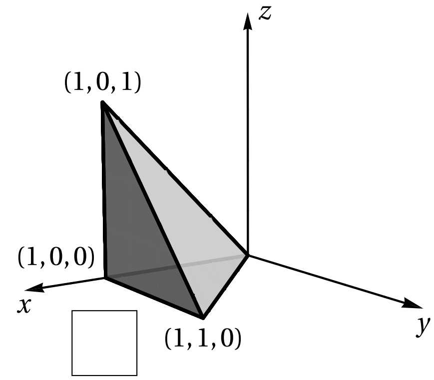




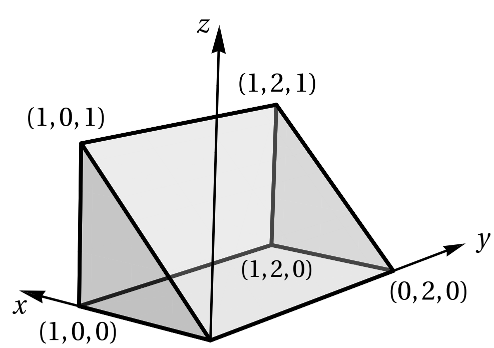






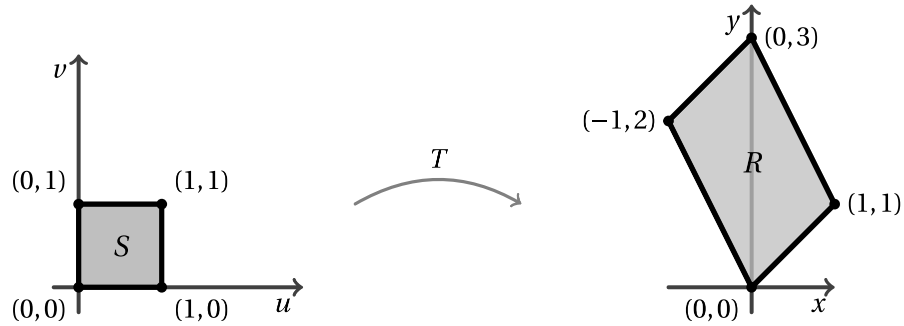


6. Compute the mass of solid region E shown at right if the mass density is $\rho(x,y,z) = z$. **(4 points)**



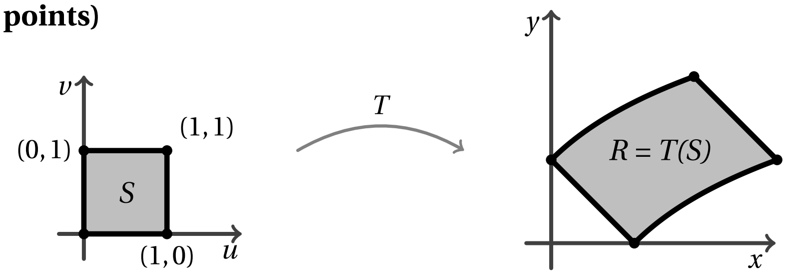
Mass =

7. (a) Let R be the region shown below right. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [0,1] \times [0,1]$ to R. **(3 points)**



$T(u, v) = \Big\langle \qquad , \qquad \Big\rangle$

(b) Consider the transformation $T(u,v) = \left(e^u - v, u + v\right)$ whose behavior is depicted below. Compute $\iint_R 3\, dA$ via an integral over S. **(3 points)**



$\iint_R 3\, dA =$

8. Let S be the surface in $\mathbb{R}^3$ which is the boundary of the solid cube $D = \{-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1\}$. For $\mathbf{F}(x, y, z) = \langle yz^2 + e^z + x,\ ze^z + x + y,\ xe^x + xy + z \rangle$, compute $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS$ by any valid method, where $\mathbf{n}$ is the outward-pointing unit normal vector field. **(4 points)**

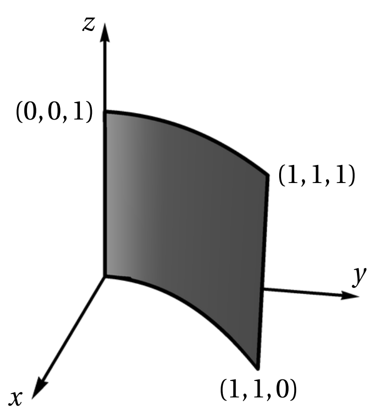
$\iint_S \mathbf{F} \cdot \mathbf{n}\ dS =$

9. Consider the region R below the surface $z = 1 - x^2 - y^2$ and above the xy-plane. Compute the volume of R. **(5 points)**

Volume =

10. For each surface S in parts (a) and (b) give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D and call your parameters u and v.

(a) The portion of the surface $x = y^2$ shown at left. **(2 points)**



$D = \{ \qquad \}$

$\mathbf{r}(u, v) = \langle \qquad , \qquad , \qquad \rangle$

(b) The portion of the cylinder $x^2 + z^2 = 1$ between the planes $y = 0$ and $y = 2$. **(3 points)**

$D = \{ \qquad \}$

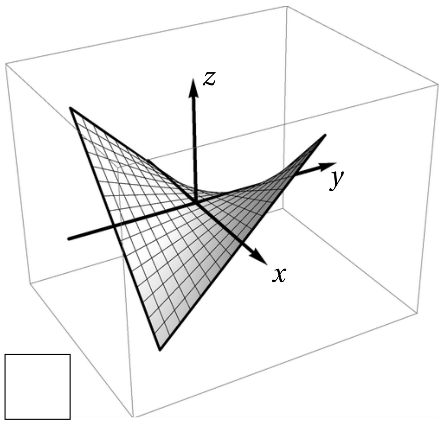
$\mathbf{r}(u, v) = \langle \qquad , \qquad , \qquad \rangle$

(c) Let M be the surface in part (b). Is the surface integral $\iint_M y\, dS$: negative zero positive

Circle your answer. **(1 point)**

11. Let S be the surface parameterized by $\mathbf{r}(u,v) = \langle u,\ uv,\ v\rangle$ for $-1 \le u \le 1$ and $-1 \le v \le 1$.

(a) Mark the picture of S below. **(2 points)**



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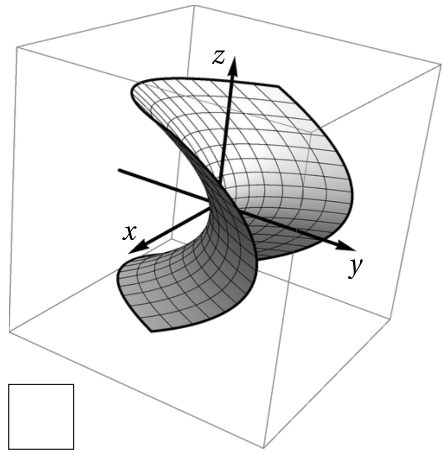

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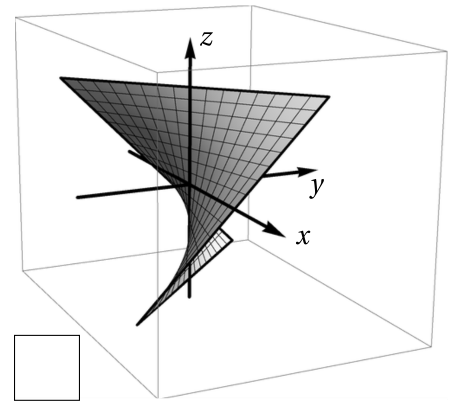

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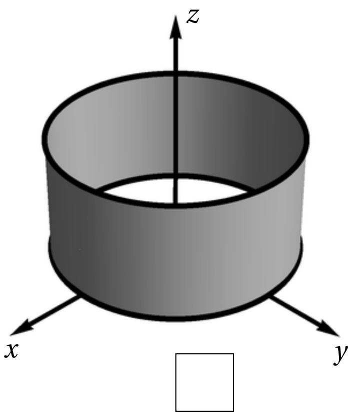
(b) Completely setup, but do not evaluate, the surface integral $\iint_S x^2\, dS$. **(5 points)**

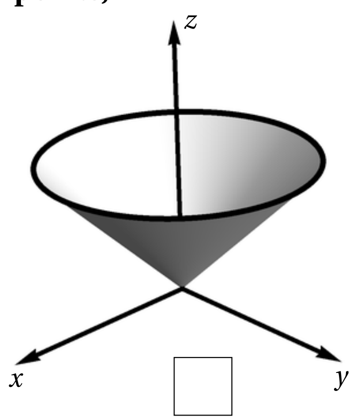
(c) Find the tangent plane to S at $(0,0,0)$. [You *must* show work that justifies your answer.] **(2 points)**

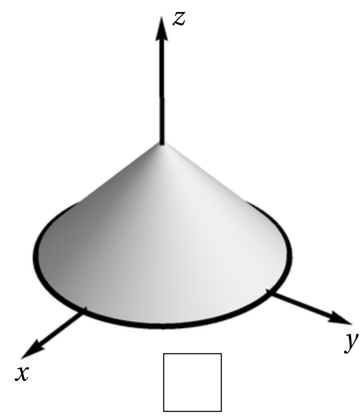
Equation: $\square\, x + \square\, y + \square\, z = \square$

12. Consider the surface S parameterized by $\mathbf{r}(u, v) = \langle \cos u,\ \sin u,\ v\rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 1$.

(a) Mark the picture of S below. **(2 points)**





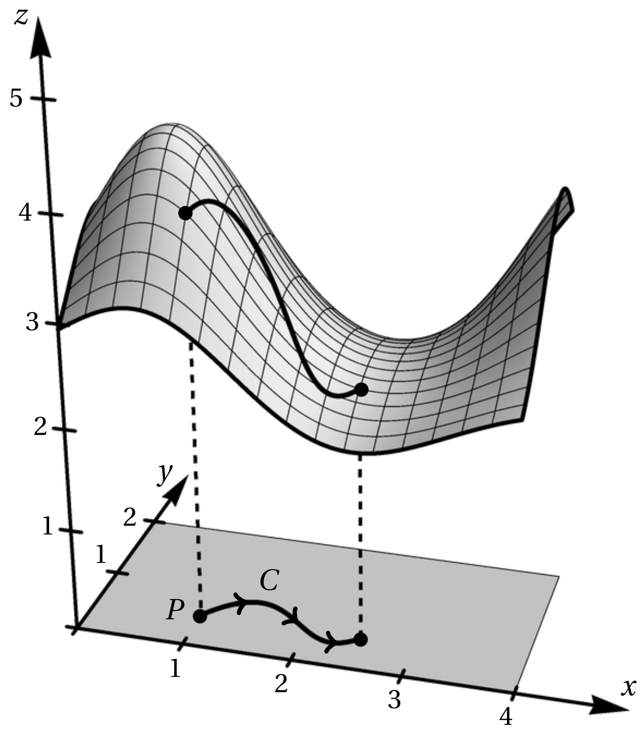


(b) Consider the vector field $\mathbf{F} = \langle yz,\ -xz,\ 1\rangle$ which has $\operatorname{curl}\mathbf{F} = \langle x,\ y,\ -2z\rangle$. Directly evaluate $\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\ dS$ via the given parameterization, where $\mathbf{n}$ is the outward normal vector field. **(4 points)**

$\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\ dS =$

(c) Check your answer in (b) using Stokes' Theorem. **(4 points)**

13. Consider the function $f(x,y)$ on the rectangle $D = \{0 \le x \le 4 \text{ and } 0 \le y \le 2\}$ whose graph is shown below right. For each part, circle the best answer. **(1 point each)**



(a) At the point $P = (1, 0.5)$ is $\dfrac{\partial f}{\partial y}$:

negative	zero	positive

(b) At P is $\dfrac{\partial^2 f}{\partial x^2}$:

negative	zero	positive

(c) How many critical points does f have in the *interior* of D?

0	1	2	3	4

(d) The integral $\iint_D f(x,y)\, dA$ is:

negative	zero	positive

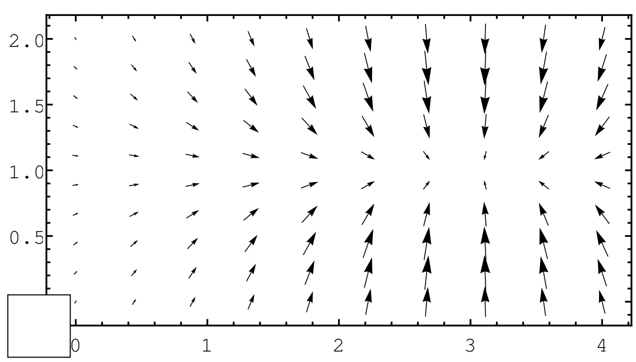
(e) For the curve C shown, the line integral $\int_C \nabla f \cdot d\mathbf{r}$ is:

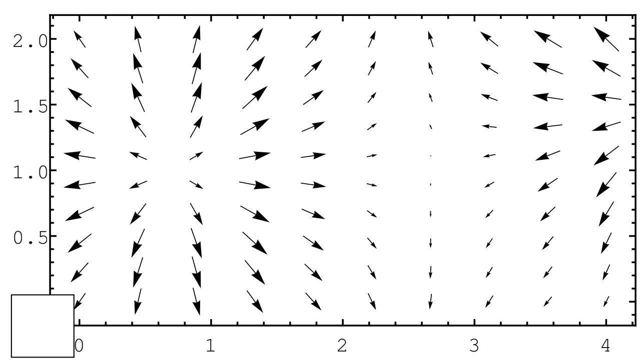
-3	-1.5	0	1.5	3

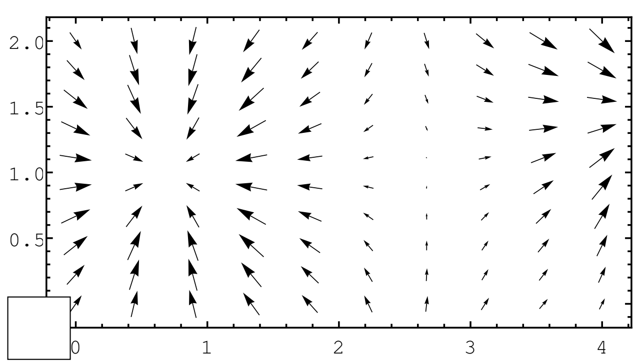
(f) The line integral $\int_C f\, ds$ is:

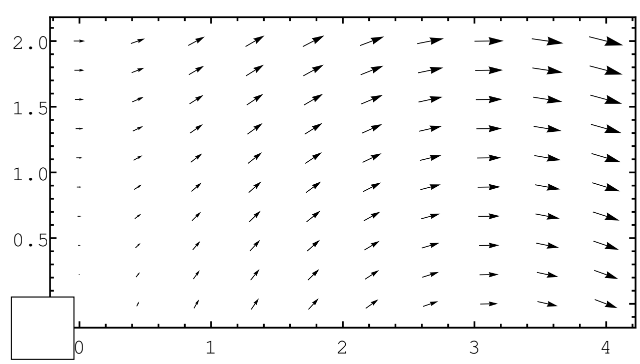
negative	zero	positive

(g) Mark the plot of the vector field ∇f.





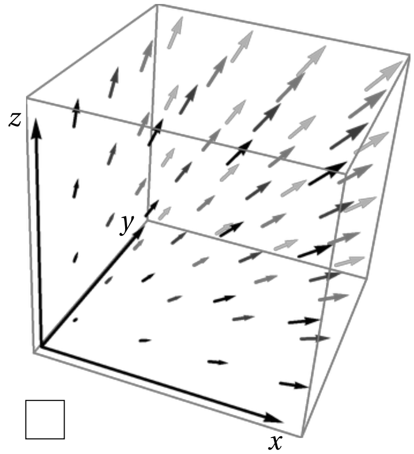


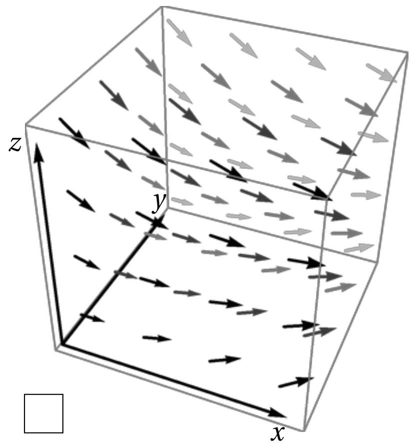


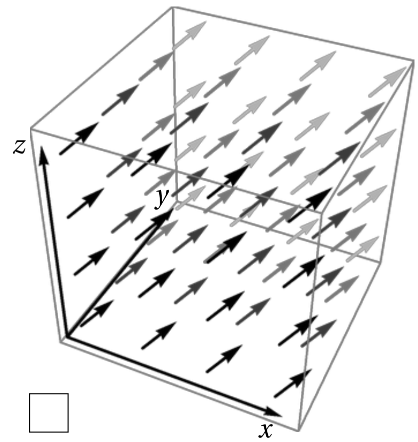
14. For each problem, circle the best answer. **(1 point each)**

(a) Consider the vector field $\mathbf{F} = \langle 1,\ x,\ -z \rangle$. The vector field $\mathbf{F}$ is: | conservative | not conservative |

(b) Mark the plot of $\mathbf{F}$ on the region where each of x, y, z is in $[0, 1]$:

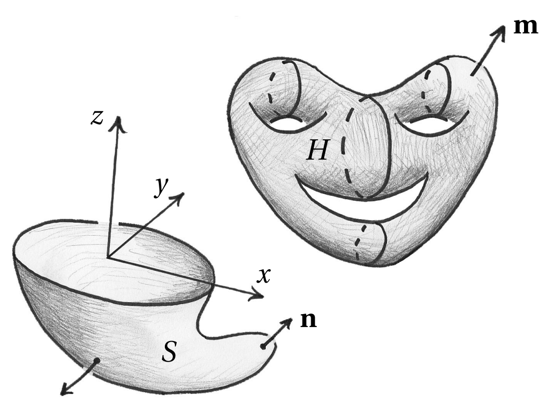






(c) For the leftmost vector field in part (b) is the divergence: | negative | zero | positive |

Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. Let $\mathbf{G} = \langle x,\ y,\ z \rangle$.



(d) The flux $\iint_H \mathbf{G} \cdot \mathbf{m}\, dS$ is:

| negative | zero | positive |

(e) The flux $\iint_S \mathbf{G} \cdot \mathbf{n}\, dS$ is: | negative | zero | positive |

(f) The flux $\iint_S (\operatorname{curl} \mathbf{G}) \cdot \mathbf{n}\, dS$ is: | negative | zero | positive |

Scratch work may go here.

Scratch work may go here, also.

1 Rubric

1. (a) 3 points: 1 point each for correctly calculating the contribution from each side. [Some work must exist and be roughly consistent with answers.]

   (b) 1 point for knowing to compute $\iint_R \left(\partial Q/\partial x - \partial P/\partial y\right)\, dA$.
   1 point for correctly computing $\partial Q/\partial x - \partial P/\partial y$.
   1 point for calculating integral which matches answer in first part.

2. 2 points for each correct answer, no partial credit.

3. (a) M/C: 2 points if **exactly** the correct four boxes are checked, 0 points otherwise.

   (b) 1 point for recognizing that ∇f should be parallel to ∇g.
   1 point for choosing a correct constraint function $g(x, y)$.
   1 point for correctly calculating $\nabla g(1,1)$.

4. 1 point for computing x_t and y_t.
   1 point for evaluating f_x and f_y at correct point.
   1 point for calculations.

5. 1 point for each correct answer, no partial credit.

6. 1 point for having z as the integrand.
   1 point for outer pair of limits.
   1 point for inner limits (i.e. finding the equation of the tilted plane).
   1 point for correct answer (only available if limits are correct).

7. (a) 1 pt for computing images of some points under T or any other reasonable approach. 1pt for each pair of bounds.

   (b) 1 pt for calculating the Jacobian matrix J.
   1 pt for including det of same in dA.
   1 pt for computations.

8. 2 points for applying Divergence Theorem.
   1 point for calculating $\text{div}(\mathbf{F}) = 3$.
   1 point for correctly evaluating $\iiint_D 3\, dV = 24$.

9. 1 point for using cylindrical coordinates or setting up in rectangular and switching to polar.
   1 point each for limits of r and z; subtract 1 point if θ limits are wrong.
   1 point for the r factor in dV.
   1 point for correct answer (only available setup is correct).

10. (a) 1 point for picking either $\{y, z\}$ or $\{x, z\}$ and solving correctly for the remaining coordinate function.
    1 point for correct D.

    (b) 1 point for taking y and angle about the y axis as the parameters.
    1 point for correct $\mathbf{r}$.
    1 point for D.
    Award 0 points for ±squareroot "parameterizations".
    The answer $\mathbf{r}(u, v) = \langle \cos u, \sin u, v\rangle$ is at most one point.

    (c) 1 point for correct answer.

11. (a) 2 points for correct answer.

(b) 1 point for trying to compute $\mathbf{r}_u \times \mathbf{r}_v$.
1 point for replacing x^2 with u^2 in integrand.
1 point for taking $dS = |\mathbf{r}_u \times \mathbf{r}_v|\, du\, dv$.
1 point for limits of integration.
1 point for computations.

(c) 1 point for using $\mathbf{r}_u \times \mathbf{r}_v$ as the normal.
1 point for answer.

12. (a) 2 points for correct answer.

(b) 1 point for computing $\mathbf{r}_u \times \mathbf{r}_v$.
1 point for knowing integrand is $\mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)$
1 point for writing $\mathbf{F}$ in terms of u and v.
1 point for computations/answer.

(c) 1 point for successfully parameterizing a circle and knowing basic formula for line integral.
1 point for finding that the line integral around the bottom is 0.
1 point for finding that the line integral around the top is $\pm 2\pi$.
1 point for correct answer.