1 Compactness

We assume there are countably many sentence symbols.

A finite set $\{\varphi_1, \cdots, \varphi_n\}$ of wff's is satisfiable iff $\varphi_1 \wedge \cdots \wedge \varphi_n$ is not a contradiction.

We say that $\{\varphi_1, \cdots, \varphi_n\}$ is consistent when it's satisfiable.

We say that $\mathcal{T}$ of wff's is consistent when $\mathcal{T}$ is finitely satisfiable.

i.e. $\varphi_1 \wedge \cdots \wedge \varphi_n$ is not contradiction for any $\varphi_i \in \mathcal{T}$.

Let $\mathcal{T}$ be a consistent set of wff's. We extend $\mathcal{T}$ to a maximal consistent set Δ of wff's.

Lemma 1.0.1 *Let S be a consistent set of wff's, and let φ a wff. Then $S \cup \{\varphi\}$ or $S \cup \{\neg\varphi\}$ is consistent.*

Lemma 1.0.2 *Let S be a consistent set of wff's, then the following are equivalent:*

1. *S is a maximal consistent set of wff's*

2. *For any wff φ either $\varphi \in S$ or $\neg\varphi \in S$*