1. Considering the symmetric difference quotient approximation of 1st order derivative
$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$
Accordingly, we have the 2nd order approximation
$$\begin{aligned} f''(x) &\approx \frac{f'(x+h) - f'(x-h)}{2h} \\ &\approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \end{aligned} \tag{1}$$
As a result, we see that
$$\begin{aligned} f'''(x) &\approx \frac{f''(x+h) - f''(x-h)}{2h} \\ &\approx \frac{\frac{f(x+2h)-2f(x+h)+f(x)}{h^2} - \frac{f(x)-2f(x-h)+f(x-2h)}{h^2}}{2h} \\ &= \frac{f(x+2h) - 2f(x+h) + f(x) - (f(x) - 2f(x-h) + f(x-2h))}{2h^3} \\ &= \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3} \end{aligned} \tag{2}$$
Thus, we see that it is an approximation of 3rd order derivative by symmetric difference quotient.

2. By Taylor's Theorem, we have that
$$f(x+h) = f(x) + f'(x)h + f''(x)h^2/2 + f'''(x)h^3/6 + f^{(4)}(x)h^4/24 + f^{(5)}(\theta)h^5/120$$
then we can calculate (2) by
$$\begin{aligned} (2) &= \frac{[f(x+2h) - f(x+h)] + [f(x-h) - f(x+h)] + [f(x-h) - f(x-2h)]}{2h^3} \\ &= \frac{2f'''(x)h^3 + Mh^5/2}{2h^3} \\ &= f'''(x) + Mh^2/4 (\text{M is the upper bound of } f^{(5)}(\theta) \text{ term}) \end{aligned}$$
Thus, we see that the truncation error is $O(h^2)$.

3. Assume this algorithm has error of ε, then the round off would be

$$\text{error} \leq 6\varepsilon/2h^3 = 3\varepsilon/h^3$$

4. According solutions to question of question 2 and 3, we have total error

$$E = Mh^2/4 + 3\varepsilon/h^3$$

so we let

$$E' = 0 \Rightarrow \frac{Mh}{2} - 9\varepsilon h^{-4} = 0 \Rightarrow h = (\frac{18\varepsilon}{M})^{-5}$$

5.

$$E = (\frac{18 \cdot 2^{-52}}{M})^{-5}$$