Math 241 Formula sheet

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Contents

1 Vector Function

Limit

$$\lim_{t\to\infty}(x(t), y(t), z(t)) = (\lim_{t\to\infty} x(t), \lim_{t\to\infty} y(t), \lim_{t\to\infty} z(t))$$

Derivative

$$\mathbf{r}'(t) = (x'(t), y'(t), z'(t))$$

Integral

$$\int_a^b (r)(t) = (\int_a^b x(t)dt, \int_a^b y(t)dt, \int_a^b z(t)dt)$$

Length

$$L = \int_a^b \sqrt{(\frac{dx}{dt})^2 + \frac{dy}{dt})^2 + \frac{dz}{dt})^2}dt = \int d\mathbf{r}$$

Curvature

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|F'(t)|^3} = \frac{|f''(x)|}{[1 + [f'(x)^2]]^{3/2}}$$

Principal normal vector

$$\mathbf{N} = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$$

Binormal vector

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

2 Partial Derivative

Continuous at (a, b)

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

Clairaut's Theorem

$$\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x}$$

Linear approximation

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

Whole Derivative

$$dz = z_x dx + z_y dy$$

Tangent Plane

$$f_x(a,b)(x-a) + f_y(a,b)(y-b) = z - f(a,b)$$

Tangent Plane at (a,b,c) of $f(x,y,z)=0$

$$\nabla f(a,b,c)_x(x-a) + \nabla f(a,b,c)_y(y-b) + \nabla f(a,b,c)_z(z-c) = 0$$

Chain Rule 1

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Chain Rule 2

$$\begin{cases} \frac{dz}{ds} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \\ \frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \end{cases}$$

Chain Rule 3

$$\frac{\partial u}{\partial t_i} = \sum \frac{\partial u}{\partial x_i}\frac{\partial x_i}{\partial t_i}$$

Directional Derivative

$$\begin{cases} D_{\mathbf{u} f(x,y) = f_x(x,y)a + f_y(x,y)b} \\ \mathbf{u} = (a,b)\text{Unitdirectionvector} \end{cases}$$

Gradient

$$\nabla f(x,y) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$$

Directional Derivative 2

$$D_{\mathbf{u}} f(x,y) = \nabla f(x,y) \cdot \mathbf{u} = |\nabla f| \cos(\theta)$$

Maximum, Minimum

- $$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \\ \vdots \end{cases} \Rightarrow \text{Critical Points(Local max, min, saddlepoint)}$$

- Second Derivative Test

$$D = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}^2(a,b)$$

- $$\Rightarrow \begin{cases} D > 0 \ \& \ f_x x > 0 \to \text{localmin} \\ D > 0 \ \& \ f_x x < 0 \to \text{localmax} \\ D < 0 \to \text{saddlepoint} \end{cases}$$

Lagrange Multiplier Solve

$$\begin{cases} \nabla f(x,y,z) - \lambda \nabla g(x,y,z) = 0 \\ g(x,y,z) = k \end{cases}$$

to get critical points

3 Double & triple integral

Polar

$$\iint f(x,y)dxdy = \iint f(r,\theta)rdrd\theta$$

Mass

$$m = \iint_D \rho dA$$

Moment

$$\begin{cases} M_x = \iint_D y\rho(x,y)dA \\ M_y = \iint_D x\rho(x,y)dA \end{cases}$$

Center of mass

$$\begin{cases} \bar{x} = \frac{M_x}{m} \\ \bar{y} = \frac{M_y}{m} \end{cases}$$

Rotational inertia

$$\begin{cases} I_x = \iint y^2 \rho(x,y) dA \\ I_y = \iint x^2 \rho(x,y) dA \\ I_z = \iint (x^2 + y^2) \rho(x,y) dA \end{cases}$$

Surface Area

$$A = \iint_D \sqrt{f_x^2 + f_y^2 + 1} dA = \iint_D d\mathbf{S}$$

Volume

$$V = \iint_\Omega dV$$

Cylindrical

$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{cases} , \iiint f(x,y,z) dxdydz = \iiint f(r,\theta,z) r dr d\theta dz$$

Spherical

$$\begin{cases} x = \rho\sin(\phi)\cos(\theta) \\ y = \rho\sin(\phi)\sin(\theta) \\ z = \rho\cos(\phi) \end{cases} , \iiint f(x,y,z) dxdydz = \iiint f(r,\phi,\theta) r^2 \sin(\phi) dr d\phi d\theta$$

Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

Change variable

$$\iint f(x,y)dA = \iint_S f(x(u,v),y(u,v))|\frac{\partial(x,y)}{\partial(u,v)}|dudv$$

4 Vector Field

Line integral 1

$$\int_c f(x,y)ds = \int_a^b f(x(t),y(t),z(t))\sqrt{dx^2(t)+dy^2(t)+dz^2(t)}$$

Line integral 2

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b F(\mathbf{r}(t))\cdot d\mathbf{r}'(t) = \int_c \mathbf{F}\cdot\mathbf{T}ds$$

Fundamental Theorem for line integral

$$\int_c \nabla f dr = f(r(b)) - f(r(a))$$

On open simply connected region,

$$\oint_C F\cdot dr = 0 \Leftrightarrow \text{conservative} \Leftrightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 \Leftrightarrow \nabla\times F = 0$$

Green's Theorem 1

$$\oint_c Pdx + Qdy = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$$

Curl

$$curlF = \nabla\times F$$

Div

$$divF = \nabla\cdot F$$

Green's Theorem 2

$$\oint_c F\cdot ndS = \iint_D divF(x,y)dA$$

Surface

$$\begin{cases} r(u,v) = (x(u,v), y(u,v), z(u,v)) \\ A = \iint_D |r_u \times r_v| dA = \iint_D \sqrt{1 + (z_x)^2 + (z_y)^2} dA \end{cases}$$

Surface integral

$$\iint_D f(x,y,z) dS = \iint f(r(u,v)) |r_u \times r_v| dA = \iint F \cdot n dS = \iint_D F \cdot (r_u \times r_v) dA$$

Strokes' Theorem

$$\oint_C F \cdot dr = \iint_S curl F \cdot dS$$

Gaussian Theorem

$$\iint_S F \cdot n dS = \iiint_E div F dV$$