

Describe a context free grammar for the following languages. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

1. $\{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0 \text{ and } i + \ell = j + k\}$.
2. $L = \{0, 1\}^* \setminus \{0^n 1^n \mid n \geq 0\}$. In other words the complement of the language $\{0^n 1^n \mid n \geq 0\}$.

Solution: 1. The context-free grammar can be designed base on the difference of $i + l$ and $j + k$:

$S \rightarrow A B$	$\{a^i b^j c^k d^\ell : i + l \neq j + k\}$
$A \rightarrow aA Ad aC Cd aD Dd$	$\{a^i b^j c^k d^\ell : i + l > j + k\}$
$B \rightarrow bB Bc bC Cc$	$\{a^i b^j c^k d^\ell : i + l < j + k\}$
$C \rightarrow \varepsilon bCc$	$\{a^i b^j c^k d^\ell : i + l = j + k\}$
$D \rightarrow \varepsilon aDd aCd$	$\{a^i b^j c^k d^\ell : i + l = j + k\}$

2. For $\{0^i 1^j : i, j \geq 0\}$, the context-free grammar can be designed base on the difference of i and j :

$S \rightarrow A B$	$\{0^i 1^j : i \neq j\}$
$A \rightarrow 1A A1 1C C1$	$\{0^i 1^j : i < j\}$
$B \rightarrow 0B B0 0C C0$	$\{0^i 1^j : i > j\}$
$C \rightarrow \varepsilon 0C1 1C0$	$\{0^i 1^j : i = j\}$

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