Lecture 5

Partial Redundancy Elimination

I Forms of redundancy
- -- global common subexpression elimination
- -- loop invariant code motion
- -- partial redundancy

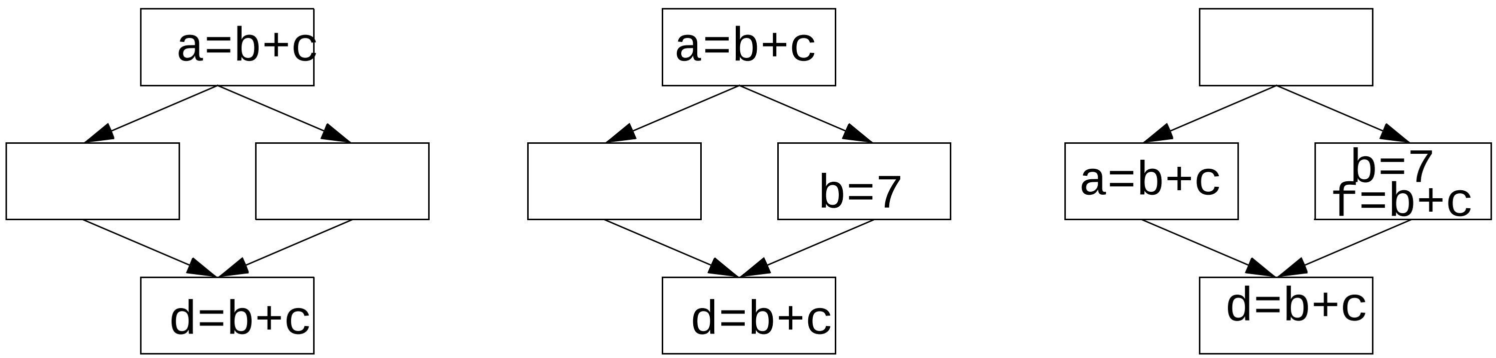
II Lazy Code Motion Algorithm
- -- Mathematical concept: a cut set
- -- Basic technique (anticipation)
- -- 3 more passes to refine algorithm

Reading: Chapter 9.5

Overview

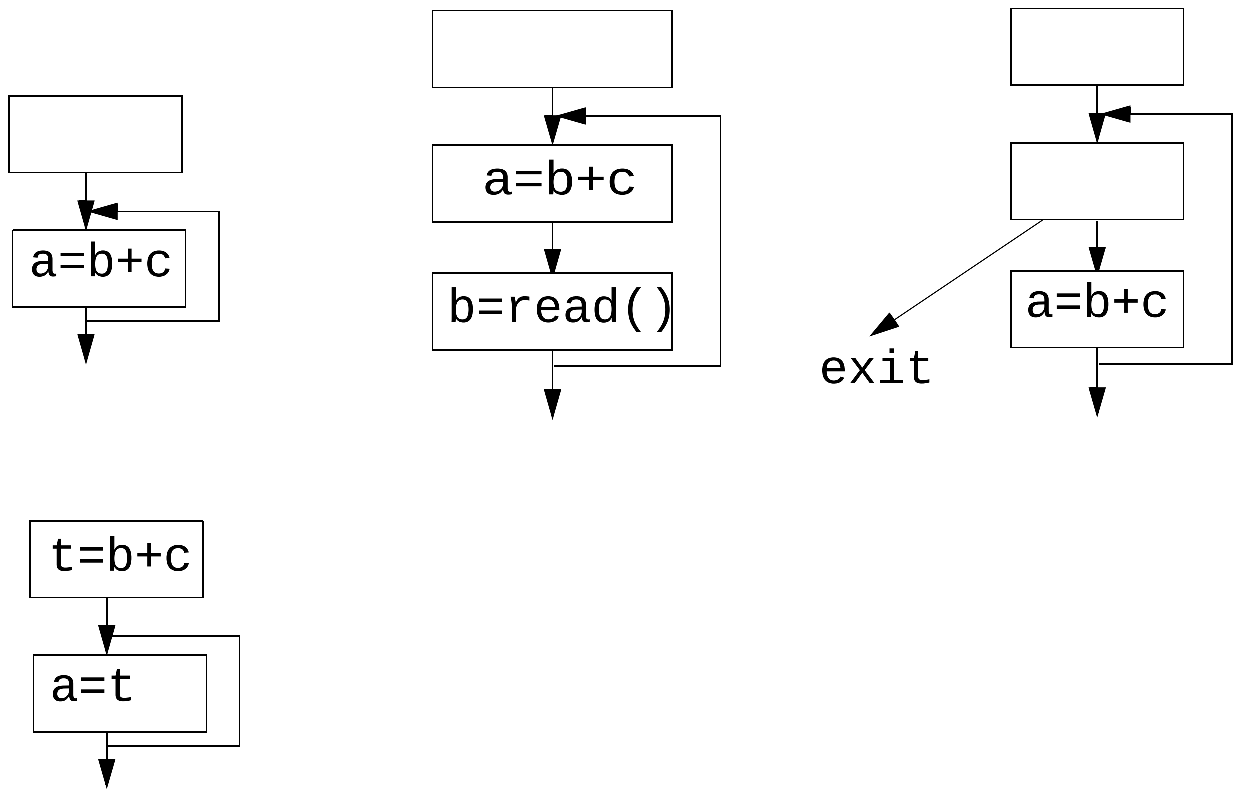
- **Eliminates many forms of redundancy in one fell swoop**
- **Originally formulated as 1 bi-directional analysis**
- **Lazy code motion algorithm**
    - formulated as 4 separate uni-directional passes (backward, forward, forward, backward)

I. Common Subexpression Elimination



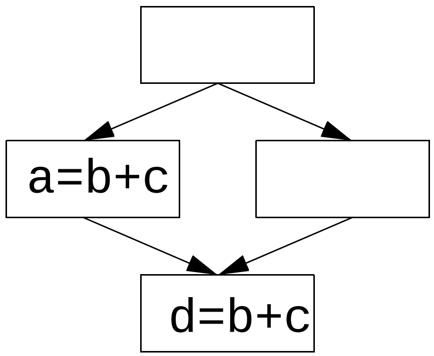
- **A common expression may have different values on different paths!**
- **On every path reaching p,**
  - expression b+c has been computed
  - b, c not overwritten after the expression

Loop Invariant Code Motion



- Given an expression (b+c) inside a loop,
  does the value of b+c change inside the loop?
  is the code executed at least once?

Partial Redundancy



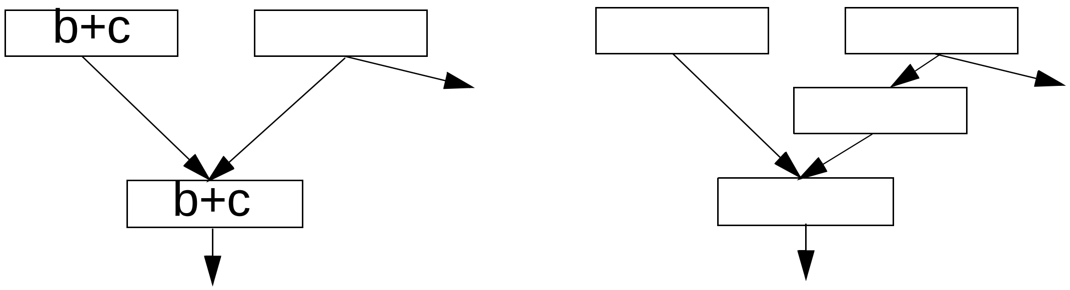
- Can we place calculations of b+c
  such that no path re-executes the same expression

- Partial redundancy elimination (PRE)
  - subsumes:
    - global common subexpression (full redundancy)
    - loop invariant code motion (partial redundancy for loops)

Unifying theory: More powerful, elegant --> but less direct.

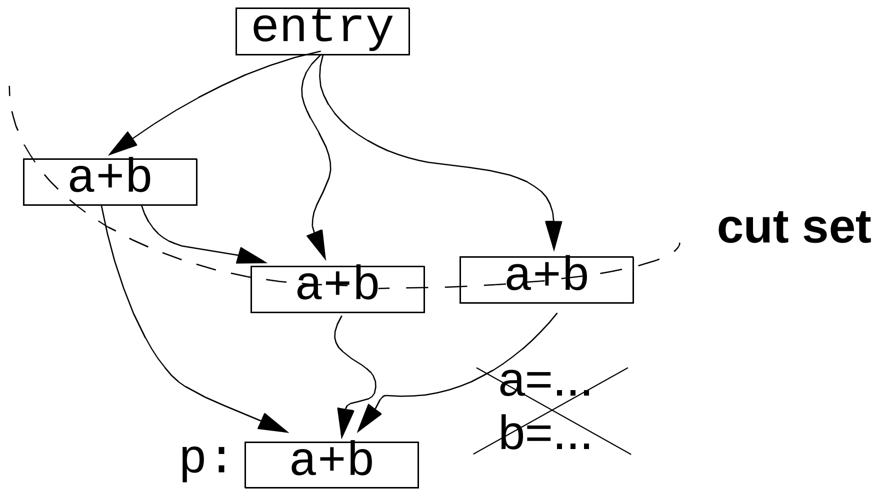
II. Preparing the Flow Graph

- **Key observation**
  - A bi-directional (!) data flowcan now be replaced with several unidirectional data flows -- much easier
  - Better result as well!



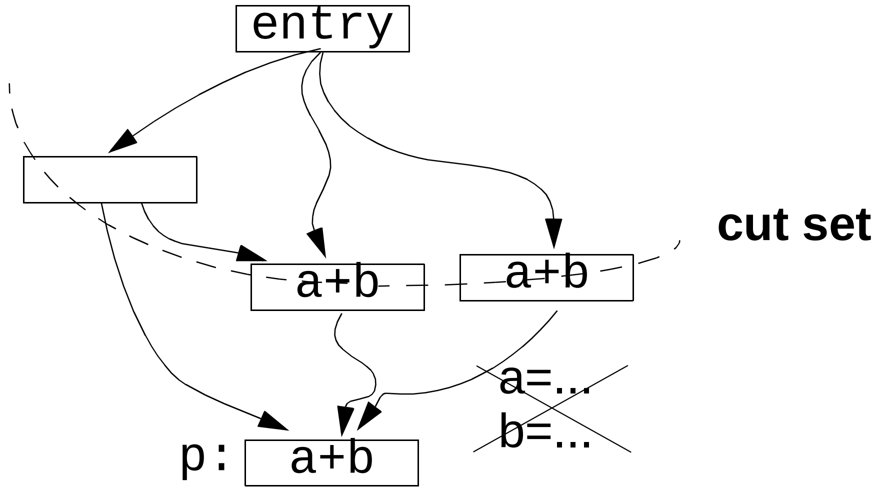
- **Definition: Critical edges**
  - source basic block has multiple successors
  - destination basic block has multiple predecessors
- **Modify the flow graph: (treat every statement as a basic block)**
  - To keep algorithm simple: restrict placement of instructions to the beginning of a basic block
  - Add a basic block for every edge that leads to a basic block with multiple predecessors (not just on critical edges)

Full Redundancy: A Cut Set in a Graph



- **Full redundancy at p: expression a+b redundant on all paths**
  - a cut set: nodes that separate entry from p
  - a cut set contains calculation of a+b
  - a, b, not redefined

Partial Redundancy: Completing a Cut Set



- **Partial redundancy at p: redundant on some but not all paths**
  - Add operations to create a cut set containing a+b
  - Note: Moving operations up can eliminate redundancy
- **Constraint on placement: no wasted operation**
  - a+b is “anticipated” at B if its value computed at B will be used along ALL subsequent paths
  - a, b not redefined, no branches that lead to exit with out use
- **Range where a+b is anticipated --> Choice**

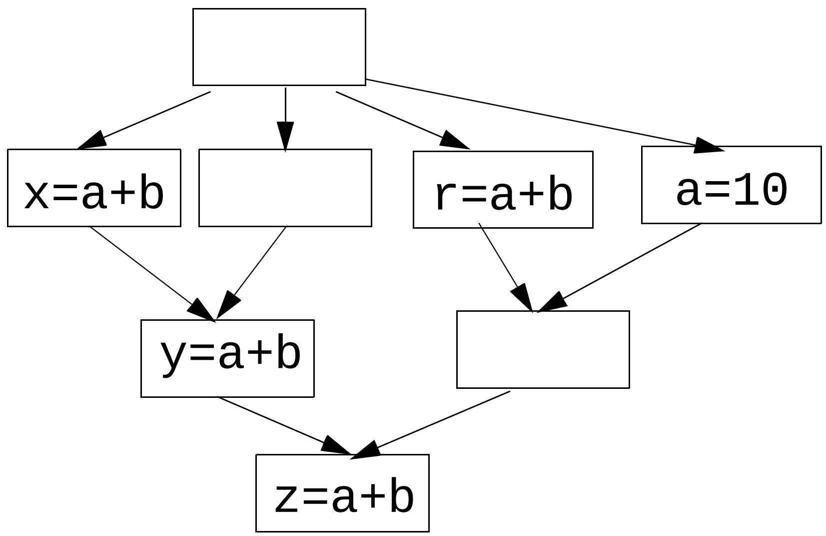
Pass 1: Anticipated Expressions

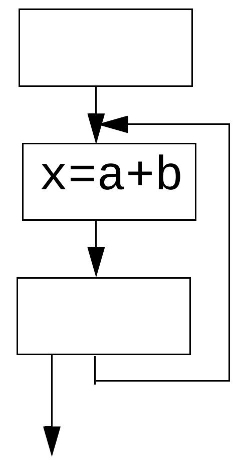
- **Backward pass: Anticipated expressions**
  Anticipated[b].in: Set of expressions anticipated at the entry of b
  - An expression is anticipated if its value computed at point p will be used along ALL subsequent paths

	Anticipated Expressions
Domain	Sets of expressions
Direction	backward
Transfer function	$f_b(x) = EUse_b \cup (x - EKill_b)$ EUse: used exp EKill: exp killed
$\wedge$	$\cap$
Boundary	in[exit] = $\emptyset$
Initialization	in[b] = {all expressions}

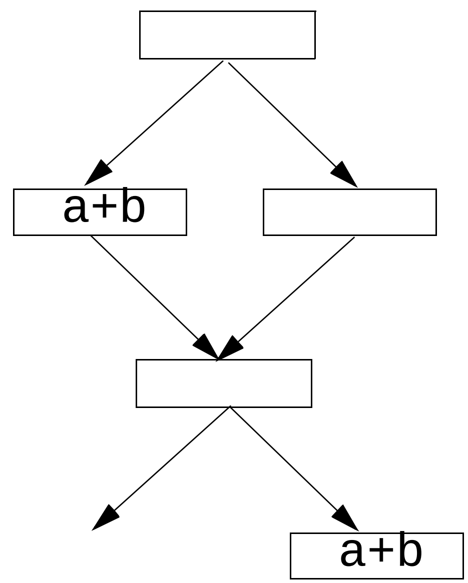
- **First approximation:**
  - place operations at the frontier of anticipation (boundary between not anticipated and anticipated)

Examples (1)



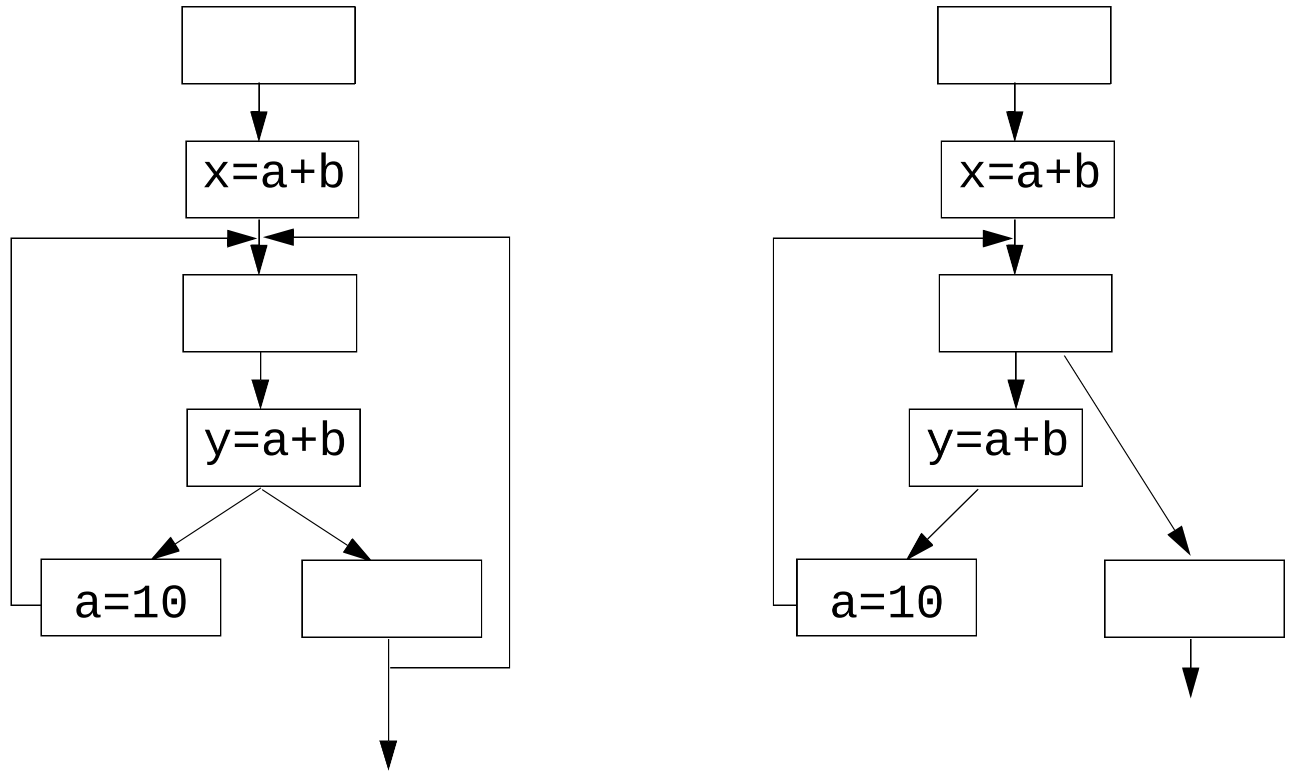


Examples (2)



Do you know how the algorithm works without simulating it?

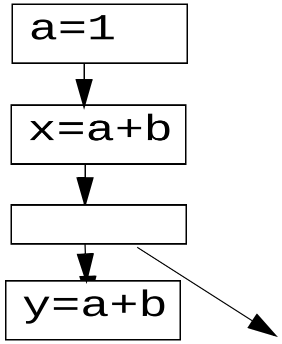
Examples (3)



There is still some redundancy left!

Pass 2: Place As Early As Possible

- **First approximation: frontier between “not anticipated” & “anticipated”**
- **Complication: Anticipation may oscillate**



- Pretend we calculate expression e whenever it is anticipated
- e will be **available** at p
  if e has been “anticipated but not subsequently killed” on all paths reaching p

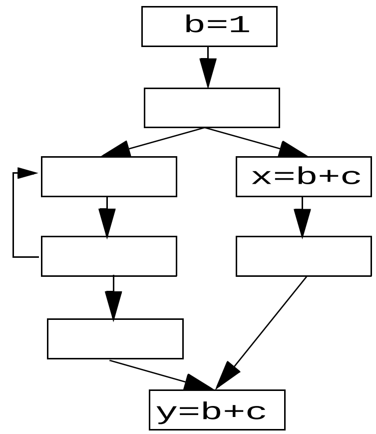
	Available Expressions
Domain	Sets of expressions
Direction	forward
Transfer function	$f_b(x)$ = (Anticipated[b].in $\cup$ x) - $EKill_b$
$\wedge$	$\cap$
Boundary condition	out[entry] = $\emptyset$
Initialization	out[b] ={all expressions}

Early Placement

- **earliest(b)**
  - set of expressions added to block b under early placement
- **Place expression at the earliest point anticipated and not already available**
  - earliest(b) = anticipated[b].in − available[b].in
- **Algorithm**
  - For all basic block b, if x+y $\in$ earliest[b]
    - at beginning of b:
      create a new variable t
      t = x+y,
      replace every original x+y by t

Pass 3: Lazy Code Motion

- **Delay without creating redundancy to reduce register pressure**

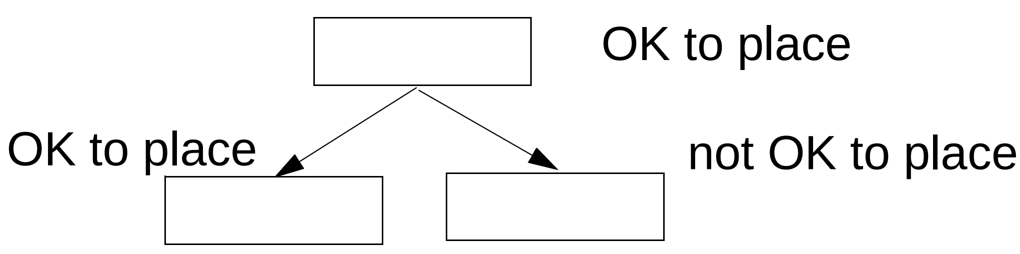


- **An expression e is postponable at a program point p if**
  - all paths leading to p
    have seen the earliest placement of e but not a subsequent use

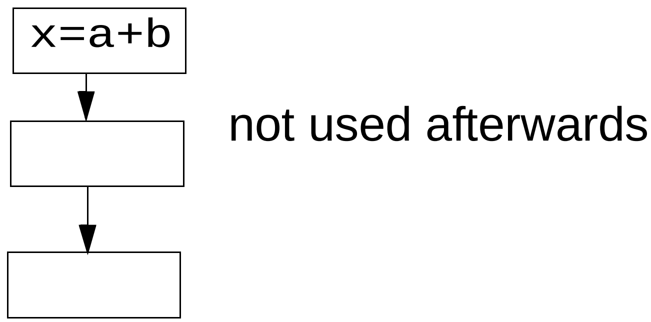
	Postponable Expressions
Domain	Sets of expressions
Direction	forward
Transfer function	$f_b(x) = (\text{earliest}[b] \cup x) - \text{EUse}_b$
$\wedge$	$\cap$
Boundary condition	out[entry] = $\emptyset$
Initialization	out[b] = {all expressions}

Latest: frontier at the end of “postponable” cut set

- latest[b] = (earliest[b] $\cup$ postponable.in[b]) $\cap$
  (EUse$_b$ $\cup \neg(\bigcap_{s \in succ[b]}$(earliest[s] $\cup$ postponable.in[s])))
  - OK to place expression: earliest or postponable
  - Need to place at b if either
    - used in b, or
    - not OK to place in one of its successors
- Works because of pre-processing step
  (an empty block was introduced to an edge if the destination has multiple predecessors)
  - if b has a successor that cannot accept postponement,
    b has only one successor
  - The following does not exist



Pass 4: Cleaning Up



- **Eliminate temporary variable assignments unused beyond current block**
- **Compute: Used.out[b]: sets of used (live) expressions at exit of b.**

	Used Expressions
Domain	Sets of expressions
Direction	backward
Transfer function	$f_b(x) = (EUse[b] \cup x) - latest[b]$
$\wedge$	$\cup$
Boundary condition	in[exit] = $\emptyset$
Initialization	in[b] = $\emptyset$

Code Transformation

- For all basic blocks b,

  if (x+y) $\in$ (latest[b] $\cap$ used.out[b])

    at beginning of b:

    add new t = x+y

    replace every original x+y by t

4 Passes for Partial Redundancy Elimination

- *Heavy lifting:* **Cannot introduce operations not executed originally**
  - Pass 1 (backward): Anticipation: range of code motion
  - Placing operations at the frontier of anticipation
    gets most of the redundancy
- *Squeezing the last drop of redundancy:*
  An anticipation frontier may cover a subsequent frontier
  - Pass 2 (forward): Availability
  - Earliest: anticipated, but not yet available
- *Push the cut set out -- as late as possible*
  To minimize register lifetimes
  - Pass 3 (forward): Postponability: move it down provided it does not create redundancy
  - Latest: where it is used or the frontier of postponability
- *Cleaning up*
  Pass 4: Remove temporary assignment

Remarks

- **Powerful algorithm**
  - Finds many forms of redundancy in one unified framework
- **Illustrates the power of data flow**
  - Multiple data flow problems