Using Static Single Assignment Form

Announcements

- Project 2 schedule due today
- HW1 due Friday

Last Time

- SSA Technicalities

Today

- Constant propagation
- Loop invariant code motion
- Induction variables

Constant Propagation

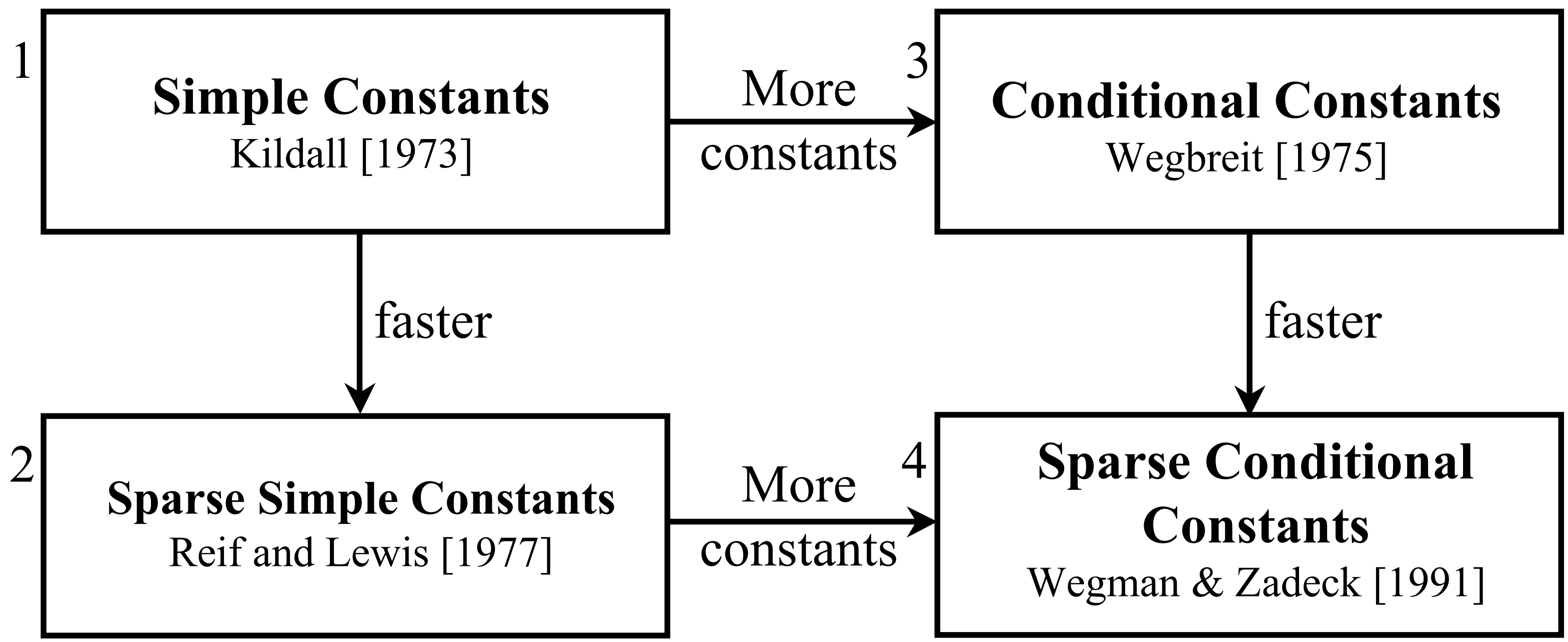
Goal

- Discover constant variables and expressions and propagate them forward through the program

Uses

- Evaluate expressions at compile time instead of run time
- Eliminate dead code (*e.g.,* debugging code)
- Improve efficacy of other optimizations (*e.g.,* value numbering and software pipelining)

Roadmap


1
Simple Constants
Kildall [1973]
More
constants
3
Conditional Constants
Wegbreit [1975]
faster
faster
2
Sparse Simple Constants
Reif and Lewis [1977]
More
constants
4
Sparse Conditional
Constants
Wegman & Zadeck [1991]

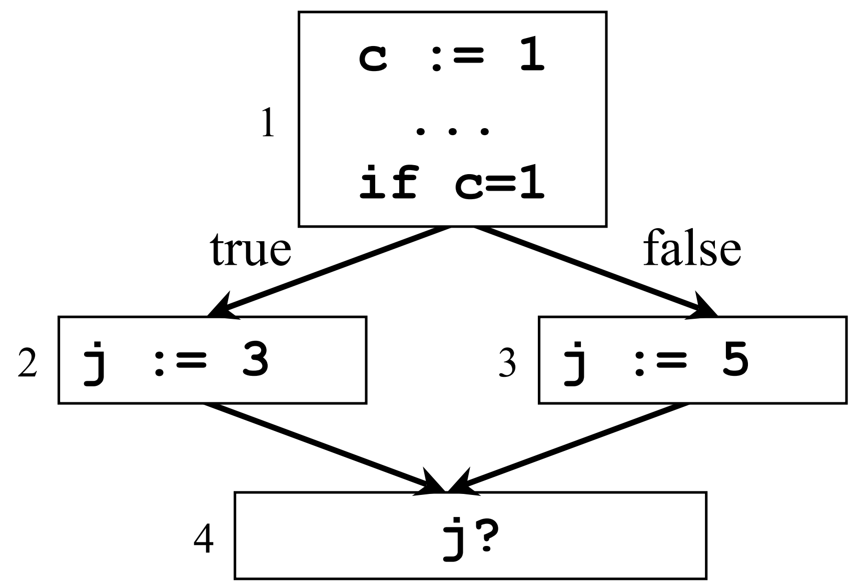
Kinds of Constants

Simple constants Kildall [1973]

– Constant for all paths through a program

Conditional constants Wegbreit [1975]

– Constant for actual paths through a program (when only one direction of a conditional is taken)



Data-Flow Analysis for Simple Constant Propagation

Simple constant propagation: analysis is "reaching constants"

- D: $\mathbf{2^{v \times c}}$
- $\sqcap$: $\cap$
- F:
  - Kill($x \leftarrow \ldots$) = $\{(x,c)\ \forall c\}$
  - Gen($x \leftarrow c$) = $\{(x,c)\}$
  - Gen($x \leftarrow y \oplus z$) = **if** $(y,c_y) \in In$ & $(z,c_z) \in In$, $\{(x, c_y \oplus c_z)\}$
  - . . .

Data-Flow Analysis for Simple Constant Propagation (cont)

Reaching constants for simple constant propagation

– D: $\{\text{All constants}\} \cup \{\top, \bot\}$

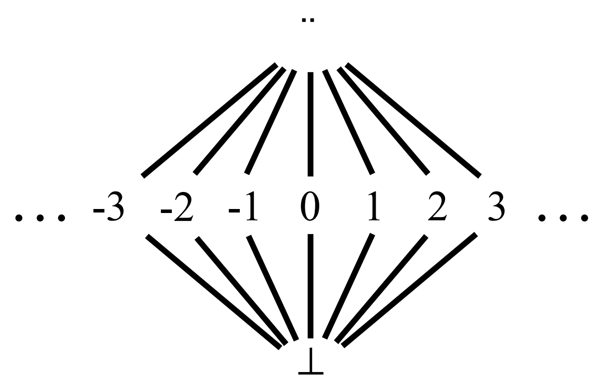
– $\sqcap$: $\mathbf{c} \sqcap \top = \mathbf{c}$

$\mathbf{c} \sqcap \bot = \bot$

$\mathbf{c} \sqcap \mathbf{d} = \bot$ **if** $\mathbf{c} \neq \mathbf{d}$

$\mathbf{c} \sqcap \mathbf{d} = \mathbf{c}$ **if** $\mathbf{c} = \mathbf{d}$

Using tuples of lattices



– F:

– $F_{x \leftarrow c}(\text{In}) =$ **c**

– $F_{x \leftarrow y \oplus z}(\text{In}) =$ **if** $c_y = \text{In}_y$ **&** $c_z = \text{In}_z$, **then** $c_y \oplus c_z$, **else** $\top$ **or** $\bot$

– . . .

Initialization for Reaching Constants

Pessimistic

- Each variable is initially set to $\perp$ in data-flow analysis
- Forces merges at loop headers to go to $\perp$ conservatively

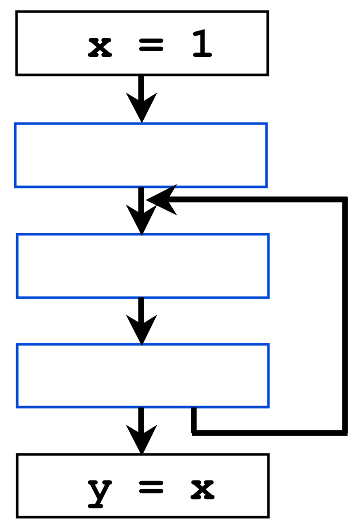
Optimistic

- Each variable is initially set to $\top$ in data-flow analysis
- What assumption is being made when optimistic reaching constants is performed?

Implementing Simple Constant Propagation

Standard worklist algorithm

- Identifies simple constants
- For each program point, maintains one constant value for each variable
- O(EV) (E is the number of edges in the CFG; V is number of variables)



Problem

- Inefficient, since constants may have to be propagated through irrelevant nodes

Solution

- Exploit a sparse dependence representation (*e.g.,* SSA)

Sparse Simple Constant Propagation

Reif and Lewis algorithm Reif and Lewis [1977]

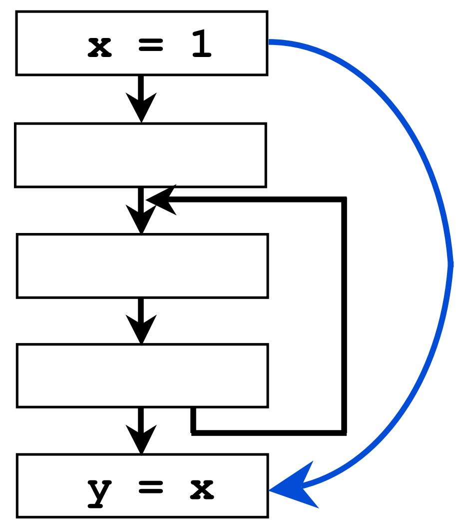
- Identifies simple constants
- Faster than Simple Constants algorithm

SSA edges

- Explicitly connect defs with uses
- How would you do this?

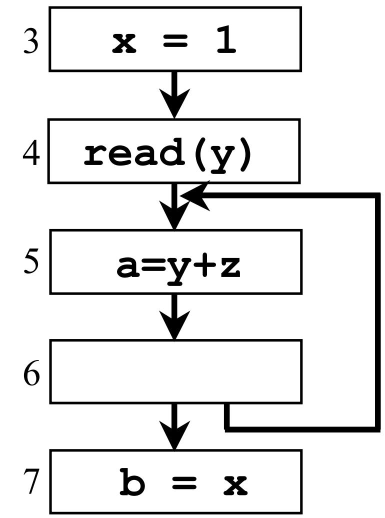
Main Idea

- Iterate over SSA edges instead of over all CFG edges



Sparse Simple Constants Algorithm (Ch. 19 in Appel)

```
worklist = all statements in SSA
while worklist ≠ ∅
    Remove some statement S from worklist
    if S is x = phi(c,c,...,c) for some constant c
        replace S with v = c
    if S is x=c for some constant c
        delete s from program
        for each statement T that uses v
            substitute c for x in T
            worklist = worklist union {T}
```



Sparse Simple Constants

Complexity

- $O(E') = O(EV)$, E' is number of SSA edges
- $O(n)$ in practice

Other Uses of SSA

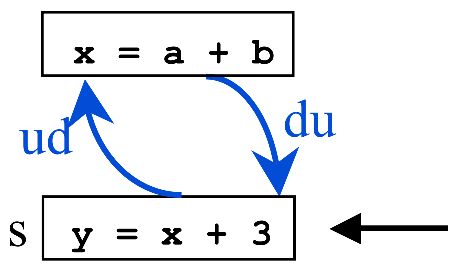
Dead code elimination

while ∃ a variable v with no uses and whose def has no other side effects

Delete the statement s that defines v

for each of s's ud-chains

Delete the corresponding du-chain that points to s



If y becomes dead and there are no other uses of x, then the assignment to x becomes dead, too

- Contrast this approach with one that uses liveness analysis
  - This algorithm updates information incrementally
  - With liveness, we need to invoke liveness and dead code elimination iteratively until we reach a fixed point

Other Uses of SSA (cont)

Induction variable identification

- Induction variables
  - Variables whose values form an arithmetic progression
  - Useful for strength reduction and loop transformations

Why bother?

- Automatic parallelization, . . .

Simple approach

- Search for statements of the form, `i = i + c`
- Examine ud-chains to make sure there are no other defs of `i` in the loop
- Does not catch all induction variables. Examples?

Induction Variable Identification (cont)

Types of Induction Variables

- **Basic** induction variables
  - Variables that are defined once in a loop by a statement of the form, `i=i+c` (or `i=i*c`), where `c` is a constant integer
- **Derived** induction variables
  - Variables that are defined once in a loop as a linear function of another induction variable
    - $j = c_1 * i + c_2$
    - $j = i / c_1 + c_2$, where c_1 and c_2 are loop invariant

Induction Variable Identification (cont)

Informal SSA-based Algorithm

- Build the SSA representation
- Iterate from innermost CFG loop to outermost loop
  - Find SSA cycles
    - Each cycle **may** be a **basic** induction variable if a variable in a cycle is a function of loop invariants and its value on the current iteration
  - Find **derived** induction variables as functions of loop invariants, its value on the current iteration, and basic induction variables

Induction Variable Identification (cont)

Informal SSA-based Algorithm (cont)

- Determining whether a variable is a function of loop invariants and its value on the current iteration
  - The ϕ -function in the cycle will have as one of its inputs a def from inside the loop and a def from outside the loop
  - The def inside the loop will be part of the cycle and will get one operand from the ϕ -function and all others will be loop invariant
  - The operation will be plus, minus, or unary minus

Next Time

Reading

– Ch 8.10, 12.4

Lecture

– Redundancy elimination