Lecture 8

Software Pipelining

I. Example of DoAll Loops

- **Machine**
  - Per instruction: 1 read, 1 write, 1 (2-stage) arithmetic op, with hardware loop op and auto-incrementing addressing mode.
- **Source code**

```
For i = 1 to n
     D[i] = A[i] * B[i]+ c
```

- **Code for one iteration**

```
1.   LD   R5,0(R1++)
2.   LD   R6,0(R2++)
3.   MUL  R7,R5,R6
4.
5.   ADD  R8,R7,R4
6.
7.   ST   0(R3++),R8
```

- **No parallelism in basic block**

Unrolling

```
 1. L: LD
 2.    LD
 3.              LD
 4.    MUL       LD
 5.              MUL       LD
 6.    ADD                 LD
 7.              ADD                 LD
 8.    ST                  MUL       LD
 9.                                  MUL
10.              ST        ADD
11.                                  ADD
12.                        ST
13.                                  ST      BL (L)
```

- Let *u* be the degree of unrolling:

  Length of u iterations = $7+2(u-1)$

  Execution time per source iteration = $(7+2(u-1)) / u = 2 + 5/u$

Software Pipelined Code

1.	LD				
2.	LD				
3.	MUL	LD			
4.		LD			
5.		MUL	LD		
6.	ADD		LD		
7.			MUL	LD	
8.	ST	ADD		LD	
9.				MUL	LD
10.		ST	ADD		LD
11.					MUL
12.			ST	ADD	
13.					
14.				ST	ADD
15.					
16.					ST

- Unlike unrolling, software pipelining can give optimal result.
- Locally compacted code may not be globally optimal
- DOALL: Can fill arbitrarily long pipelines with infinitely many iterations

Example of DoAcross Loops

```
Loop:                              1. LD
   Sum  = Sum + A[i];              2. MUL
   B[i] = A[i] * c                 3. ADD
                                   4. ST
```

Software Pipelined Code

```
1. LD
2. MUL
3. ADD    LD
4. ST     MUL
5.        ADD
6.        ST
```

Doacross loops

- Recurrences can be parallelized
- Harder to fully utilize hardware with large degrees of parallelism

II. Problem ForMULation

- **Goals**
  - maximize throughput
  - small code size
- **Find**
  - an identical relative schedule S(n) for every iteration
  - a constant initiation interval (T)

such that

  - the initiation interval is minimized

- **Complexity**
  - NP-complete in general

```
S
0    LD
1    MUL        T=2
2    ADD   LD
3    ST    MUL
           ADD
           ST
```

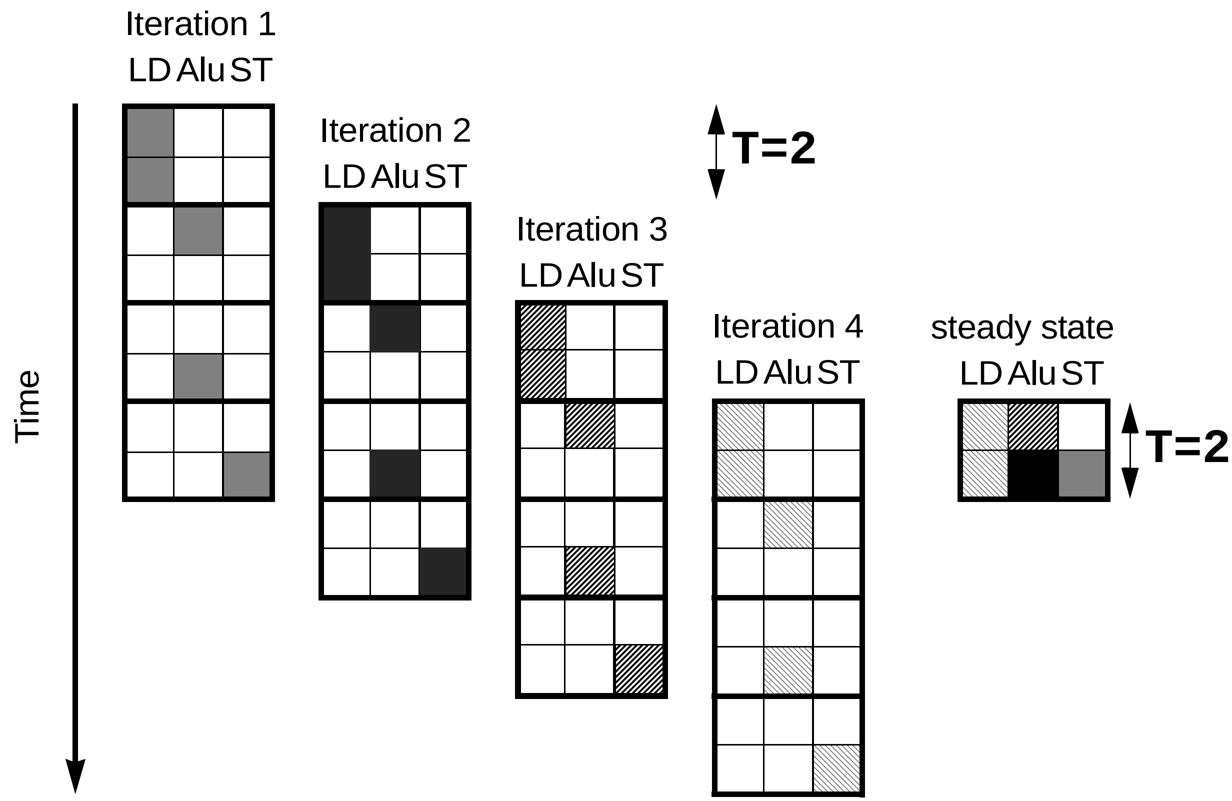
Resources on Bound on Initiation Interval

- **Example: Resource usage**

```
LD, LD, MUL, ADD, ST
```

- **Lower bound on initiation interval?**

Scheduling Constraints: Resource

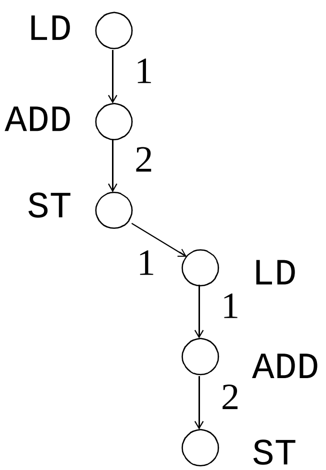


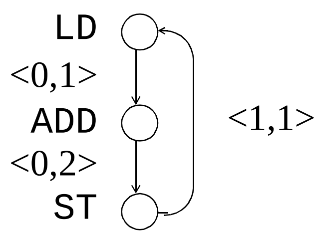
- RT: resource reservation table for single iteration
- RT_S: modulo resource reservation table

$$RT_S[i] = \Sigma_{t | t \bmod T = i} RT[t]$$

Scheduling Constraints: Precedence

```
for (i = 0; i < n; i++){
 *(p++) = *(q++) + c
}
```





T: Initiation interval

S(*n*): Schedule for operation *n* wrt beginning of its iteration

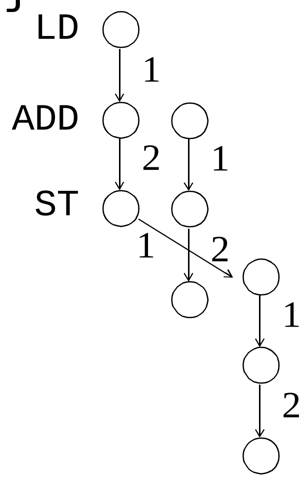
Time to execute `ST` in iteration 0: *S*(`ST`)

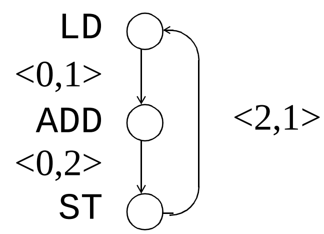
Time to execute `LD` in next iteration: *T* +*S*(`LD`)

$T + S(\mathtt{LD}) - S(\mathtt{ST}) >= 1$

$T + S(\mathtt{LD}) - S(\mathtt{LD}) >= 4$; $T >= 4$;

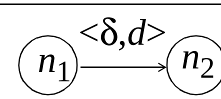
```
for (i = 2; i < n; i++){
  A[i] = A[i-2] + 1;
}
```





$2T + S(\mathtt{LD}) - S(\mathtt{ST}) >= 1$

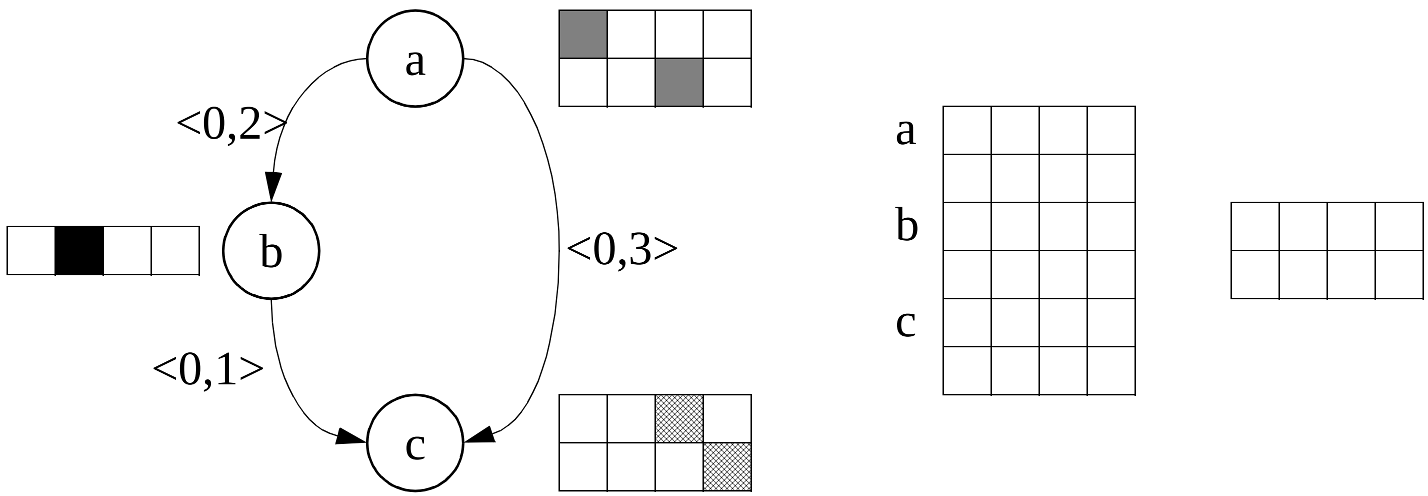
$2T + S(\mathtt{LD}) - S(\mathtt{LD}) >= 4$; $T >= 2$



Iteration for n_2 starts δx*T* cycles later.

Minimum Initiation Interval Due to Precedence?

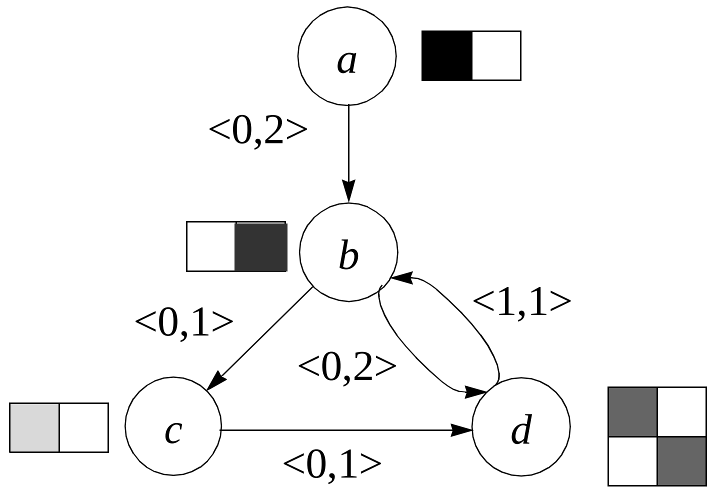
III. Example: Acyclic Graph



Algorithm for Acyclic Graphs

- **Find lower bound of initiation interval:** T_0
  - based on resource constraints
- For $T = T_0, T_0{+}1, ...$ until all nodes are scheduled
  - For each node n in topological order
    - s_0 = earliest n can be scheduled
    - For each $s = s_0\ , s_0\ {+1}, ..., s_0\ {+T{-}1}$ // find the first slot
      - if NodeScheduled (n, s) break;
    - If n cannot be scheduled break; // none found
- **NodeScheduled (n, s)**
  - Check resources of n at s
    in modulo resource reservation table

Cyclic Graphs



- **No such thing as "topological order"**
- **b - > c; c -> b**

$$S(c) - S(b) \geq 1$$

$$T + S(b) - S(c) \geq 2$$

- **Schedule b, constrains c and vice versa**

$$S(b) + 1 \leq S(c) \leq S(b) - 2 + T$$

$$S(c) - T + 2 \leq S(b) \leq S(c) - 1$$

Strongly Connected Components

- **A strongly connected component SCC**
  - Set of nodes such that every node can reach every other nodes
- **Every node constrains all others from above and below**
  - Finds longest paths between every pair of nodes
  - As each node scheduled,
    find lower and upper bounds of all other nodes in SCC
- **SCCs are hard to schedule**
  - Critical cycle: no slack
    - Backtrack starting with the first node in SCC
  - increases T, increases slack
- **Edges between SCCs are acyclic**
  - Acyclic graph: every node is a separate SCC

Algorithm Design

- **Find lower bound of initiation interval:** T_0
  - based on resource constraints and precedence constraints
- For $T = T_0, T_0+1, \ldots$, until all nodes are scheduled
  - E^*= longest path between each pair
  - For each SCC c in topological order
    - s_0 = Earliest c can be scheduled
    - For each $s = s_0 , s_0 +1, \ldots, s_0 +T-1$ // find the first slot
      - if SCCScheduled (c, s) break;
    - If c cannot be scheduled return false; // none found
  - return true;

Scheduling a SCC

- **Can SCC c be scheduled starting at time s**
- **SCCScheduled (c, s)**
  - Schedule first node at s, return false if fails
  - For each remaining node n in c
    - s_l = lower bound on n based on E*
    - s_u = upper bound on n based on E*
    - For each $s = s_l, s_l + 1, \min(s_l + T - 1, s_u)$ // find the first slot
      - if NodeScheduled (n, s) break;
    - If n cannot be scheduled return false; // none found
  - return true;

Modulo Variable Expansion

- **Software-pipelined code**

```
1.   LD    R5,0(R1++)
2.   LD    R6,0(R2++)
3.   MUL   R7,R5,R6
4.
5.   ADD   R8,R7,R4
6.
7.   ST    0(R3++),R8
```

```
  1. LD
  2. LD
  3. MUL      LD
  4.          LD
  5.          MUL      LD
  6. ADD               LD
L:7.                   MUL      LD
  8. ST       ADD               LD       BL L
  9.                            MUL      LD
 10.          ST       ADD               LD
 11.                                     MUL
 12.                   ST       ADD
 13.
 14.                            ST       ADD
```

Modulo Variable Expansion

```
  1.  LD R5,0(R1++)
  2.  LD R6,0(R1++)
  3.  LD R5,0(R1++) MUL R7,R5,R6
  4.  LD R6,0(R1++)
  5.  LD R5,0(R1++) MUL R17,R5,R6
  6.  LD R6,0(R1++) ADD R8,R7,R4
L 7.  LD R5,0(R1++) MUL R7,R5,R6
  8.  LD R6,0(R1++) ADD R8,R17,R4  ST 0(R3++),R8
  9.  LD R5,0(R1++) MUL R17,R5,R6
 10.  LD R6,0(R1++) ADD R8,R7,R4   ST 0(R3++),R8 BL L
 11.                MUL R7,R5,R6
 12.                ADD R8,R17,R4  ST 0(R3++),R8
 13.
 14.                ADD R8,R7,R4   ST 0(R3++),R8
 15.
 16.                               ST 0(R3++),R8
```

Algorithm

- **Normally, every iteration uses the same set of registers**
  - introduces artificial anti-dependences for software pipelining
- **Modulo variable expansion algorithm**
  - schedule each iteration ignoring artificial constraints on registers
  - calculate life times of registers
  - unroll the steady state of software pipelined loop to use different registers
- **Code generation**
  - generate one pipelined loop with only one exit
    (at beginning of steady state)
  - generate one unpipelined loop to handle the rest
  - code generation is the messiest part of the algorithm!
- **HW support: rotating register files (Cydrome Cydra/iA64)**
  - indexed register access (e.g. reg. no + contents of special reg)

Conclusions

- **Numerical Code**
  - Software pipelining is useful for machines with a lot of pipelining and instruction level parallelism
  - Compact code
  - Limits to parallelism: dependences, critical resource