Tiling: A Data Locality Optimizing Algorithm

Announcements

- Monday November 28th, Dr. Sanjay Rajopadhye is talking at BMAC
- Friday December 2nd, Dr. Sanjay Rajopadhye will be leading CS553

Last Monday

- Kelly & Pugh transformation framework
- Loop fusion and fission
- Brief intro to scheduling Alpha programs

Today

- "Unroll and Jam" and Tiling
- Review of the paper "A Data Locality Optimizing Algorithm" by Michael E. Wolf and Monica S. Lam

Loop Unrolling

Motivation

- Reduces loop overhead
- Improves effectiveness of other transformations
  - Code scheduling
  - CSE

The Transformation

- Make n copies of the loop: n is the **unrolling factor**
- Adjust loop bounds accordingly

Loop Unrolling (cont)

Example

```
do i=1,n
   A(i) = B(i) + C(i)
enddo
```

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```
do i=1,n by 2
   A(i)   = B(i) + C(i)
   A(i+1) = B(i+1) + C(i+1)
enddo
```

Details

- When is loop unrolling legal?
- Handle end cases with a cloned copy of the loop
  - Enter this special case if the remaining number of iteration is less than the unrolling factor

Loop Balance

Problem

- We'd like to produce loops with the right balance of memory operations and floating point operations
- The ideal balance is machine-dependent
  - *e.g.* How many load-store units are connected to the L1 cache?
  - *e.g.* How many functional units are provided?

Example

```
do j = 1,2*n
   do i = 1,m
      A(j) = A(j) + B(i)
   enddo
enddo
```

- The inner loop has 1 memory operation per iteration and 1 floating point operation per iteration
- If our target machine can only support 1 memory operation for every two floating point operations, this loop will be memory bound

What can we do?

Unroll and Jam

Idea

– Restructure loops so that loaded values are used many times per iteration

Unroll and Jam

– Unroll the outer loop some number of times
– Fuse (Jam) the resulting inner loops

Example

```
do j = 1,2*n
   do i = 1,m
      A(j) = A(j) + B(i)
   enddo
enddo
```

Unroll the Outer Loop

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
   enddo
   do i = 1,m
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```


Unroll and Jam Example (cont)

Unroll the Outer Loop

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
   enddo
   do i = 1,m
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```

Jam the inner loops

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```

– The inner loop has 1 load per iteration and 2 floating point operations per iteration
– We reuse the loaded value of **B(i)**
– The Loop Balance matches the machine balance

Unroll and Jam (cont)

Legality

– When is Unroll and Jam legal?

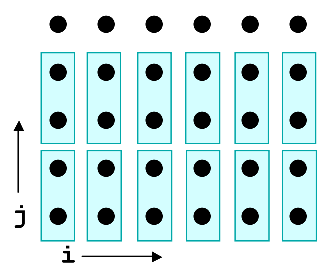
Disadvantages

– What limits the degree of unrolling?

Unroll and Jam IS Tiling (followed by inner loop unrolling)

Original Loop

```
do j = 1,2*n by 2
  do i = 1,m
    A(j)= A(j) + B(i)
  enddo
enddo
```



After Tiling

```
do jj = 1,2*n by 2
  do i = 1,m
    do j = jj, jj+2-1
      A(j)= A(j)+B(i)
    enddo
  enddo
enddo
```

After Unroll and Jam

```
do jj = 1,2*n by 2
  do i = 1,m
      A(j)= A(j)+B(i)
      A(j+1)= A(j+1)+B(i)
  enddo
enddo
```

Paper Critique and Presentation Format

Critique: 1-2 pages with a paragraph answering each of the following questions

- What problem did the paper address?
- Is the problem important/interesting?
- What is the approach used to solve the problem?
- How does the paper support or justify the conclusions it reaches?
- What problems are explicitly or implicitly left as future research questions?

Presentation: 10-15 slides that present the answers to the critique questions (example for the paper "A Data Locality Optimizing Algorithm" by Michael E. Wolf and Monica S. Lam follows)

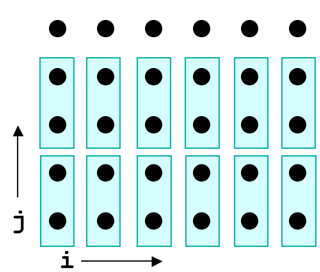
What is the problem the paper addresses?

How can we apply loop interchange, skewing, and reversal to generate

- a loop that is legally tilable (ie. fully permutable)
- a loop that when tiled will result in improved data locality

Original Loop

```
do j = 1,2*n by 2
  do i = 1,m
    A(j)= A(j) + B(i)
  enddo
enddo
```



Is the problem important/interesting?

Performance improvements due to tiling can be significant

- For matrix multiply, 2.75 speedup on a single processor
- Enables better scaling on parallel processors

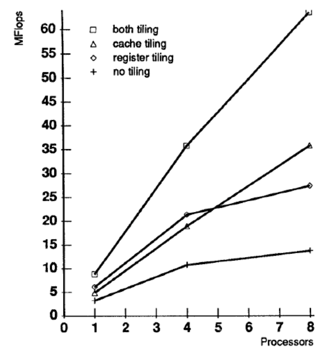


Figure 1: Performance of 500 × 500 double precision matrix multiplication on the SGI 4D/380. Cache tiles are 64 × 64 iterations and register tiles are 4 × 2.

Tiling Loops More Complex than MM

- requires making loops permutable
- goal is to make loops exhibiting reuse permutable

What is the approach used to solve the problem?

Create a unimodular transformation that results in loops experiencing reuse becoming fully permutable and therefore tilable

Formulation of the data locality optimization problem (the specific problem their approach solves)

- For a given iteration space with
  - a set of dependence vectors, and
  - uniformly generate reference sets

  the data locality optimization problem is to find the unimodular and/or tiling transform, subject to data dependences, that minimizes the number of memory accesses per iteration.

The problem is hard

- Just finding a legal unimodular transformation is exponential in the number of loops.

Terminology

Dependence vector - a generalization of distance and direction vectors

Reuse versus Locality

Localized vector space

Uniformly generated reference sets

Heuristic for solving data locality optimization problem

Perform reuse analysis to determine innermost tile (ie. localized vector space)

- only consider elementary vectors as reuse vectors

For the localized vector space, break problem into all possible tiling combinations

Apply SRP algorithm in an attempt to make loops fully permutable

- (S)kew transformations, (R)eversal transformation, and (P)ermutation
- Definitely works when dependences are lexicographically positive distance vectors
- $O(n^2*d)$ where n is the loop nest depth and d is the number of dependence vectors

How does the paper support the conclusion it reaches?

"The algorithm ... is successful in optimizing codes such as matrix multiplication, successive over-relaxation (SOR), LU decomposition without pivoting, and Givens QR factorization".

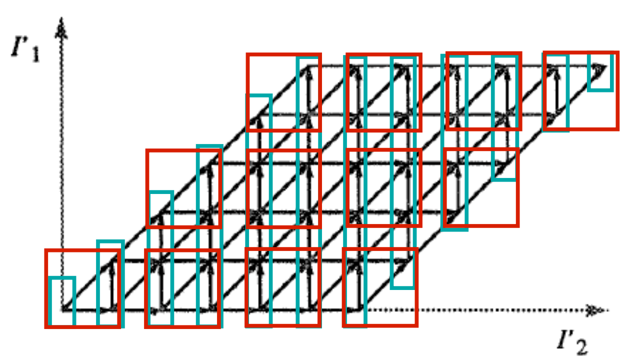
- They implement their algorithm in the SUIF compiler
- They have the compiler generate serial and parallel code for the SGI 4D/380
- They perform some optimization by hand
  - register allocation of array elements
  - loop invariant code motion
  - unrolling the innermost loop
- Benchmarks and parameters
  - LU kernel on 1, 4, and 8 processors using a matrix of size 500x500 and tile sizes of 32x32 and 64x64
  - SOR kernel on 500x500 matrix, 30 time steps

SOR Transformations

Variations of 2D (data) SOR

- wavefront version, theoretically great parallelism, but bad locality
- 2D tiling, better than wavefront, doesn't exploit temporal reuse
- 3D tile version, best performance

Picture for 1D (data) SOR



What problems are left as future research?

Explicitly stated future work

- The authors suggest that their SRP algorithm may have its performance improved with a branch and bound formulation.

Questions left unanswered in the paper

- How should the tile sizes be selected?
- After performing tiling, what algorithm should be used to determine further transformations for improved performance?
  - They perform inner loop unrolling and other, but do not perform a model for which transformations should be performed and what their parameters should be.
- What is the relationship between storage reuse, data locality, and parallelism?

Concepts

Unroll and Jam is the same as Tiling with the inner loop unrolled

Tiling can improve ...

- loop balance
- spatial locality
- data locality

Next Time (November 28th)

Student Surveys

Lecture

– Compiling object-oriented languages