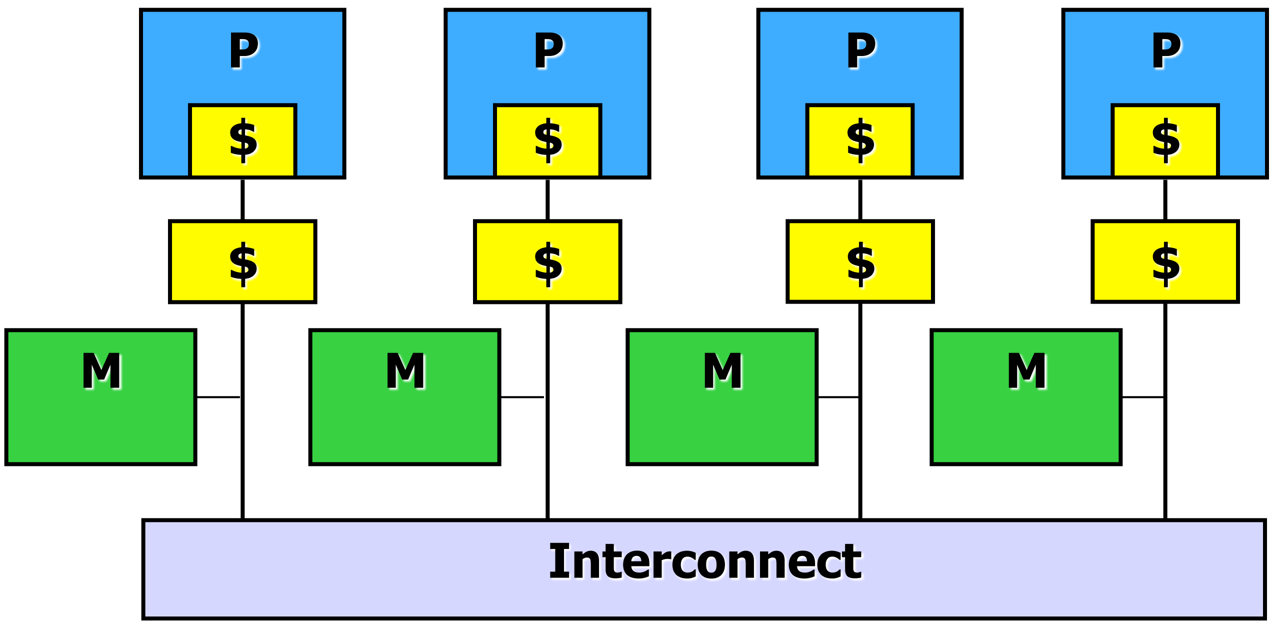
CS 243
Lecture 13
Loop Transformations for Parallelism and Locality

1. Blocking
2. Pipelining
3. Affine Partitioning: Communication-free
4. Affine Partitioning: with Communication

Readings: Chapter 11–11.3, 11.6–11.7.4, 11.9-11.9.6

Shared Memory Machines

Performance on Shared Address Space Multiprocessors:
Parallelism & Locality



Parallelism and Locality

- Parallelism DOES NOT imply speed up!

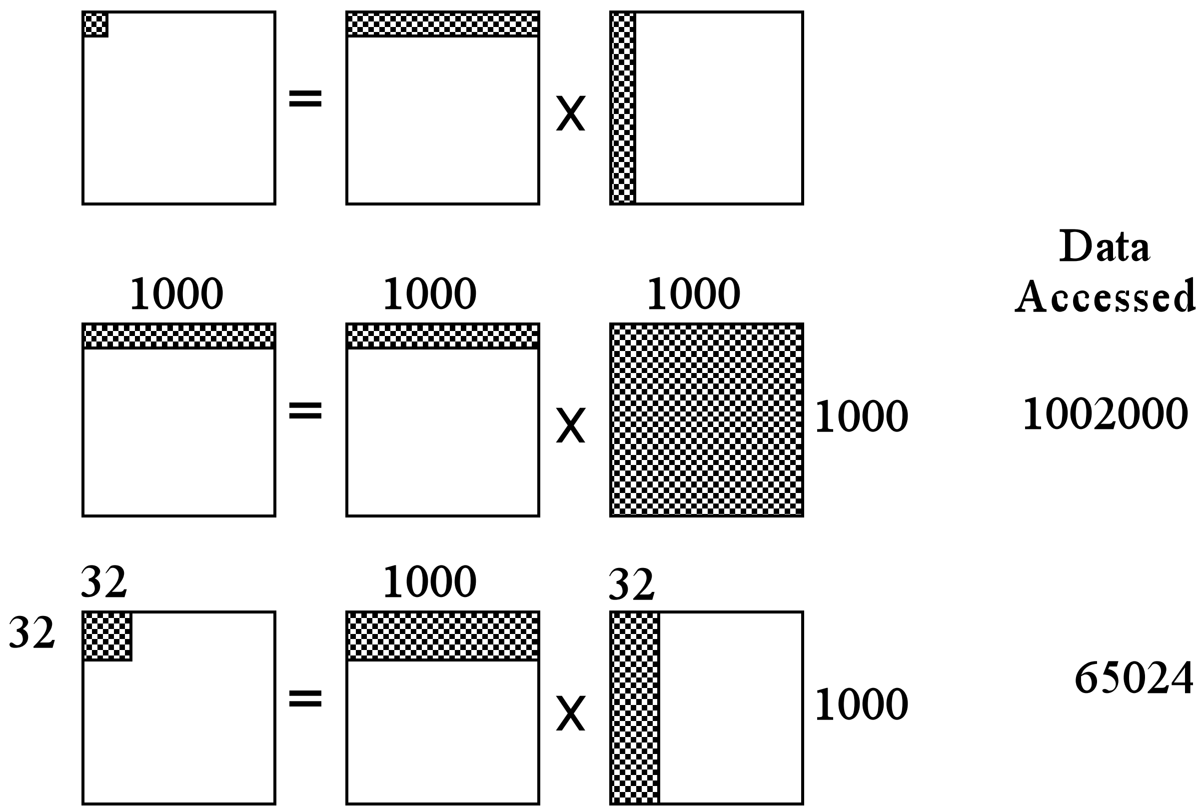
- Parallel performance:
  Improve locality with loop transformations
  - Minimize communication
  - Operations using the same data are executed on the same processor

- Sequential performance:
  Improve locality with loop transformations
  - Minimize cache misses
  - Operations using the same data are executed close in time.

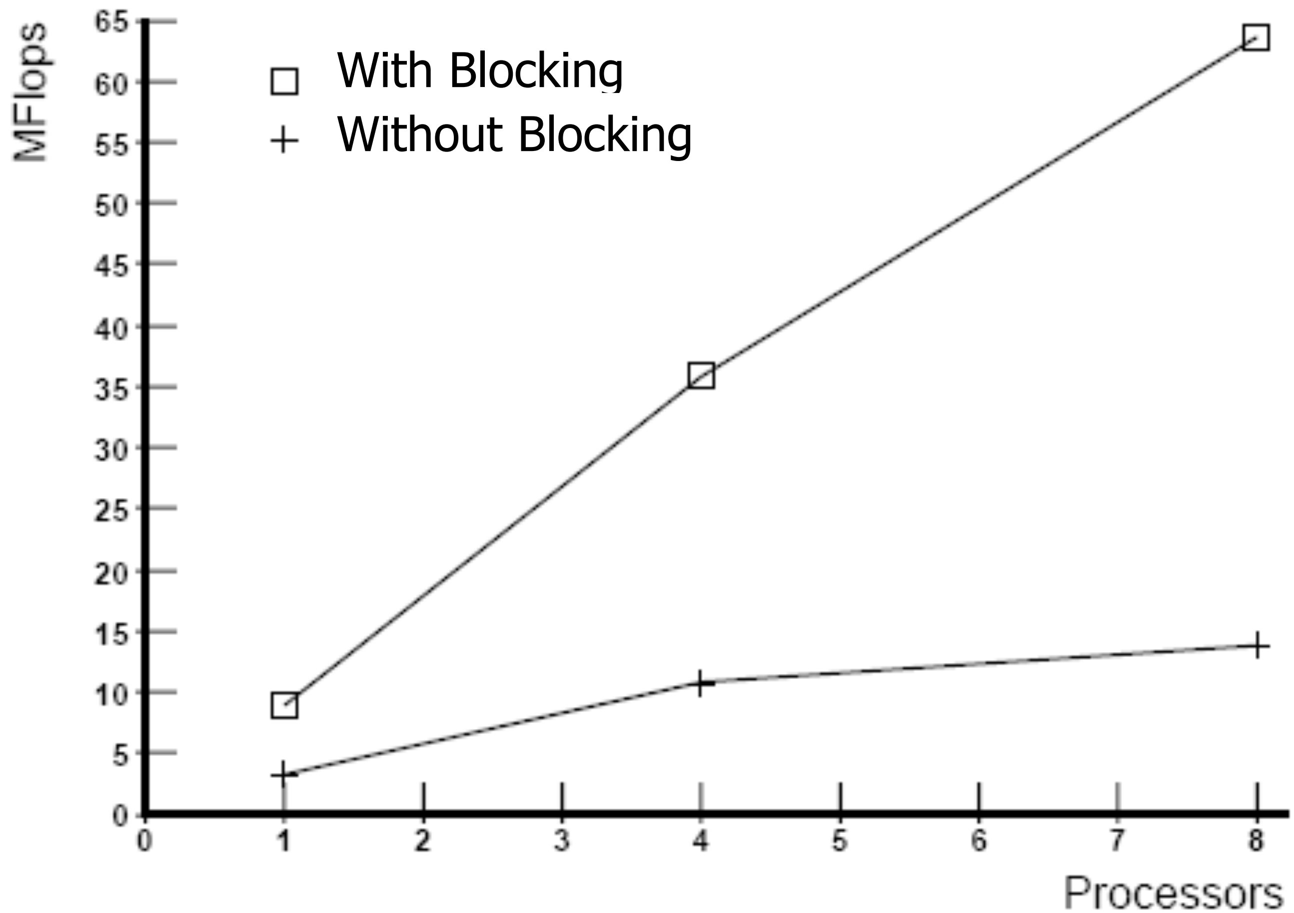
Important Concepts in Parallelization & Locality Opt.

- Two kinds of loop transforms
  - Blocking
  - Affine partitioning
- Two kinds of parallelism
  - Do-all loops
  - Pipelining

1. Blocking Example: Matrix Multiplication



Experimental Results



Code Transform

- Before

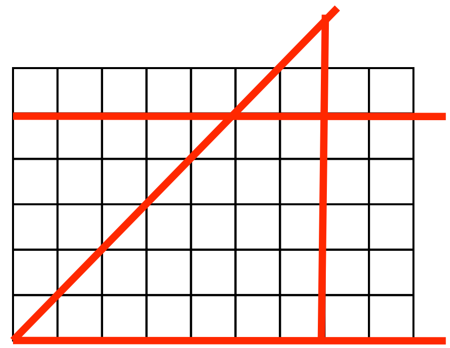
```
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++) {
    for (k = 0; k < n; k++) {
      Z[i,j] = Z[i,j] + X[i,k]*Y[k,j];
    }}}
```

- After

```
for (ii = 0; ii < n; ii = ii+B) {
  for (jj = 0; jj < n; jj = jj+B) {
    for (kk = 0; kk < n; kk = kk+B) {
      for (i = ii; i < min(n,kk+B); i++) {
        for (j = jj; j < min(n,kk+B); j++) {
          for (k = kk; k < min(n,kk+B); k++) {
            Z[i,j] = Z[i,j] + X[i,k] * Y[k,j];
          }}}}}}
```

2. Iteration Space

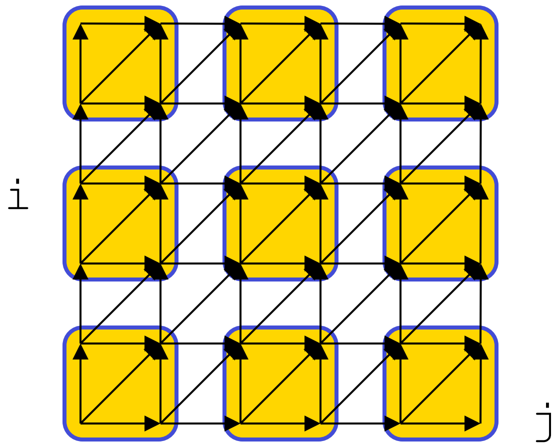
```
FOR i = 0 to 5
  FOR j = i to 7
    …
```



- n-deep loop nests: n-dimensional polytope
- Iterations: coordinates in the iteration space
- Assume: iteration index is incremented in the loop
- Sequential execution order: lexicographic order
  - [0,0], [0,1], …, [0,6], [0,7],
    [1,1], …, [1,6], [1,7], …

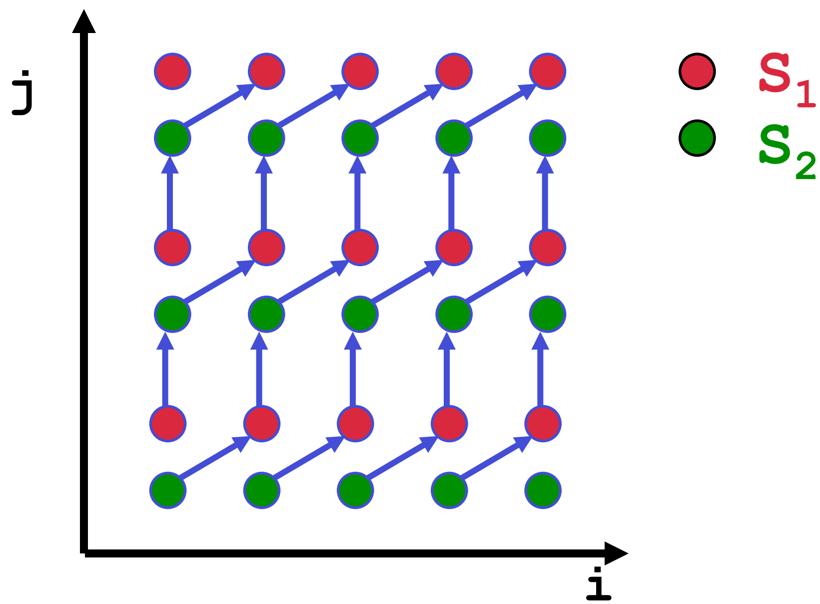
Pipelining Example: SOR (Successive Over-Relaxation)

```
for i = 0 TO m
  for j = 0 to n
    X[j+1] = 1/3 * (X[j] + X[j+1] + X[j+2])
```



3. Affine Partitioning: An Contrived but Illustrative Example

```
FOR j = 1 TO n
    FOR i = 1 TO n
        A[i,j] = A[i,j]+B[i-1,j];          (S1)
        B[i,j] = A[i,j-1]*B[i,j];          (S2)
```



Best Parallelization Scheme

Algorithm finds affine partition mappings for each instruction:

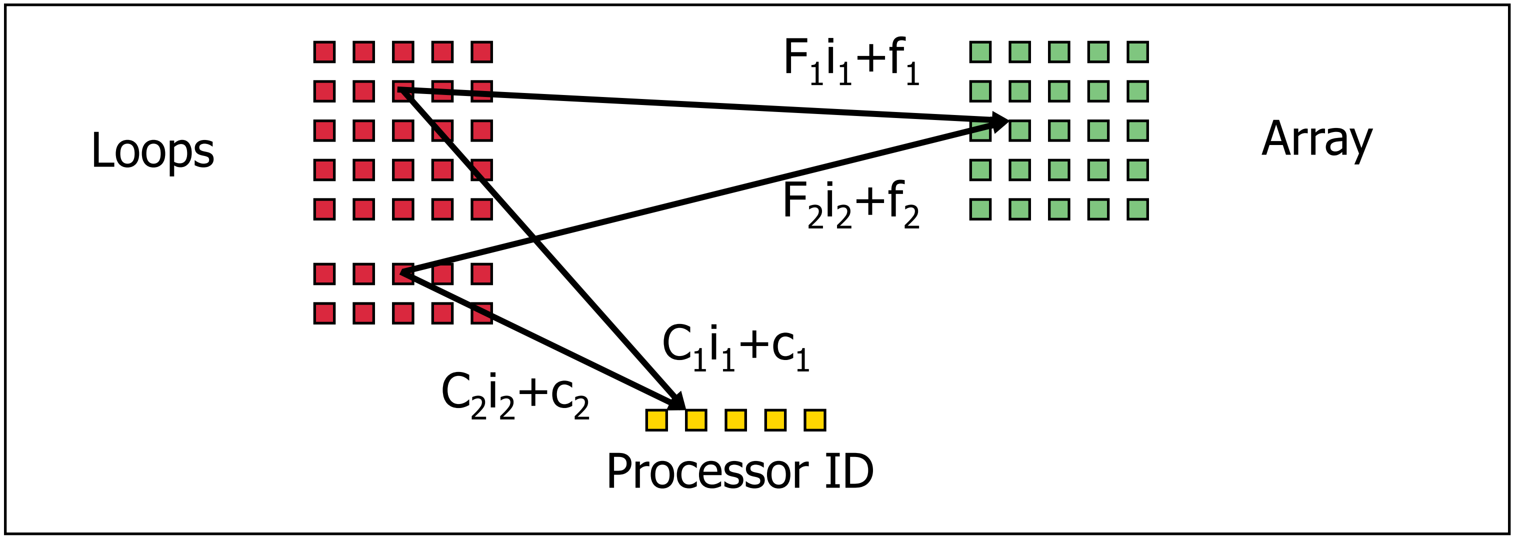
S1: Execute iteration (i, j) on processor i-j.

S2: Execute iteration (i, j) on processor i-j+1.

SPMD code: Let p be the processor's ID number

```
if (1-n <= p <= n) then
  if [1 <= p) then
   B[p,1] = A[p,0] * B[p,1];                  (S2)
  for i1 = max[1,1+p) to min[n,n-1+p) do
   A[i1,i1-p] = A[i1,i1-p] + B[i1-1,i1-p];    (S1)
   B[i1,i1-p+1] = A[i1,i1-p] * B[i1,i1-p+1];  (S2)
  if (p <= 0) then
   A[n+p,n] = A[n+p,N] + B[n+p-1,n];          (S1)
```

Maximum Parallelism & No Communication



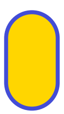
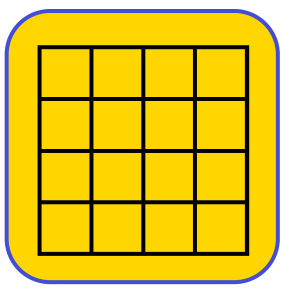
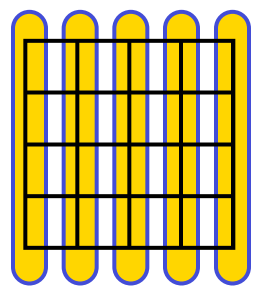
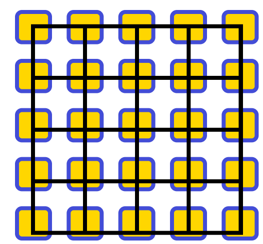
For every pair of data dependent accesses $F_1i_1+f_1$ and $F_2i_2+f_2$

Find C_1, c_1, C_2, c_2:

$\forall\ i_1, i_2 \quad F_1 i_1 + f_1 = F_2 i_2 + f_2 \rightarrow C_1i_1+c_1 = C_2i_2+c_2$

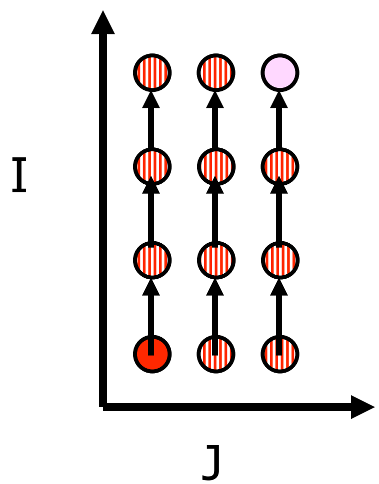
with the objective of maximizing the rank of C_1, C_2

Rank of Partitioning = Degree of Parallelism

Affine Mapping	$\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}i\\j\end{bmatrix}$	$\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}i\\j\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}i\\j\end{bmatrix}$
Rank	0	1	2
 Mapped to same processor			

Code Generation

```
for I = 1 to 4
  for J = 1 to 3
    Z[I,J] = Z[I-1,J]
```



$p = j$

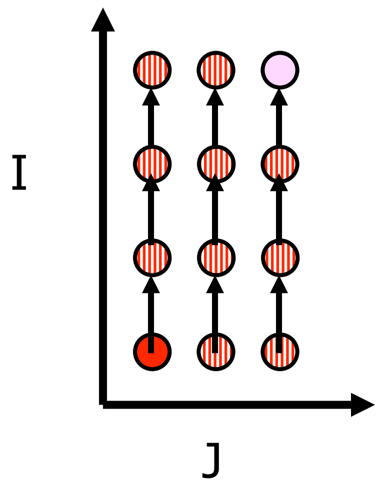
```
for P = 1 to 3
  for I = 1 to 4
    for J = 1 to 3
      if (j == P)
        Z[I,J] = Z[I-1,J]

for P = 1 to 3
  for I = 1 to 4
    Z[I,P] = Z[I-1,P]
```

SPMD (single program multiple data) code:

```
for I = 1 to 4
    Z[I,P] = Z[I-1,P]
```

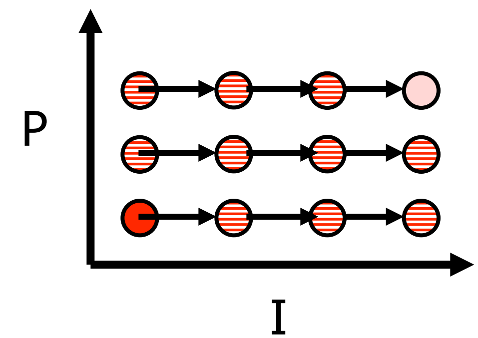
Loop Permutation (Loop Interchange)

```
for I = 1 to 4
  for J = 1 to 3
    Z[I,J] = Z[I-1,J]
```



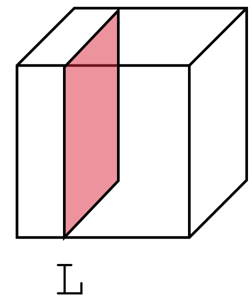
$$\begin{bmatrix} p' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}$$

```
for P = 1 to 3
  for I = 1 to 4
    Z[I,P] = Z[I-1,P]
```

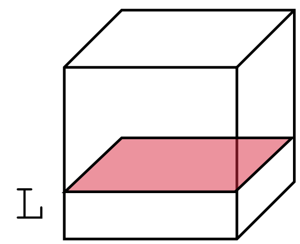


Optimizing Arbitrary Loop Nesting Using Affine Partitions (chotst, NAS)

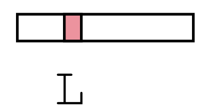
```
      DO 1 J = 0, N
         I0 = MAX ( -M, -J )
          DO 2 I = I0, -1
             DO 3 JJ = I0 - I, -1
                DO 3 L = 0, NMAT
3                 A(L,I,J) = A(L,I,J) - A(L,JJ,I+J) * A(L,I+JJ,J)
             DO 2 L = 0, NMAT
2              A(L,I,J) = A(L,I,J) * A(L,0,I+J)
          DO 4 L = 0, NMAT
4           EPSS(L) = EPS * A(L,0,J)
          DO 5 JJ = I0, -1
             DO 5 L = 0, NMAT
5              A(L,0,J) = A(L,0,J) - A(L,JJ,J) ** 2
          DO 1 L = 0, NMAT
1           A(L,0,J) = 1. / SQRT ( ABS (EPSS(L) + A(L,0,J)) )

         DO 6 I = 0, NRHS
          DO 7 K = 0, N
             DO 8 L = 0, NMAT
8              B(I,L,K) = B(I,L,K) * A(L,0,K)
             DO 7 JJ = 1, MIN (M, N-K)
                DO 7 L = 0, NMAT
7                 B(I,L,K+JJ) = B(I,L,K+JJ) -  A(L,-JJ,K+JJ) * B(I,L,K)
          DO 6 K = N, 0, -1
             DO 9 L = 0, NMAT
9              B(I,L,K) = B(I,L,K) * A(L,0,K)
             DO 6 JJ = 1, MIN (M, K)
                DO 6 L = 0, NMAT
6                 B(I,L,K-JJ) = B(I,L,K-JJ) - A(L,-JJ,K) * B(I,L,K)
```



A



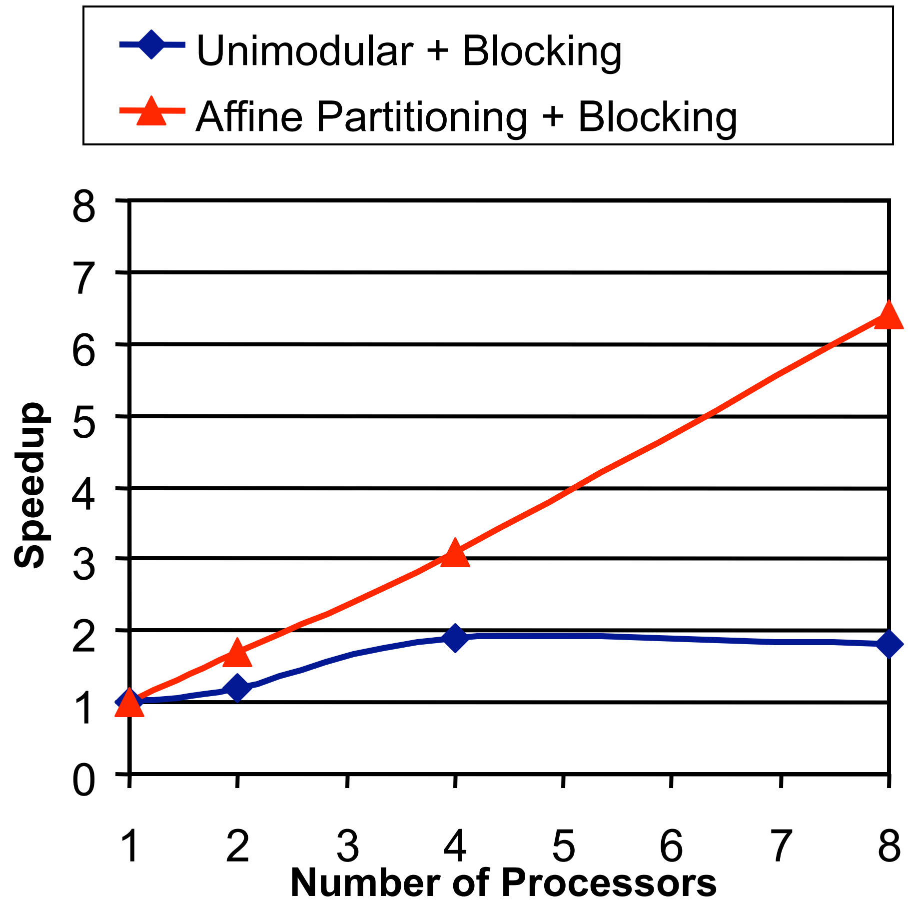
B



EPSS

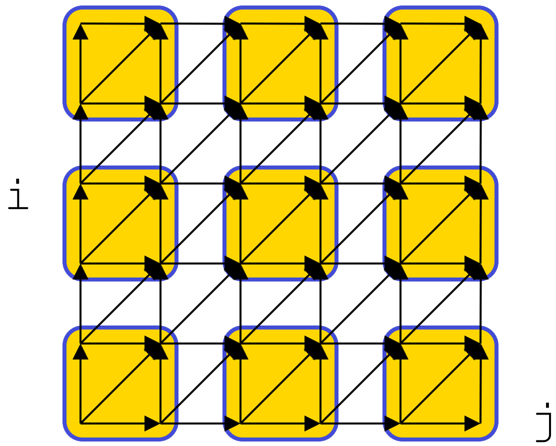
Chotst: Results with Affine Partitioning + Blocking

(Unimodular: a subset of affine partitioning for perfect loop nests)

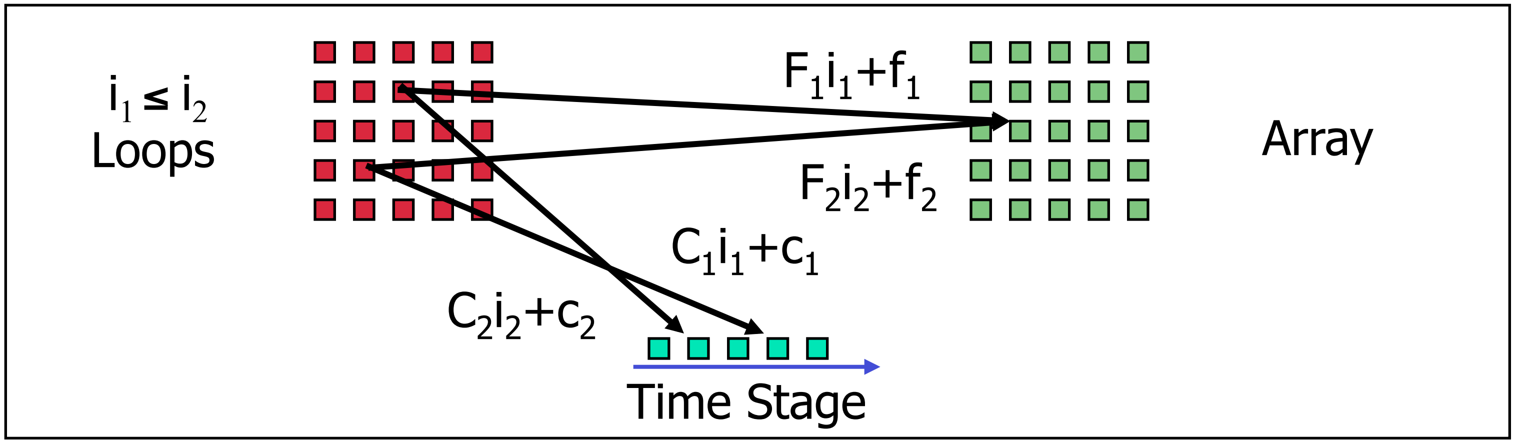


4. Advanced topic: Pipelining
SOR (Successive Over-Relaxation): An Example

```
for i = 0 TO m
  for j = 0 to n
    X[j+1] = 1/3 * (X[j] + X[j+1] + X[j+2])
```



Finding the Maximum Degree of Pipelining



For every pair of data dependent accesses $F_1 i_1 + f_1$ and $F_2 i_2 + f_2$

Let $B_1 i_1 + b_1 \geq 0$, $B_2 i_2 + b_2 \geq 0$ be the corresponding loop bound constraints,

Find C_1, c_1, C_2, c_2:

$\forall\ i_1, i_2 \quad B_1 i_1 + b_1 \geq 0,\ B_2 i_2 + b_2 \geq 0$

$$(i_1 \leq i_2) \wedge (F_1 i_1 + f_1 = F_2 i_2 + f_2) \rightarrow C_1 i_1 + c_1 \leq C_2 i_2 + c_2$$

with the objective of maximizing the rank of C_1, C_2

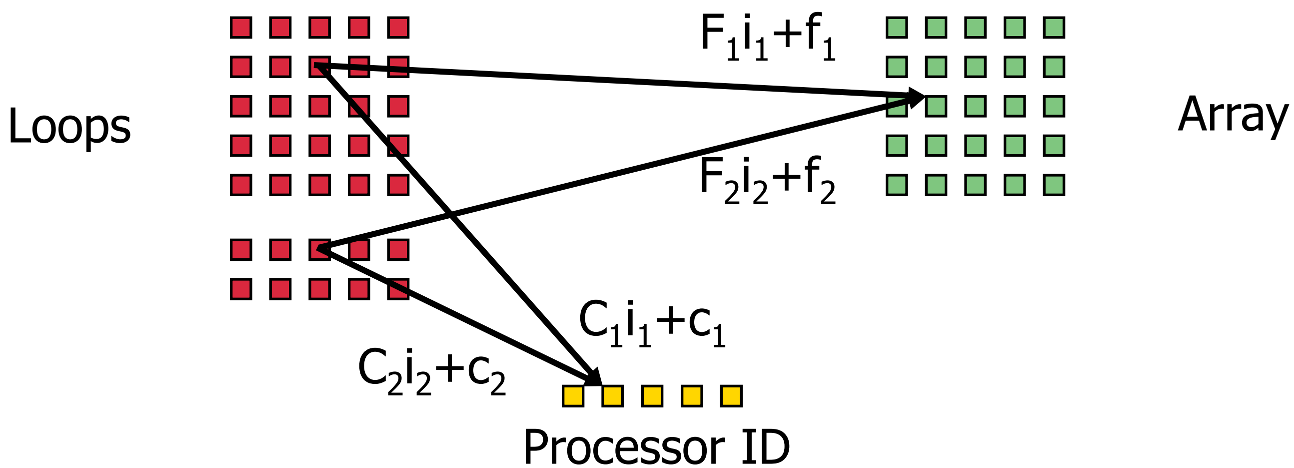
Key Insight

- Choice in time mapping => (pipelined) parallelism
- Rank(C) – 1 degree of parallelism with
  1 degree of synchronization
- Can create blocks with Rank(C) dimensions

- Find time partitions is not as straightforward as space partitions
  - Need to deal with linear inequalities
  - Solved using Farkas Lemma – no simple intuitive proof

Summary

Communication-Free



Pipelining