A Market Implied Valuation of Loans and their Embedded Optionality

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1 Introduction

Recent years have seen a gradual movement in accounting standards and regulation towards valuing loans at fair value in a more forward-looking manner. For example, IFRS 9 accounting rules require that loans use a forward-looking expected credit loss model to increase the timeliness of loss recognition and maximise the use of relevant observable market data. However, this is a difficult task as loans are typically not traded and consequently there are no directly observable market prices.

A related problem is that loans are often long-dated and have embedded derivatives which – on their own – would typically be valued in a risk-neutral framework. Most methods for discounting loan cashflows treat effects such as default and prepayment only approximately. It is therefore interesting to consider the valuation of loans in a risk-neutral framework to evaluate both forward-looking expected losses and the embedded optionality in a consistent model that accounts for the specifics of the cashflows, the default and recovery process and the embedded optionality.

2 Basic Loan Pricing

The valuation of loans can be separated into the following distinct components:

- *Principal repayments.* These are repayments on the principal amount, which are determined by the change in the principal schedule at a given time. Whilst some loans are bullet-like with a single repayment at maturity, others are amortising, with repayment occurring throughout the lifetime.
- *Coupon payments.* These are the coupon payments that depend on whether the underlying loan is fixed or floating:
    - *Fixed.* In this case, there will be a series of fixed payments determined by a fixed-rate and the principal schedule.
    - *Floating.* In this case, the payments are not yet fixed, and for valuation purposes, it is necessary to calculate the appropriate expected floating payment from the discount factors on the projection curve. This is then multiplied by the principal amount at the time in question to give the appropriate expected cashflow. The first payment will usually already have been fixed.
- *Spread payments.* For floating-rate loans, there is typically a deterministic spread that is added to the floating payments which will be similar to the fixed payments above.

A risky valuation of loans must take into account the relevant interest rates for default-free valuation and also the appropriate survival probabilities. It is also necessary to consider the recovery amount that would be applicable in the event of a default of the borrower.

A market implied valuation of loans requires an assessment of:

- the interest rate curve for discounting;
- the interest rate curve for projection (where relevant for floating rate loans which may differ from the above);
- the appropriate credit spread curve (for example, via credit default swap (CDS) data); and
- the loan recovery rate.

Making use of observable data where possible might involve using CDS quotes for the obligor in question where available (e.g. loans to sovereigns or large corporations) and the use of proxies (e.g. peer or comparable market information) otherwise. This market implied or 'risk-neutral' approach to determining survival probabilities (and equivalently default probabilities) is standard for valuation even though the underlying CDS market is not especially liquid. Relative generic methods to estimate CDS proxy spreads (e.g. Chourdakis et al. 2012) have emerged in relation to the estimation of fair value. An alternative approach is to use more dynamic structural models for default probability estimation.

The precise recovery rate assumptions are also important. Most commonly, recovery approaches use the obligation principal which is in line with the most common bankruptcy procedures where voting rights are usually based upon nominal amounts. Most historical recovery statistics are based on (a percentage of) par value and recovery of face value (**RFV**) is the most common modelling assumption used in credit risk pricing. An alternative to the RFV approach is a recovery of market value (**RMV**) approach where the recovery is assumed to be a proportion of the present value of the remaining cashflows at the time of default. This is less commonly used but has some nice properties such as leading to default-free valuation as the recovery tends to 100% and creating fungibility between different components (e.g. bonds and strips).

There is also the question of the relative seniority of the loans compared to the underlying CDS market. Corporate and sovereign CDS generally reference senior unsecured debt whereas loans are usually more senior. Recovery rates used in the CDS market are not tradable and are not even consensus values but are rather conventions used for valuing transactions in relation to unwinds or novations. Indeed, the recovery rate has a minimal impact on CDS valuation, unless the underlying credit is particularly distressed. An approximate assessment of the loan recovery should consider that it is really the ratio of the loan loss given default (**LGD**) to the CDS LGD that drives the loan value. This would ensure, for example, that a change in the CDS market convention would not impact the value of loans significantly. It should be noted that in this paper we are not explicitly considering the special case of collateralised loans where the precise collateral available may enter directly into the analysis.

3 Optionality

The embedded optionality in loans may exist due to two contractual features:

- *Prepayment.* Typically, obligors have the right to prepay loans with a specified notice period. The lender is therefore short this prepayment option which often is of Bermudan style since the obligor can prepay at multiple points. This relates to both interest rates and credit spreads which can both drive the decision to repay.
- *Remuneration.* For floating rate loans, future reference rates may be negative or have significant non-zero probability of being negative. However, lenders will typically have clauses in documentation that prevents them being required to remunerate the borrower when rates are negative. This optionality may relate to just the floating rate (more commonly used) or the floating rate plus the spread. The lender therefore has a benefit which is European style and relates only to interest rates.

Note that the above optionalities are also linked. For example, the remuneration floor increases the value of the liability for the loan obligor, which may, in turn, make them more likely to prepay the loan. It is therefore not possible to separate the above components (although in practice they may be mutually exclusive). The optionality therefore needs to be valued jointly in a hybrid interest rate and credit model due to the repayment option having sensitivity to both factors and the linkage between this and the remuneration floor optionality.

In the examples below, we use a one-factor Hull-White model for interest rates with a deterministic basis assumption (the latter assumption is relevant for floating rate loans with a reference rate that differs from the discount rate). This model is analytically tractable and allows negative interest rates. The volatility term structure can be calibrated to caps, floors or swaptions.

For hazard rates, we use a Cox-Ingersoll-Ross (CIR) model which is also quite analytically tractable. The parameters can be calibrated to the CDS curve and credit spread options (if available). This model prevents negative credit spreads if the so-called 'Feller condition' is satisfied for hazard rates to be strictly positive. The so-called CIR++ approach can be used to match survival probabilities exactly.

The two-dimensional option model is implemented via a lattice (grid/tree). The advantage of this approach is that it is generally efficient and accurate (as long as a sufficient number of steps are taken). An alternative implementation approach would be the so-called American Monte Carlo approach that would use regression or other approaches to value the Bermudan-style optionality.

4 Examples

The following examples will consider two hypothetical loans with features broadly outlined in Table 1. Since the loans are long-dated and the borrowers are relatively risky, then the credit component is very important. The risk-free and risky valuations are shown (from the point of view of the lender) in Table 2. Note that there is not much difference between the different recovery assumptions (RFV or RMV). Note also that Loan 2 has significant probability of negative floating rates which is seen by the overall value of the coupons (ignoring the optionality) being slightly negative.

Table 1: *Example loans.*

	Currency	Type	Coupon	Notional	Maturity
Loan 1	USD	Fixed	2%	175,000,000	35 years
Loan 2	EUR	Floating	Floating plus 1%	1,000,000,000	25 years

Table 2: *Basic loan valuation (no optionality).*

	Loan 1			Loan 2		
	Risk-free	Risky (RFV)	Risky (RMV)	Risk-free	Risky (RFV)	Risky (RMV)
Principal	179,424,776	38,669,251	38,669,251	1,028,902,553	341,182,437	341,182,437
Coupons	78,671,705	33,512,578	33,512,578	-4,529	-6,543,895	-6,543,895
Spread				150,695,872	79,756,199	79,756,199
Recovery		107,405,810	106,101,000		605,391,118	598,178,343
Total	258,096,481	179,587,639	178,282,829	1,179,593,896	1,019,785,859	1,012,573,084

Turning to the value of the embedded optionality, we first consider Loan 1 which has only prepayment-related option value.

Table ***3*** shows the value related to stochastic interest rates and credit only and the overall valuation. Note that both interest rate and credit spreads are important in determining the option value which is a non-linear combination of both. Note also that the recovery assumption now has a much more significant impact on the value. The RMV assumption leads to a higher option value, even though the loan value is slightly higher in this case (Table 2) which alone would suggest a lower value in relation to prepayment. The higher option value in the RMV case is driven by the sensitivity of the recovery to interest rates (lower interest rates will lead to a higher recovery).

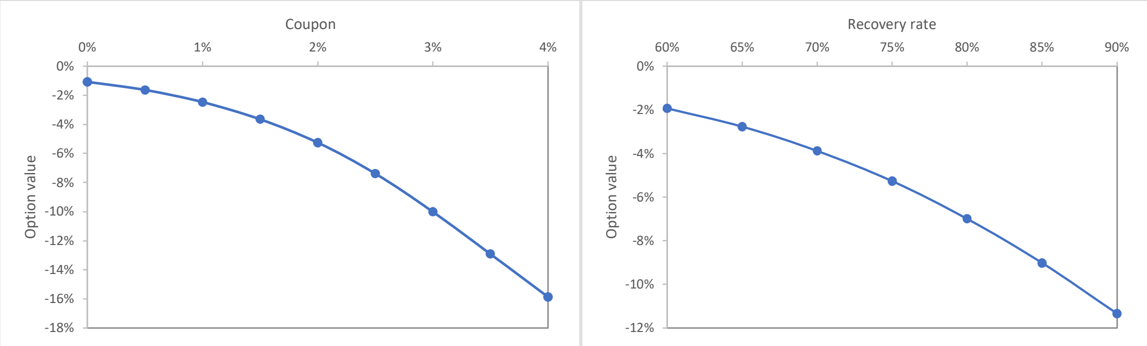
Table 3: *Option value related to Loan 1.*

	Risky (RFV)	Risky (RMV)
Interest rates only	-8,939,461	-6,664,594
Credit only	-3,387,974	-5,317,195
Both	-9,411,052	-7,883,793

Not surprisingly, the option value is also sensitive to contractual terms and model parameters.

Figure 1 shows the impact of a different coupon and recovery rate on the option value.

Figure 1. *Sensitivity of option value (as a percentage of loan principal) to coupon (left) and recovery rate (right).*



We now look at the valuation of the embedded optionality for Loan 2 which has both prepayment and remuneration option value. Under the assumption of independence between interest rates and credit, the prepayment optionality on its own can be valued via a series of zero-strike credit-contingent floorlets which are easily valued analytically in the Hull-White model (Figure 2). Note, however, that the overall option values (***Table*** 4) are not additive with respect to the different components.

Figure 2. *Valuing of remuneration optionality via zero-strike credit contingent floorlets.*

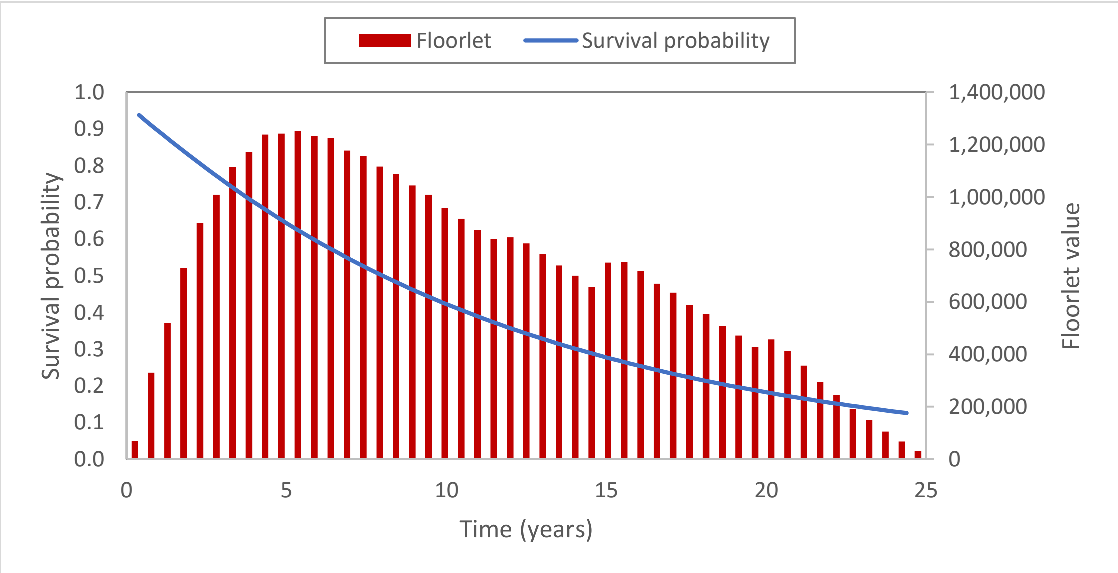
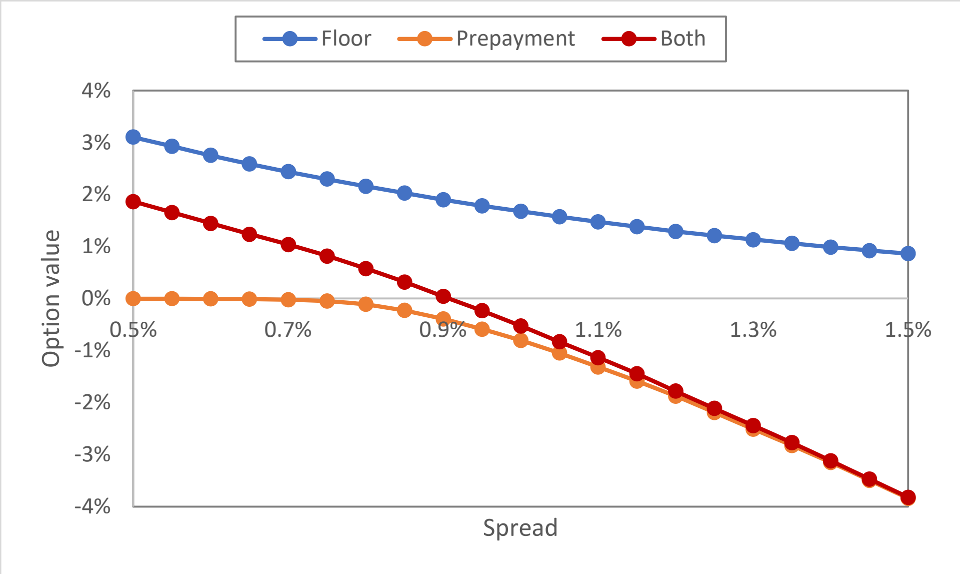


Table 4: *Option value related to Loan 2.*

	Risky (RFV)	Risky (RMV)
Remuneration only	17,820,228	17,524,570
Prepayment only	-6,635,222	-6,367,791
Prepay only (credit contingent floorlets)	-6,650,454	-6,650,454
Option	-5,222,987	-3,768,351

Where there is both prepayment and remuneration optionality, the overall effect can be quite subtle. For example, Figure 3 shows the value of the optionality in Loan 2 as a function of the spread on the loan. As the spread increases, the value from both the remuneration and prepayment optionality moves against the lender (the former option becomes less positive and the latter more negative).

Figure 3. *Option value (as a percentage of loan principal) for Loan 2 as a function of the loan spread.*



5 Conclusion

This paper has outlined a risk-neutral valuation of loans, maximising the use of available market data (for example, yield curves, CDS curves and interest rate option volatilities). This approach can be used to consistently value loans taking into account the cashflows, default and recovery process and prepayment and remuneration optionality. Changes in regulation and accounting, such as the implementation of IFRS 9, together with market changes, such as significant negative interest rate probability, are likely to lead to an increase in the use and applicability of such approaches.

6 References

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