The Problem

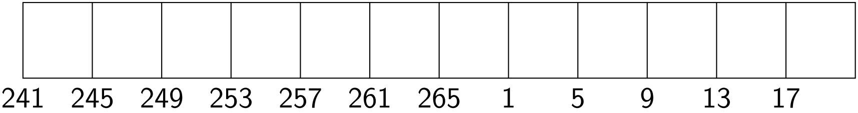
Amagqirha writes 268 numbers around the circumference of a circle such that the sum of any 20 consecutive numbers is 75. The numbers 3, 4 and 9 are in the positions 17, 83 and 144 respectively. What number is in position 210?

$$a_1 + a_2 + a_3 + \ldots + a_{20} = 75$$

$$a_1 + a_2 + a_3 + \ldots + a_{20} = 75$$
$$a_2 + a_3 + a_4 + \ldots + a_{21} = 75$$

$$a_1 + a_2 + a_3 + \ldots + a_{20} = 75$$
$$a_2 + a_3 + a_4 + \ldots + a_{21} = 75$$
$$a_1 - a_{21} = 0$$

The numbers cycle every 20 positions.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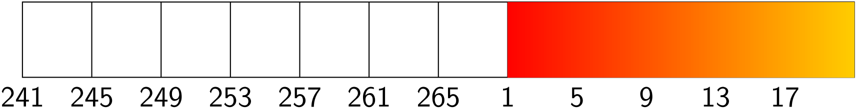
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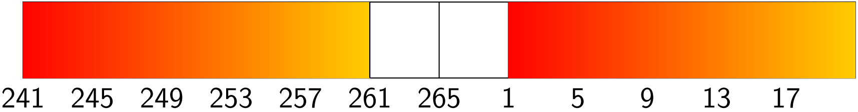
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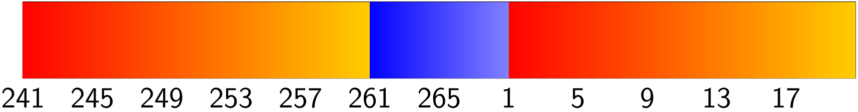
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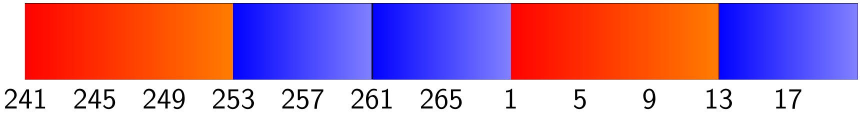
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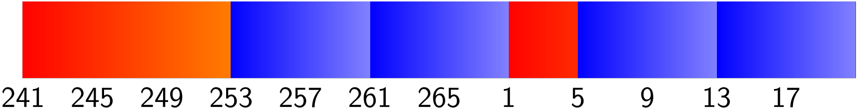
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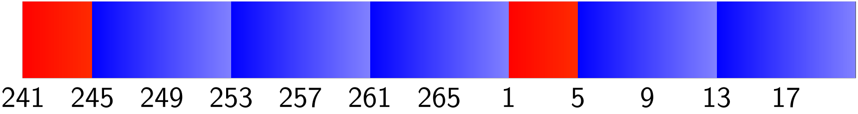
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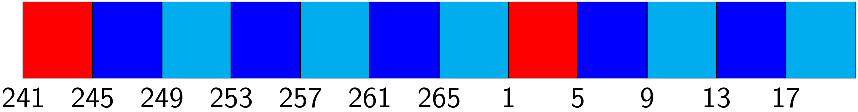
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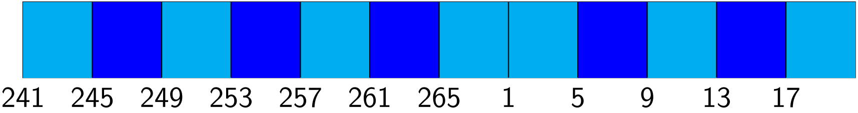
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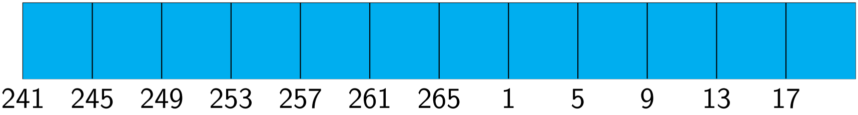
241 245 249 253 257 261 265 1 5 9 13 17

3 in position 17 $\rightarrow$ 3 in position 13 $\rightarrow \dots$ 3 in position 1.

3 in position 17 $\rightarrow$ 3 in position 13 $\rightarrow \ldots$ 3 in position 1.
4 in position 3

3 in position 17 $\rightarrow$ 3 in position 13 $\rightarrow$ ... 3 in position 1.
4 in position 3
9 in position 4

3	x	4	9

3 in position 17 $\to$ 3 in position 13 $\to \ldots$ 3 in position 1.
4 in position 3
9 in position 4

$$\boxed{3 \qquad x \qquad 4 \qquad 9}$$

$$5 \cdot (3 + x + 4 + 9) = 75$$

3 in position 17 $\to$ 3 in position 13 $\to \dots$ 3 in position 1.
4 in position 3
9 in position 4

3	x	4	9

$$5 \cdot (3 + x + 4 + 9) = 75$$

$$x = -1$$

Theorem

Suppose a sequence (a_n) has a cycle of length a and a cycle of length b. Then it also has a cycle of length $\gcd(a, b)$.