# Chapter 1

# Preliminaries: Set theory and categories

## 1 Naive set theory

### Exercises

**1.1.** Locate a discussion of Russell's paradox, and understand it.

*Solution.* □

**1.2.** ▷ Prove that if $\sim$ is a relation on a set $S$, then the corresponding family $\mathscr{P}_\sim$ defined in §1.5 is indeed a partition of $S$: that is, its elements are nonempty, disjoint, and their union is $S$. [§1.5]

*Solution.* □

**1.3.** ▷ Given a partition $\mathscr{P}$ on a set $S$, show how to define an equivalence relation $\sim$ on $S$ such that $\mathscr{P}$ is the corresponding partition. [§1.5]

*Solution.* content... □

**1.4.** How many different equivalence relations may be defined on the set $\{1, 2, 3\}$?

*Solution.* content... □

**1.5.** Give an example of a relation that is reflexive and symmetric but not transitive. What happens if you attempt to use this relation to define a partition on the set? (Hint: Thinking about the second question will help you answer the first one.)

*Solution.* content... □

**1.6.** ▷ Define a relation $\sim$ on the set $\mathbb{R}$ of real numbers by setting $a \sim b \iff b - a \in \mathbb{Z}$. Prove that this is an equivalence relation, and find a 'compelling' description for $\mathbb{R}/\sim$. Do the same for the relation $\approx$ on the plane $\mathbb{R} \times \mathbb{R}$ defined by declaring $(a_1, a_2) \approx (b_1, b_2) \iff b_1 - a_1 \in \mathbb{Z}$ and $b_2 - a_2 \in \mathbb{Z}$. [§II.8.1, II.8.10]

*Solution.* content... □

# 2 Functions between sets

## Exercises

# 3 Categories

## Exercises

# 4 Morphisms

## Exercises

# 5 Universal properties

## Exercises