6) $\chi^2(a_0, a_1) = \sum_{i=1}^{n} \left(y_i - (a_0 + a_1 x_i)\right)^2 = \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i\right)^2$

Para minimizar $\chi^2(a_0, a_1)$ se deriva respecto a a_0 y a_1. Para cada caso se iguala a 0 y se despeja a_0 y a_1.

- $$\frac{\partial(\chi^2(a_0, a_1))}{\partial a_0} = \sum_{i=1}^{n} -2(y_i - a_0 - a_1 x_i) = 0$$

$$\sum_{i=1}^{n} y_i - a_0 \sum_{i=1}^{n} 1 - a_1 \sum_{i=1}^{n} x_i = 0$$, Vease que: $\sum_{i=1}^{n} 1 = n$

$$\sum_{i=1}^{n} y_i - a_1 \sum_{i=1}^{n} x_i = a_0 n$$

$\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$:= Promedio de x

$$a_0 = \frac{\sum_{i=1}^{n} y_i}{n} - a_1 \frac{\sum_{i=1}^{n} x_i}{n}$$, $\bar{y} = \dfrac{\sum_{i=1}^{n} y_i}{n}$:= Promedio de y

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$\frac{\partial\left(\chi^2(a_0,a_1)\right)}{\partial a_1} = \sum_{i=1}^{n} -2x_i\left(y_i - a_0 - a_1 x_i\right) = 0$$

$$\sum_{i=1}^{n} x_i y_i - a_0 \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^{2} = 0$$

Reemplazando a_0

$$\sum_{i=1}^{n} x_i y_i - \left(\bar{y} - a_1 \bar{x}\right) \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^{2} = 0$$

Aplicando distributiva y "devolviendo" las definiciones de $\bar{x}$, $\bar{y}$

$$\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n} - a_1\left(-\frac{\sum_{i=1}^{n} x_i^{2}}{n} + \sum_{i=1}^{n} x_i^{2}\right) = 0$$

$$a_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^{2} - \frac{\sum_{i=1}^{n} x_i^{2}}{n}}$$

Realizamos el proceso descrito anteriormente, pero ahora para:

$$x^2(a_0, a_1, a_2) = \sum_{i=1}^{n} (y_i - (a_0 + a_1x_i + a_2x_i^2))^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1x_i - a_2x_i^2)^2$$

- $$\frac{\partial(x^2(a_0,a_1,a_2))}{\partial a_0} = \sum_{i=1}^{n} -2(y_i - a_0 - a_1x_i - a_2x_i^2) = 0$$

$$\sum_{i=1}^{n} y_i - a_0\sum_{i=1}^{n} - a_1\sum_{i=1}^{n} x_i - a_2\sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} \left[y_i = a_0 + a_1x_i + a_2x_i^2\right]$$

- $$\frac{\partial(x^2(a_0,a_1,a_2))}{\partial a_1} = \sum_{i=1}^{n} -2x_i(y_i - a_0 - a_1x_i - a_2x_i^2) = 0$$

$$\sum_{i=1}^{n} x_iy_i - a_0x_i - a_1x_i^2 - a_2x_i^3 = 0$$

$$\sum_{i=1}^{n} \left[x_iy_i = a_0x_i + a_1x_i^2 + a_2x_i^3\right]$$

- $$\frac{\partial(x^2(a_0,a_1,a_2))}{\partial a_2} = \sum_{i=1}^{n} -2x_i^2(y_i - a_0 - a_1x_i - a_2x_i^2) = 0$$

$$\sum_{i=1}^{n} x_i^2y_i - a_0x_i^2 - a_1x_i^3 - a_2x_i^4 = 0 \Rightarrow \sum_{i=1}^{n}\left[x_i^2y_i = a_0x_i^2 + a_1x_i^3 + a_2x^4\right]$$

¿Nota alguna regularidad?

Al observar el sistema de ecuaciones obtenido, es evidente que para la derivada de cada parametro a_n el termino que acompaña a y_i es x^n. Por tanto, este grado aumenta en 1 para cada derivada.