6.8.12 Parte teórica

$E = n_0 \epsilon_0 + n_1 \epsilon_1 \qquad N = n_0 + n_1$

a) Para saber la cantidad de micro-estados, se puede hacer usando la fórmula de combinaciones sin repetición, es decir:

$C^n_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ Al aplicarlo al problema tenemos

$\Omega(N, n_0) = \frac{N!}{n_0!(N-n_0)!} \Leftrightarrow N - n_0 = n_1$ luego $\Omega = \frac{N!}{n_0! n_1!}$

o $\Omega(N, n_1) = \frac{N!}{n_1!(N-n_1)!} = \frac{N!}{n_1! n_0!}$

b) $S(N, n_0) = k_b \ln(\Omega) \qquad \ln(N!) \approx N\ln(N) - N$

$S = k_B \ln\left(\frac{N!}{n_0! n_1!}\right) = k_B \ln(N!) - k_b \ln(n_0!) - k_b \ln(n_1!)$

$\approx k_b (N\ln(N) - N - n_0\ln(n_0) + n_0 - n_1 \ln(n_1) + n_1)$

$\approx k_b (N\ln(N) - n_0 \ln(n_0) - n_1 \ln(n_1))$

$\approx k_b (N\ln(N) - \sum_{i=0}^{1} n_i \ln(n_i))$

c) $x = n_1/N \qquad S(N, x) = k_b \ln(\Omega)$

$\Omega = \frac{N!}{n_1!(N-n_1)!} = \frac{N!}{(xN)!(N(1-x))!}$

$S(n, x) = k_b (N\ln(N) - N - xN\ln xN + xN - N(1-x)\ln(N(1-x)) + N(1-x))$

$S(N, x) = k_b (N\ln(N) - xN\ln(xN) - N(1-x)\ln(N(1-x)))$

$S(N, x) = k_b (N\ln(N) - xN\ln(x) + xN\ln(N) - N\ln(N) + N\ln(1-x) - xN\ln$
$+ \cancel{xN\ln(1-x)} \quad (1-x)\ln(1-x)$

$S(N, x) = \cancel{k_b\, xN\ln} - k_b N(x\ln x + (1-x)\ln(1-x))$

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e) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N = \left(\frac{\partial S}{\partial x}\right)_N \left(\frac{\partial x}{\partial E}\right)$

$$\frac{\partial x}{\partial E} = \frac{1}{N(\epsilon_1 - \epsilon_0)} = \frac{1}{N\Delta E}$$

$$\frac{\partial S}{\partial x} = -k_b N \left[1 + \ln(x) - \frac{1}{1-x} + \frac{x}{1-x} - \ln(1-x)\right]$$

$$\frac{\partial S}{\partial x} = -k_b N \left[\ln\left(\frac{x}{1-x}\right)\right]$$

$$\frac{\partial S}{\partial x} \cdot \frac{\partial x}{\partial E} = -\frac{k_b}{\Delta E} \ln\left(\frac{x}{1-x}\right) = \frac{1}{T}$$

$$\ln\left(\frac{x}{1-x}\right) = -\frac{\Delta E}{k_b T} \iff \frac{x}{1-x} = e^{-\Delta E/k_b T}$$

$$x\left(1 + e^{-\Delta E/k_b T}\right) = e^{-\Delta E/k_b T}$$

$$x = \frac{e^{-\Delta E/k_b T}}{\left(1 + e^{-\Delta E/k_b T}\right)} = \frac{1}{1 + e^{\Delta E/k_b T}}$$

f)

$$\cancel{f(\Delta\epsilon)} \qquad x(T) = \frac{1}{1+e^{-\Delta E/k_BT}} \qquad \lim_{T\to 0} x(T) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+1} = \frac{1}{2}$$

$$\lim_{T\to\infty} x(T) = \frac{1}{1+e^{-0}} = \frac{1}{1+1} = \frac{1}{2}$$

$$S(N, x) = -k_B N \left(\frac{1}{2}\ln\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\ln\left(\frac{1}{2}\right)\right) = -k_B N \left(\ln\left(\frac{1}{2}\right)\right) = -k_B N \ln 2^{-1}$$

$$S(N, x) = k_B N \ln 2$$

g)

$$V_i = V \qquad V_f = 2V \qquad \Omega_F = \Omega_i\, 2^N$$

por cada particula hay 2 estados más posibles.

$$S_f = k_b \ln \Omega_f \qquad S_i = k_b \ln \Omega_i$$

$$\Delta S = \cancel{k_b \ln} \; k_b \ln\left(\frac{\Omega_f}{\Omega_i}\right) = k_b \ln\left(\frac{\Omega_i 2^N}{\Omega_i}\right) = k_b N \ln 2$$