

# 11.12 Truth tables

## Truth table basics

A Boolean function can be represented in various ways, like an equation, a circuit, or a truth table. A **truth table** lists all possible variable value combinations on the left, and lists the function's value for each combination on the right. Each row corresponds to a possible minterm. Generating all combinations is done by counting up in binary.

Note: Minterms are sometimes written as m0, m1, ..., indicating their row's decimal equivalent: a'b'c' is 000 or m0, a'b'c is 001 or m1, etc..

A function with N variables will have a truth table with  $2^N$  rows:

- 2 variables yields  $2^2 = 4$  rows
- 3 variables yields  $2^3 = 8$  rows
- 4 variables yields  $2^4 = 16$  rows
- (And so on)

PARTICIPATION  
ACTIVITY

11.12.1: Truth table basics.

Start ☐ 2x speed

		a	b	f(a, b)
m0	a'b'	0	0	1
m1	a'b	0	1	0
m2	ab'	1	0	0
m3	ab	1	1	1

	a	b	c	f(a, b, c)
m0	0	0	0	
m1	0	0	1	
	0	1	0	
	0	1	1	
...	1	0	0	
	1	0	1	
	1	1	0	
m7	1	1	1	

PARTICIPATION  
ACTIVITY

11.12.2: 2-input truth table.

Consider the following incomplete truth table.

a	b	f(a, b)
0	0	M
K	J	
1	0	N
1	L	

1) What should J be?

☐ 1

☐ 0

2) What should K be?

☐ 1

☐ 0

3) What should L be?

☐ 1

☐ 0

4) Row 00 corresponds to what possible minterm?

☐ a'b'

☐ ab

5) Row 10 corresponds to what possible minterm?

☐ a'b

☐ ab'

6) Function f(a, b) = ab' is to be represented on the above table. What value should be written for N?

☐ 1

☐ 0

7) Function f(a, b) = ab' is to be represented on the above table. What value should be written for M?

☐ 1

☐ 0

8) A function f(a, b, c, d, e) has 5 variables. How many rows will the function's truth table have?

☐ 5

☐ 10

## Converting a truth table to an equation

A function captured as a truth table can be transformed to a sum-of-minterms equation by summing the minterms in rows having a 1. That equation can then be converted to a circuit.

### PARTICIPATION ACTIVITY

11.12.3: Converting a truth table to an equation, and then a circuit.

Start ☐ 2x speed

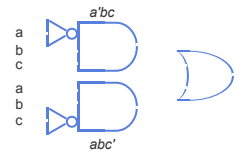
#### Truth table

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

#### Equation

$$f = a'bc + abc'$$

#### Circuit



### PARTICIPATION ACTIVITY

11.12.4: Converting a truth table to an equation.

Consider the following truth table:

e	f	y
0	0	0
0	1	1 (a)
1	0	0
1	1	1 (b)

- 1) Type the minterm corresponding to row (a).

Check [Show answer](#)

- 2) Type the minterm corresponding to row (b).

Check [Show answer](#)

- 3)  $y = ?$

Check [Show answer](#)

### PARTICIPATION ACTIVITY

11.12.5: Converting a truth table to an equation and then circuit.

Consider the following truth table. An equation will be  $y = \_ + \_$ .

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

- 1) Which is one of the minterms in the equation?

- ☐  $a'b'$   
☐  $a'b$

- 2) Which is one of the minterms in the equation?

- ☐  $ab'$   
☐  $ab$

- 3) Considering y's equation, how many AND gates will exist in a circuit derived directly from that equation?

- ☐ 1  
☐ 2  
☐ 3

4) Considering y's truth table, how many AND gates will exist in a circuit derived directly from the equation derived from that table?

- ☐ 2  
☐ 4

5) A 3-input function's truth table has 5 1's. How many AND gates will exist in a circuit derived directly from the equation derived from that table?

- ☐ 3  
☐ 5  
☐ 8

### Capturing behavior as a truth table

Some functions are more easily captured as a truth table, others as an equation.

**PARTICIPATION ACTIVITY** 11.12.6: Some functions are more easily captured as a truth table, others as an equation.

Start ☐ 2x speed

Three parking spaces exist (a, b, c).  
A parked car causes the space's variable to be 1.

a	b	c
0	1	0

y is 1 if two cars are parked adjacently

z is 1 if a car is parked on an edge

a	b	c	y	$y = a'bc + abc' + abc$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	0	
1	1	0	1	
1	1	1	1	

$z = a + c$

**PARTICIPATION ACTIVITY** 11.12.7: Capturing behavior as a truth table or equation.

Consider the above parking space example.

1) Function y could have been captured directly as an equation.

- ☐ True  
☐ False

2) Converting a truth table to a sum-of-minterms equation involves much thought and tradeoffs.

- ☐ True  
☐ False

3) A function has 12 inputs. Which is a designer more likely to try first when capturing the function's behavior?

- ☐ Truth table  
☐ Equation

4) Function z could have been captured directly as a truth table.

- ☐ True  
☐ False

5) If function z was captured as a truth table, how many rows would have an output 1 for z?

- ☐ 2  
☐ 6

### Example: Windows-open detector

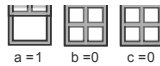
**PARTICIPATION ACTIVITY** 11.12.8: Windows-open detector: A truth table easily captures the behavior.

Start ☐ 2x speed

Three windows, each with sensor a, b, c. 1 means open.  
Sound alarm (set  $y = 1$ ) if two or more windows are open



a	b	c	y
0	0	0	0



a = 1

b = 0

c = 0

0	0	1	0	
0	1	0	0	
0	1	1	1	Two windows are open
1	0	0	0	
1	0	1	1	Two windows are open
1	1	0	1	Two windows are open
1	1	1	1	Three windows are open

$$y = a'bc + ab'c + abc' + abc$$

#### PARTICIPATION ACTIVITY

11.12.9: Windows-open example.

Consider the above windows-open example.

1) Could the behavior have been captured directly as an equation?

- ☐ Yes  
☐ No

2) How many truth table rows have 1's in the output column?

- ☐ 4  
☐ 8

3) The equation also includes specific minterms for truth table rows with 0's.

- ☐ True  
☐ False

4) The functionality of y differs depending on whether the designer captured behavior using a truth table or equation.

- ☐ True  
☐ False

#### Converting an equation to a truth table

Sometimes a designer wants to convert an equation to a truth table. Such conversion can be achieved by first transforming the equation to sum-of-minterms (discussed in an earlier section). Then, the designer can simply place a 1 in each minterm's row in the truth table.

Like sum-of-minterms form, a truth table is a canonical representation (discussed earlier) of a function.

#### CHALLENGE ACTIVITY

11.12.1: Convert the table to a sum-of-minterms.

Start

a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

y = Ex:  $ab' + a'b$

1

Next