

## 12.3 3- and 4-variable K-maps

### 3-variable K-map

A K-map for three variables has two variables across the top. For adjacent columns to differ in only one variable, note that the columns don't count up in binary (00, 01, 10, 11), but rather are 00, 01, 11, 10. Cells on the far left and far right also differ by one variable so are also "adjacent"; the K-map wraps like a bracelet.

#### PARTICIPATION ACTIVITY

12.3.1: 3-variable K-map basics.

Start ☐ 2x speed

$$y = ab'c' + ab'c + a'bc + abc'$$

a \ bc	00	01	11	10
0		$a'b'c$	$a'bc$	
1	$ab'c'$		$abc$	$abc'$

a \ bc	00	01	11	10
0	0	0	1	1
1	1	1	0	0

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

#### PARTICIPATION ACTIVITY

12.3.2: Click a cell to show/hide the corresponding minterm.

a \ bc	00	01	11	10
0				
1				

#### PARTICIPATION ACTIVITY

12.3.3: 3-variable K-map.

a \ bc	00	01	11	10
0	1(J)	0	0(L)	0(M)
1	0	0	1(K)	0

Given function  $y = a'b'c' + abc + a'bc$ , represented in the above figure's K-map.

1) (J) corresponds to which minterm?

Check [Show answer](#)

2) (K) corresponds to which minterm?

Check [Show answer](#)

3) (L) should have what value (0 or 1)?

Check [Show answer](#)

4) (M) should have what value (0 or 1)?

Check [Show answer](#)

5) Cells (L) and (K) differ in what variable:  
a, b, or c?

Check [Show answer](#)

6) Cells (L) and (M) differ in what variable:  
a, b, or c?

Check [Show answer](#)

7) Cells (L) and (J) differ in how many variables?

Check [Show answer](#)

**CHALLENGE  
ACTIVITY**

12.3.1: Select the shown minterm(s).

Start

a b c

a \ bc	00	01	11	10
0				
1				

1

2

3

4

5

6

Check

Next

**Simplification with a 3-variable K-map**

**PARTICIPATION  
ACTIVITY**

12.3.4: Simplification with a 3-variable K-map:  $i(j + j')$  opportunities are obvious.

Start ☐ 2x speed

$$y = ab'c' + ab'c + a'bc + a'bc'$$
$$y = ab' + a'b$$

a \ bc	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$ab'$

$ab'c' + ab'c$   
 $ab'(c' + c)$   
 $ab'(1)$   
 $ab'$

$a'bc + a'bc'$   
 $a'b(c + c')$   
 $a'b(1)$   
 $a'b$

**PARTICIPATION  
ACTIVITY**

12.3.5: Simplification with 3-variable K-maps.

a \ bc	00	01	11	10
0	(J) 1	1	0	1 (K)
1	0	0	0	1

a \ bc	00	01	11	10
0	1 (M)	0	0	1
1	0	1 (P)	0	0

1) Circle (J) corresponds to what simplified term?

Check [Show answer](#)

2) Circle (K) corresponds to what simplified term?

Check [Show answer](#)

3) Circle (M) corresponds to what simplified term?

Check [Show answer](#)

4) Circle (P) is what term?

Check [Show answer](#)

**PARTICIPATION  
ACTIVITY**

12.3.6: More simplification with 3-variable K-maps.

	bc	00	01	11	10
a	0	(L)1	1	0	0
	1	0	1(M)	0	0

	bc	00	01	11	10
a	0	(R)1	1	0	1(S)
	1	0	(P)1	1(Q)	0

1) Circle (L) is what simplified term?

Check [Show answer](#)

2) Circle (M) is what simplified term?

Check [Show answer](#)

3) Is circle (P) necessary? Type: yes or no

Check [Show answer](#)

4) Is circle (Q) necessary? Type: yes or no

Check [Show answer](#)

5) Is circle (S) a good circle? Type: yes or no

Check [Show answer](#)

#### PARTICIPATION ACTIVITY

12.3.7: 3-variable K-map simplification.



#### CHALLENGE ACTIVITY

12.3.2: Add fewest and largest circles to cover all the 1s.



- 1
- 2
- 3
- 4
- 5
- 6
- 7

#### CHALLENGE ACTIVITY

12.3.3: Write the simplified term or terms.



- 1
- 2
- 3
- 4
- 5
- 6
- 7

Larger circles

A circle may encompass four adjacent 1's, which removes two variables rather than just one.

**PARTICIPATION  
ACTIVITY**

12.3.8: Circling four 1's removes two variables.

Start ☐ 2x speed

a	bc			
	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$y = ab'c' + ab'c + abc + abc'$$

$$y = a(b'c' + b'c + bc + bc')$$

$$y = a(b'(c' + c) + b(c + c'))$$

$$y = a(b'(1) + b(1))$$

$$y = a(b + b')$$

$$y = a(1)$$

$$y = a$$

a	bc			
	00	01	11	10
0				
1				

a	bc			
	00	01	11	10
0				
1				

a	bc			
	00	01	11	10
0				
1				

**PARTICIPATION  
ACTIVITY**

12.3.9: Drawing largest circle with 3-variable K-maps.

a	bc			
	00	01	11	10
0	m0 1	m1 1	m3 1	m2 1
1	m4 0	m5 1	m7 0	m6 0

1) A circle covering which cells should be drawn?

- ☐ m0, m1
- ☐ m3, m2
- ☐ m0, m1, m3, m2

2) A circle covering which cells should be drawn?

- ☐ m1, m5
- ☐ m5
- ☐ m5, m7

### 4-variable K-map

A K-map can be drawn for four variables. Top and bottom rows are adjacent (as are left and right columns). Valid circle sizes are 1, 2, 4, 8, or 16 cells.

Figure 12.3.1: Four-variable K-map example.

ab	cd			
	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	1	1	1	1
10	1	0	0	1

**PARTICIPATION  
ACTIVITY**

12.3.10: Simplification with 4-variable K-maps.

ab	cd			
	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	1	1	1
10	0	1	1	1

Match the simplified term.

(N) (M) (J) (K)

$a'd'$

$ab$

$ac'd$

$c$

Reset

**PARTICIPATION  
ACTIVITY**

12.3.11: 4-variable K-map simplification.



### Expression simplification in programming

*K-maps usage is not restricted to just digital design. Computer programmers sometimes simplify expressions using K-maps. Ex: A program may make a decision based on an expression: If ( (isRed AND !isBlue) OR (!isRed AND isBlue) OR (isRed AND isBlue) ) then take action X. (! means NOT).*

*The expression represents a function with two variables. Using a two-variable K-map, the programmer simplifies the expression, yielding: If (isRed OR isBlue) then take action X. The resulting program is easier to read.*

Exploring further:

- [Online K-map tool](#) (T. Thormaehlen)

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