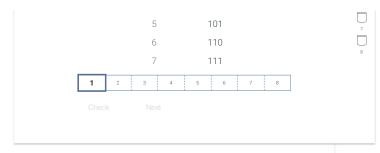# 11.10 Binary and counting

Decimal and binary numbers

Humans have ten fingers so humans use a base ten number system. Ex: 452 means $4 \cdot 10^2 + 5 \cdot 10^1 + 2 \cdot 10^0$. Digital systems have two-valued signals (high, low) so digital systems use a base two number system. Ex: 1101 means $1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$. A number in base ten is called a *decimal* number (from Latin "decem" meaning ten).

Base ten has ten symbols for a digit: 0, 1, ..., 9. When counting up and reaching 9, the digit resets to 0 and a 1 carries to the next digit. Ex: 008, 009, 010, 011, or 098, 099, 100, 101. Base two has only two symbols for a digit: 0 and 1. So counting up results in frequent carries. Ex: 000, 001, 010, 011, 100, 101, 110, 111. A number in base two is called a *binary* number (from Latin "bini" meaning two together).
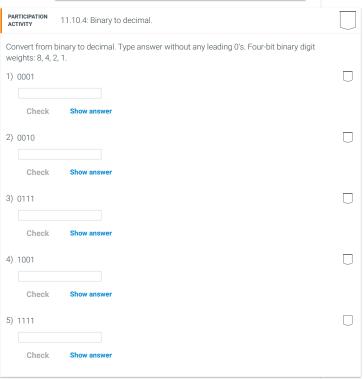
PARTICIPATION ACTIVITY   11.10.1: Counting up.

Start   ☐ 2x speed

–	0	0	
*	1	1	
**	2	10	Reset, carry
***	3	11	
****	4	100	Reset, carry and reset, carry
*****	5	101	
******	6	110	Reset, carry
*******	7	111	
********	8	1000	Reset, carry and reset, carry and reset, carry
*********	9	1001	
**********	10	Reset digit to 0, carry 1	1010	Reset, carry
***********	11	1011	
	...		
	98		
	99		
	100	Reset digit to 0, carry 1 to next digit which is 9 so reset to 0, carry 1 to next digit	

PARTICIPATION ACTIVITY   11.10.2: Counting in binary.

What binary number comes next? In explanations, "first digit" is the rightmost digit.

1) 000
   ○ 001
   ○ 100

2) 001
   ○ 011
   ○ 010

3) 011
   ○ 010
   ○ 100

4) 100
   ○ 101
   ○ 001

5) 101
   ○ 110
   ○ 111

CHALLENGE ACTIVITY   11.10.1: Counting up with 3 bits.

Can you count from 000 to 111 in binary in 20 seconds?

Start

Decimal	Binary (3 bits)
0	000
1	001
2	010
3	011
4	100

5	101
6	110
7	111

7

8

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Check   Next

Converting binary to decimal

Digital designers benefit from being able to quickly convert between binary and decimal numbers. Given a binary number, each digit's weight is summed to form a decimal number. Ex: $1101 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 8 + 4 + 0 + 1 = 13$.

11.10.3: Binary number tool.

0	0	0	0	**0**
8	**4**	**2**	**1**	(decimal value)
2^3	2^2	2^1	2^0	

11.10.4: Binary to decimal.

Convert from binary to decimal. Type answer without any leading 0's. Four-bit binary digit weights: 8, 4, 2, 1.

1) 0001

Check   Show answer

2) 0010

Check   Show answer

3) 0111

Check   Show answer

4) 1001

Check   Show answer

5) 1111

Check   Show answer

Converting decimal to binary

Given a decimal number, starting from the leftmost binary digit (greater than the decimal number), a 1 is placed in each digit as long as the resulting binary number doesn't exceed the decimal number.

Figure 11.10.1: Converting decimal to binary.

Converting 9 to 4-bit binary

1...	8 is less
11..	12 is too much, so use 10..
101.	10 (ten) is too much, so use 100.
1001	9 is equal

11.10.5: Decimal to four-bit binary.

Convert from decimal to binary. Type a four-bit answer: 0101, not 101. Four-bit binary digit weights: 8, 4, 2, 1.

1) 3

Check   Show answer

2) 4

Check   **Show answer**

3) 5

Check   **Show answer**

4) 13

Check   **Show answer**

Seen on a t-shirt

There are 10 kinds of people in this world:
Those who get binary, and those who don't.

Provide feedback on this section