Part 1: Symmetric Pascal Matrix

X-axis is size of Pascal Matrix. Y-axis is error

Part 1 (f)

(i) It is justified to use the LU or QR-factorizations as opposed to calculating the inverse matrix because it is faster and more accurate. Inverting matrices scales horribly to larger sized matrices.

(ii) The benefit of doing QR factorizations in this way is that the condition number of household reflections and givens rotations is 1 so the numbers are stable and stay similar magnitudes. The condition number for intermediate row operations is also close to one, so LU factorization is stable as well.

Note: A log scale was not used for the y-axis because LU error was always zero, because Pascal LU decomposition always gave perfect integers, so the computer had no error. Zero is undefined on the log scale. If it were on the log scale, the shape would be increasing with a relatively constant positive slope.

Part 2: Convergence of the Iterative Methods

X-axis is the maximum norm of the initial guess from exact solution.

Y-axis is number of iterations required to converge within error bound

The effect of initial vector position on iterations required is small, but there is a slight increase in iterations needed when the initial guess is further away, which makes sense because the initial error is within about the same order of magnitude each time. The steps needed for Jacobi compared to the steps needed for Gauss-Seidal is about 4.3 times as much, and the reason is because Gauss-Seidal’s largest Eigenvalue is about ¼ that of Jacobi’s largest Eigenvalue. Notice the scale on the side of Jacobi’s graph is much larger than Gauss-Seidal’s graph, which explains why there’s a more continuous looking shape on the left graph and a more discreet looking pattern on the right; it’s because Gauss-Seidal is that much more efficient to the point where there’s only a few possible values for the number of iterations needed.