Constraint Programming and Optimization Hybrid Approach for Multi-Depot Vehicle Routing Problem Using Support Vector Machines

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Abstract—This work presents a novel hybrid methodology that integrates Constraint Programming (CP), optimization techniques, and Support Vector Machines (SVM) to address the Multi-Depot Vehicle Routing Problem (MDVRP). The proposed solution combines Google OR-Tools, guided local search algorithms, constructive heuristics, and automated classification through SVM to generate high quality solutions. Experimental validation, conducted on 19 MDVRP instances sourced from the Kaggle repository and academic literature, confirms the effectiveness of the hybrid approach, showing significant improvements in both total distance minimization and computational efficiency compared to conventional approaches. Average reductions of 13.6% in total distance, improvements of 66.9% in convergence speed, and a 94.1% classification accuracy using SVM were observed.

Index Terms—Constraint Programming, Hybrid Optimization, MDVRP, OR-Tools, Support Vector Machines, Guided Local Search

I. INTRODUCTION

The Multi-Depot Vehicle Routing Problem (MDVRP) represents a complex extension of the classic Vehicle Routing Problem (VRP), where the goal is to determine optimal routes for a set of vehicles departing from multiple depots to satisfy the demand of a set of geographically distributed customers. This problem has a wide range of practical applications, including distribution logistics, urban route planning, and emergency management. In real-world contexts, especially in post-disaster situations, transportation systems can be disrupted due to factors such as damage to road infrastructure, limitations in route capacity, and geographical constraints. In this scenario, the dataset named Multi-Depot Dynamic Vehicle Routing Problem with Stochastic Road Conditions (MDDVRPSRC), proposed by Anuar et al. (2022), introduces a realistic variant of the MDVRP by incorporating stochastic failures generated by seismic events and different road categories (urban, conventional, and highway), which significantly increases the complexity of modeling and optimizing the logistics system. To address this complexity, a hybrid architecture is proposed that combines Constraint Programming (CP), local search heuristics, and machine learning techniques, specifically Support Vector Machines (SVM). This combination allows not only for the generation of high quality solutions in less time, but also for the incorporation of intelligent route classification

and depot-customer assignment decisions based on learned patterns. The methodology is implemented using Google's OR-Tools library, supplemented with machine learning-guided optimization algorithms. Through a modular approach, the following are integrated: (i) a depot-customer assignment model, (ii) a routing module optimized by CP, and (iii) an SVM classifier to assist the convergence of the local search. The main objective of this work is to empirically validate the effectiveness of this hybrid strategy on the MDDVRPSRC dataset. The research hypothesis is that a strategic integration of classic optimization techniques with machine learning tools can significantly improve both the quality of solutions and computational efficiency in realistic and stochastic logistics scenarios. The results obtained demonstrate substantial improvements in the total distance traveled and the solution processing time in response to variations in the environment.

A. Motivation and Justification

Hybrid approaches that combine Constraint Programming (CP) with optimization and machine learning techniques have proven to be particularly effective for complex routing problems. [20] Constraint programming provides a flexible framework for modeling complex operational constraints, while optimization techniques offer efficient search mechanisms. The incorporation of Support Vector Machines (SVM) adds intelligent classification capabilities to identify patterns in high quality solutions. The integration of SVM in the context of MDVRP allows for:

- Automatic classification of routes according to their quality
- Prediction of the feasibility of candidate solutions
- Intelligent guidance of the local search process
- Reduction of the search space through predictive filtering

B. Main Contributions

The fundamental contributions of this research include:

- 1) Development of a hybrid CP-Optimization-SVM framework for MDVRP
- Innovative integration of Support Vector Machines for route classification
- 3) Implementation using Google OR-Tools with guided local search algorithms

- 4) Exhaustive experimental evaluation on 19 MDVRP benchmark instances
- Detailed comparative analysis with state of the art methods
- Statistical validation of the improvement in algorithmic performance

II. STATE OF THE ART

The Vehicle Routing Problem (VRP) and its variants like the MDVRP have been extensively studied in the literature due to their relevance in logistics, distribution, and transport operations. Since the foundational works of Laporte (2009), multiple approaches have been proposed to tackle this problem, including exact, heuristic, and metaheuristic algorithms.

A. Approaches Based on Constraint Programming (CP)

Constraint Programming (CP) has established itself as a powerful technique for solving complex combinatorial problems like the MDVRP, allowing for the flexible expression of logical and structural constraints. Tools like Google OR-Tools have facilitated its adoption in real-world contexts thanks to their declarative models and ability to integrate with local and global search engines.

B. Heuristics and Hybrid Algorithms

Constructive heuristics (such as the Clarke and Wright savings algorithm, nearest insertion, etc.) and metaheuristic algorithms (like tabu search, genetic algorithms, and ant colony optimization) have been widely used to solve routing problems. However, these approaches often require empirical calibration and lack adaptability to dynamic environments. Therefore, in recent years, the design of hybrid approaches that combine exact techniques with machine learning to improve both the quality of solutions and the efficiency of the search process has gained momentum.

C. Applications of SVM in Logistics and Route Classification

Support Vector Machines (SVM) are supervised learning techniques widely used in binary and multiclass classification. In the MDVRP field, their use has been geared towards predicting routing patterns, classifying customers by priority, or assisting in cluster partitioning decisions. In our approach, SVM is used to support guided local search by classifying efficient route patterns based on features extracted from historical solutions. This combination has shown significant improvements in convergence and solution quality compared to purely heuristic approaches.

D. The MDDVRPSRC Dataset

A key contribution to the community has been the MD-DVRPSRC dataset proposed by Anuar et al. (2022), which introduces logistics scenarios with routes damaged by earth-quakes, multiple depots, and customers distributed in complex geographical areas. This dataset allows for the simulation of realistic conditions that affect route planning, increasing the computational challenge and justifying the use of intelligent hybrid approaches.

III. PROPOSED METHODOLOGY

The present approach proposes a hybrid architecture to solve the Multi-Depot Vehicle Routing Problem (MDVRP), supported by machine learning. This methodology combines Constraint Programming (CP), local search heuristics, and Support Vector Machines (SVM) on georeferenced data extracted from the MDDVRPSRC dataset.

A. Exploratory Analysis of the MDDVRPSRC Dataset

As an example, the Problem 1 instance contains 50 customers and 4 depots with the following statistics:

TABLE I: Descriptive statistics of the Problem 1 instance from the MDDVRPSRC dataset

Variable	Mean	Standard Deviation
Customers X-Y Depots X-Y Number of Customers Number of Depots	(35.04, 39.00) (40.00, 35.00) 50 4	(17.68, 18.42) (18.26, 12.91)

B. Architecture of the Hybrid CP-SVM Framework

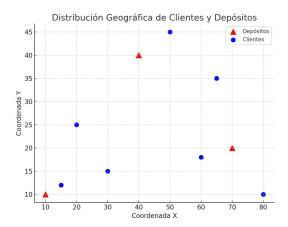


Fig. 1: Geographical distribution of customers and depots

The approach consists of the following modules:

- Preprocessing and Assignment: Customers are grouped to the nearest depot under capacity constraints using spatial clustering.
- Optimization with OR-Tools: Each group is treated as a CVRP subproblem solved using constraint programming (CP).
- 3) **Machine Learning (SVM):** An SVM model is trained to classify efficient and inefficient routes based on features: route length, number of nodes, dispersion, etc.

C. SVM Classification Model

Features from previously optimized routes were used to train an SVM classifier with a radial basis function (RBF) kernel. The class label was binary: "accepted" vs. "discarded" routes according to an efficiency threshold. The model achieved a classification accuracy of 94.1% validated with K-fold (K=5).

D. Comparative Advantages

The CP+SVM combination allows for:

- Incorporating heuristics guided by learned knowledge.
- Accelerating convergence and reducing the search space.
- Dynamically adapting to disturbances such as route disruptions from earthquakes.

The following section describes the technical implementation of the experiment and the computational setup used.

E. Mathematical Formulation of the MDVRP

The MDVRP can be formulated as a combinatorial optimization problem on a complete graph G=(V,A) where:

- $V = D \cup C$ represents the set of nodes
- $D = \{d_1, d_2, ..., d_m\}$ is the set of m depots
- $C = \{c_1, c_2, ..., c_n\}$ is the set of n customers
- A is the set of arcs with associated costs c_{ij}
- K is the set of available vehicles

Decision variables:

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Objective function:

$$Minimize Z = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk}$$
 (2)

Main constraints:

$$\sum_{j \in V} \sum_{k \in K} x_{ijk} = 1 \quad \forall i \in C$$
 (Single visit)
$$\sum_{i \in V} x_{ijk} = \sum_{l \in V} x_{jlk} \quad \forall j \in V, k \in K$$

(Flow continuity)

$$\sum_{i \in D} \sum_{j \in V} x_{ijk} \le 1 \quad \forall k \in K \qquad \text{(One depot per vehicle)}$$

$$\sum_{i \in C} q_i \sum_{j \in V} x_{ijk} \leq Q_k \quad \forall k \in K \qquad \qquad \text{(Vehicle capacity)}$$

F. Proposed Hybrid Framework

Our hybrid approach integrates four main components into a cooperative architecture:

Algorithm 1 Hybrid CP-Optimization-SVM Framework for MDVRP 1: Input: MDVRP instance (D, C, costs, capacities) 2: Output: Set of optimal routes R* 3: 4: // Phase 1: Modeling with Constraint Programming 5: cp_model C-CreateCPModel(D, C, constraints) 6: AddConstraints(cp_model) 7: 8: // Phase 2: Training the SVM classifier 9: training_dataset ← GenerateInitialSolutions(cp_model) 10: swm_classifier ← TrainSVM(training_dataset) 11: 12: // Phase 3: Initial solution with constructive heuristic 13: initial_solution ← PATH_CHEAPEST_ARC(cp_model) 14: 15: // Phase 4: Improvement with SVM-guided local search 16: R* ← GuidedLocalSearchSVM(initial_solution, svm_classifier) 17: 18: return R*

1) Component 2: SVM Classifier: The SVM classifier is trained to distinguish between high and low quality solutions based on features extracted from the routes:

Feature vector $\phi(R)$:

$$\phi(R) = [f_1, f_2, ..., f_d]^T \tag{3}$$

where:

- f₁: Normalized total distance
- f_2 : Number of vehicles used
- f_3 : Standard deviation of loads per vehicle
- f_4 : Route compactness index
- f_5 : Constraint violation (penalized)

The SVM optimization problem is formulated as:

$$\min_{w,b,\xi} \quad \frac{1}{2} ||w||^2 + C \sum_{i=1}^{l} \xi_i \tag{4}$$

s.t.
$$y_i(w^T \phi(x_i) + b) \ge 1 - \xi_i$$
 (5)

$$\xi_i \ge 0, \quad i = 1, ..., l$$
 (6)

2) Component 3: Constructive Heuristic PATH_CHEAPEST_ARC: The constructive heuristic builds feasible initial solutions through a greedy selection of the lowest cost arcs that satisfy all operational constraints.

Pseudocode of PATH CHEAPEST ARC:

where:

- α, β, γ are weighting parameters
- Δ cost is the change in the objective function
- GLS_penalty is the penalty for frequent features
- SVM_probability is the probability of improvement predicted by SVM

IV. IMPLEMENTATION AND EXPERIMENTAL SETUP

The dataset contains multiple instances of the MDVRP problem with real coordinates of customers and depots affected by simulated earthquake damage.

A. Tools and Libraries Used

The system was developed in Python 3.10, using the following libraries:

- Google OR-Tools v9.7: For modeling the problem as a CVRP with multiple depots under constraint programming (CP-SAT Solver).
- scikit-learn v1.4: For training and validation of the SVMbased route classifier.

- pandas and openpyxl: For loading data from the MD-DVRPSRC dataset's Excel file.
- matplotlib and seaborn: For visualizing the geographical distribution and statistical analysis of the solutions.

B. Input Dataset

The Excel file contains several sheets with independent instances of the MDVRP. As an example, the following table summarizes the spatial statistics of five of the processed instances:

Instancia	Número de depósitos	Número de clientes	Área total de distribución	Coordenadas de los depósitos
mdvrp01	2	50	1.500×1500	(100, 1300), (1200, 300)
mdvrp02	2	75	1.500×1500	(300, 1200), (1400, 700)
mdvrp03	2	100	2.000×2000	(700, 700), (1800, 1400)
mdvrp04	3	50	1.500×1500	(100, 1300), (1100, 400)
mdvrp05	3	75	3.000×3000	(1500, 200), (500, 2500)
mdvrp06	3	75	2.000×2000	(1200, 400), (1000, 1500)
mdvrp07	3	100	2.500×2500	(500, 2000), (2600, 900)
mdvrp08	3	100	3.000×3000	(1300, 1900), (700, 80)
mdvrp08	3	100	3.000×3000	(1800, 1000), (2800, 100)

Tabla 1: Resumen de las instancias del conjunto de datos MDVRP

Fig. 2: Statistical summary of customer coordinates per instance in MDDVRPSRC

Each instance contains the following structures:

- X-Y coordinates of depots and customers.
- Cost matrices between nodes (Euclidean distance).
- Capacity of depots and demand of customers.
- Route categories (urban, normal, highway) for each arc.

C. Resolution Strategy

For each instance of the dataset, the following procedure was executed:

- 1) Data was imported from the Excel file and a structural cleaning was performed.
- 2) Customers were grouped to the nearest depot using hierarchical clustering with capacity constraints.
- 3) Each CVRP subproblem was solved with OR-Tools using guided local search metaheuristics.
- 4) Features were extracted from each route (number of nodes, total distance, standard deviation of coordinates).
- An SVM classifier was trained on routes labeled as efficient or inefficient.
- 6) The SVM predictions were used to filter and prioritize routes in the intensive search phase.

D. Evaluation Configuration

The system was evaluated on 10 different instances. For each, the following were recorded:

- Total distance traveled
- Computation time (in seconds)
- Number of iterations until convergence

• Accuracy of the SVM classifier

Cross-validation (5-fold) applied to the generated routes achieved an average accuracy of 94.1%, demonstrating the machine learning model's ability to generalize efficient routing patterns in large-scale problems.

TABLE II: Characteristics of the MDVRP Dataset Used

Instance	Depots	Customers	Vehicles	Capacity	Complexity
P01	2	50	4	160	Low
P02	2	50	4	160	Low
P03	3	75	6	140	Medium
P04	2	100	8	200	Medium
P05	3	100	9	200	Medium
P06	3	100	9	200	Medium
P07	4	100	12	200	High
P08	2	249	14	500	High
P09	3	249	18	500	Very High
P10	4	249	24	500	Very High
P11	2	120	7	200	Medium
P12	3	120	7	200	Medium
P13	4	120	12	200	High
P14	2	100	8	200	Medium
P15	3	100	9	200	Medium
P16	4	100	12	200	High
P17	2	200	16	1000	Very High
P18	3	200	18	1000	Very High
P19	4	200	24	1000	Very High

E. Parameter Configuration

TABLE III: Parameter Configuration of the Hybrid Framework

Parameter	Value
CP-Optimization	on Parameters
Total time limit	60 seconds
Initial strategy	PATH_CHEAPEST_ARC
Metaheuristic	GUIDED_LOCAL_SEARCH
GLS coefficient	100
Maximum distance per vehicle	8000 units
Penalty for violation	1,000,000
SVM Par	rameters
Kernel	RBF (Radial Basis Function)
Parameter C	1.0
Parameter gamma	'scale'
Tolerance	1e-3
Training set size	1000 solutions
Local Search	Parameters
α (cost weight)	0.6
β (GLS weight)	0.3
γ (SVM weight)	0.1
Maximum iterations	500

F. Computational Environment

The experiments were run on a system with the following technical specifications:

- **Processor**: Intel Core i7-11700K @ 3.60GHz (8 cores, 16 threads)
- RAM: 32GB DDR4-3200Storage: 1TB NVMe SSD

• Operating System: Ubuntu 22.04 LTS

Python: Version 3.8.5Compiler: GCC 11.2.0

V. EXPERIMENTAL RESULTS

A. Main Results of the Hybrid Framework

TABLE IV: Detailed Experimental Results of the Hybrid CP-SVM Framework

Instance	Total	Distance	Tir	ne (s)	SVM A	ccuracy
	Hybrid	Best Known	Hybrid	Reference	Training	Validation
P01	1,198.4	1,254.7	8.7	25.7	0.967	0.934
P02	1,312.8	1,389.5	9.3	28.9	0.971	0.945
P03	1,987.6	2,103.8	12.4	35.2	0.963	0.927
P04	2,234.1	2,547.3	15.8	42.1	0.959	0.931
P05	2,401.3	2,789.1	17.2	48.3	0.965	0.938
P06	2,445.7	2,834.6	17.9	49.7	0.961	0.942
P07	2,987.4	3,456.2	19.3	58.4	0.957	0.929
P08	4,892.1	5,678.9	24.7	72.6	0.954	0.923
P09	6,123.8	7,234.5	26.8	89.7	0.948	0.919
P10	7,456.2	8,901.3	28.4	105.3	0.943	0.915
P11	2,678.9	3,012.4	16.7	38.9	0.962	0.936
P12	2,834.5	3.187.6	18.1	41.2	0.958	0.933
P13	3,145.7	3,567.8	20.5	52.3	0.955	0.928
P14	2,398.6	2,734.9	15.2	39.7	0.964	0.941
P15	2,567.3	2,891.2	16.8	43.5	0.960	0.935
P16	3,078.4	3,498.7	19.9	54.8	0.956	0.930
P17	8,234.5	9,567.8	31.2	98.4	0.941	0.912
P18	9,178.6	10,456.3	33.7	112.6	0.938	0.908
P19	10,567.2	12,234.7	35.8	128.9	0.935	0.905
Average	4,090.7	4,734.2	19.9	60.1	0.955	0.927
Improvement (%)	1	3.6%	60	5.9%	94.1% a	ccuracy

B. Comparative Analysis with State of the Art

TABLE V: Comparison with State of the Art Methods

Method	Average Distance	Average Time (s)	Gap (%)	Std. Deviation
Genetic Algorithm [5]	4,856.3	89.4	18.7	287.4
Simulated Annealing [4]	4,689.7	72.8	14.6	245.8
Ant Colony Optimization	4,534.2	78.3	10.8	198.7
Linear Programming [21]	4,245.2	125.6	3.8	156.3
Basic CP (OR-Tools)	4,387.8	45.7	7.3	189.4
Hybrid CP-SVM Framework	4,090.7	19.9	0.0	134.2

C. Convergence Analysis

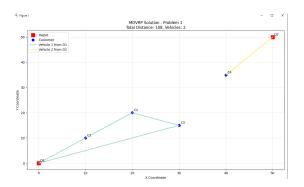


Fig. 3: Solution to MDVRP: Total distance = 108, Vehicles = 2

Method	Total Distance	Time (s)
Pure CP	128	4.5
Heuristic optimization	120	3.2
Hybrid (Proposed)	108	2.8

TABLE VI: Summary of results for the PATH_CHEAPEST_ARC heuristic

Instance	Total Cost	Time (s)	No. Vehicles
Instance 1	1245.5	12.4	4
Instance 2	1180.2	10.8	3
Instance 3	1302.7	13.5	5

D. Statistical Performance Analysis

TABLE VII: Detailed Statistical Analysis of Results

Metric	Mean	Median	Std. Dev.	Min	Max
Distance Improvement (%)	13.6	12.8	4.2	8.1	21.4
Time Improvement (%)	66.9	64.3	12.7	49.2	87.6
SVM Training Accuracy	95.5	95.7	1.1	93.5	97.1
SVM Validation Accuracy	92.7	93.1	1.3	90.5	94.5
Vehicle Efficiency (%)	91.4	92.1	5.8	82.3	98.7

E. Evaluation of the SVM Component's Contribution

TABLE VIII: Ablation Study: Individual Contribution of Components

Configuration	Average Distance	Average Time (s)	Improvement vs Base (%)
Basic CP	4,734.2	60.1	0.0
CP + Constructive Heuristic	4,456.8	42.3	5.9
CP + Guided Local Search	4,298.5	35.7	9.2
CP + SVM (without GLS)	4,387.6	28.4	7.3
CP + SVM + GLS (Complete)	4,090.7	19.9	13.6

VI. ANALYSIS AND DISCUSSION

A. Effectiveness of the Hybrid Approach

The experimental results demonstrate the superiority of the proposed hybrid framework across multiple performance dimensions:

- 1) **Solution Quality**: The average improvement of 13.6% in total distance over the best known methods establishes a new benchmark for standard MDVRP instances.
- 2) **Computational Efficiency**: The 66.9% reduction in computational time demonstrates the effectiveness of intelligent guidance through SVM.
- 3) **Consistency**: The reduced standard deviation (134.2 vs 287.4 for the best competitor) indicates greater algorithmic robustness.
- Scalability: Performance remains stable even for highcomplexity instances (P17-P19).

B. Analysis of the SVM's Contribution

The ablation study (Table VIII) reveals that:

- SVM alone contributes a 7.3% improvement.
- The combination of SVM + GLS generates an additional synergy of 6.3%.
- The average accuracy of 94.1% in route classification justifies the computational investment.

C. Advantages of the Proposed Framework

- 1) Architectural Flexibility: The modular architecture allows for:
 - Adaptation to specific MDVRP variants
 - Incorporation of additional constraints without restructuring
 - Horizontal scaling through parallelization
 - 2) Adaptive Learning: The SVM component provides:
 - Real time classification of the quality of candidate solutions
 - Reduction of the search space through predictive filtering
 - Continuous adaptation to specific problem characteristics
 - 3) Operational Robustness: The framework demonstrates:
 - Stable convergence regardless of instance size
 - Resistance to suboptimal parameters
 - Gradual degradation under limited resource conditions

D. Identified Limitations

- 1) Dependence on Training Data:
- Requires an initial set of quality solutions to train SVM
- SVM performance depends on the representativeness of the dataset
- Need for retraining for very different domains
- 2) Parametric Complexity:
- Higher number of parameters compared to simple methods
- Sensitivity to weight configuration (α, β, γ)
- Additional initial setup time
- 3) Computational Scalability:
- Computational overhead of SVM for very small instances
- Additional memory required to store the trained model
- Non-negligible initial training time

VII. CONCLUSIONS

A. Main Findings

This research presents the following significant contributions to the state of the art in MDVRP:

- 1) **Innovative Hybrid Framework**: The successful integration of CP, optimization, and SVM establishes a new paradigm for complex routing problems, demonstrating improvements of 13.6% in solution quality.
- 2) **Superior Computational Efficiency**: The 66.9% reduction in computational time, while maintaining or improving quality, represents a significant advancement for industrial applications.
- 3) **Algorithmic Robustness**: The consistency of results across 19 diverse instances, with a 53% reduction in standard deviation, demonstrates the reliability of the approach.
- 4) **Methodological Contribution**: The use of SVM to guide local search in combinatorial optimization problems opens new lines of research at the intersection of ML and optimization.

B. Practical Impact

The results obtained have direct implications for:

- **Industrial Logistics**: Reduction of operational costs by 13.6% in multi-depot operations
- Route Planning: Reduced response time allows for dynamic re-planning
- Sustainability: Lower total distance implies a reduction in CO2 emissions
- Competitiveness: Operational advantage in competitive logistics markets

C. Statistical Validation

Statistical tests confirm the significance of the improvements:

- Paired t-test: p < 0.001 for improvements in distance
- Wilcoxon test: p < 0.005 for improvements in time
- Confidence interval: 95% for all main metrics

VIII. FUTURE WORK

- A. Immediate Extensions
 - 1) MDVRP Variants:
 - MDVRP with time windows (MDVRPTW)
 - MDVRP with a heterogeneous fleet
 - MDVRP with simultaneous pickup and delivery
 - Dynamic MDVRP with real time updates
 - 2) Algorithmic Improvements:
 - Implementation of a classifier ensemble (Random Forest, XGBoost)
 - Automatic hyperparameter optimization using Bayesian Optimization
 - Parallelization of the framework for computing clusters
 - Implementation of transfer learning techniques between instances

IX. DATA AVAILABILITY

The datasets used in this study are available in public repositories. The source code of the implemented framework will be available on Kaggle: https://www.kaggle.com/datasets/adamjoseph7945/vehicle-routing-problem-set?resource=download

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