

# Quadratic Residue



When we say "modulo  $p=29$ ", it means we're working within a system where numbers "wrap around" every 29 steps. For example, 30 in this system is equivalent to 1, 31 is equivalent to 2, and so on.

## Squaring in Modular Arithmetic:

If we have a number  $a$  and we square it modulo  $p$ , denoted as  $a^2 \bmod p$ , we're finding the remainder when  $a^2$  is divided by  $p$ .

## Finding Square Roots Modulo $p$ :

Now, if we're given a number  $x$  and we want to find a number  $a$  such that  $a^2 \equiv x \pmod{p}$ , it's like trying to solve for  $a$  where  $a^2$  wraps around to give us  $x$  when divided by  $p$ .

## Quadratic Residues and Non-Residues:

- **Quadratic Residue:** If there exists an  $a$  such that  $a^2 \equiv x \pmod{p}$ , then  $x$  is a quadratic residue.
- **Quadratic Non-Residue:** If no such  $a$  exists, then  $x$  is a quadratic non-residue.

## Two Solutions for Quadratic Residues:

If  $x$  is a quadratic residue, there are always two solutions for  $a$ , because if  $a^2 \equiv x \pmod{p}$ , then  $(-a)^2 \equiv x \pmod{p}$  as well.

So, when working with modular arithmetic, some numbers have square roots modulo  $p$  (quadratic residues), and some don't (quadratic non-residues). And for those that do, there are always two solutions.