

Homework 1. Markov chains

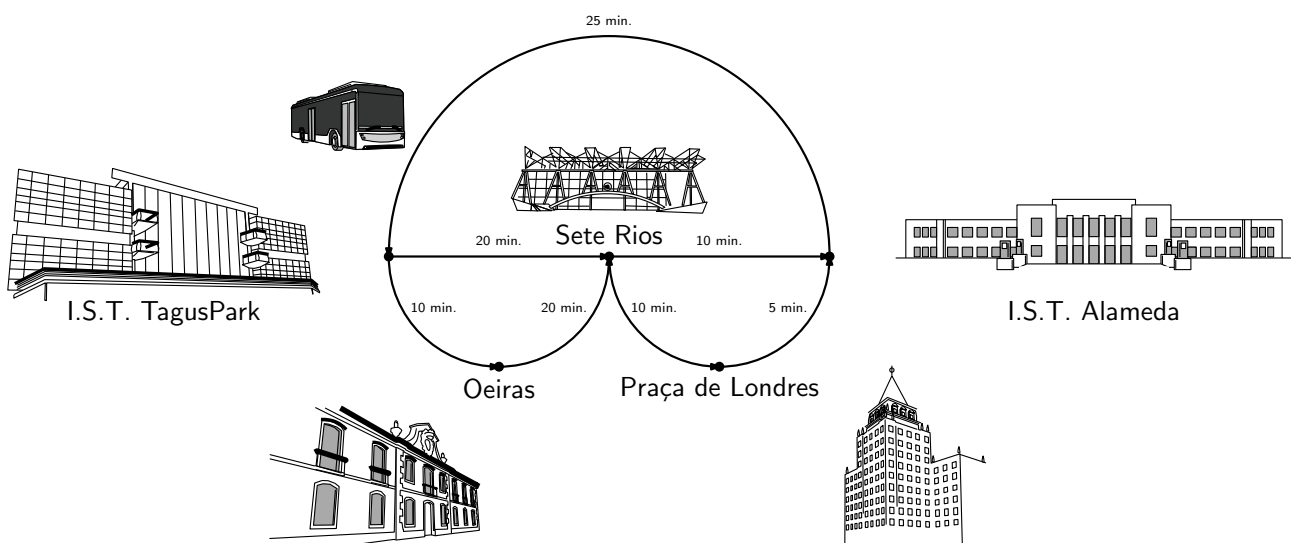


Figure 1: I.S.T. shuttle network, including 5 stops and traveling times.

Consider the diagram depicted in Fig. 1, representing a (hypothetical) set of routes that the I.S.T. shuttle may follow. In this homework, you will describe the motion of the shuttle using a Markov chain. To that purpose, consider the following information:

- All stops are considered similar, in terms of shuttle behavior;
- At each moment, there is a single shuttle traversing the city;
- The travel time between any two consecutive stops is indicated in the diagram.
- When at I.S.T. TagusPark, the shuttle will go directly to Sete Rios with a probability of 70%, and to Oeiras with a 30% probability. Similarly, when at Sete Rios, the shuttle will go directly to I.S.T. Alameda with a 50% probability, and through Praça de Londres with a 50% probability.

Exercise 1.

- (a) Write down the Markov chain model (state space and transition probabilities) representing the motion of the shuttle. Note that, for the purpose of the Markov chain, a “time step” corresponds to a moment when the shuttle is in one of the stops, not to real time.
- (b) Suppose that the shuttle departs from I.S.T. TagusPark at time step $t = 0$. Compute the probability of the shuttle being in each stop at time step $t = 3$.
- (c) Consider a passenger arriving at Oeiras just as the shuttle leaves. What is the expected waiting time for such a passenger? Assume that the shuttle is exactly 2 minutes in each stop.