# HW 1

27 September 2020 15:35

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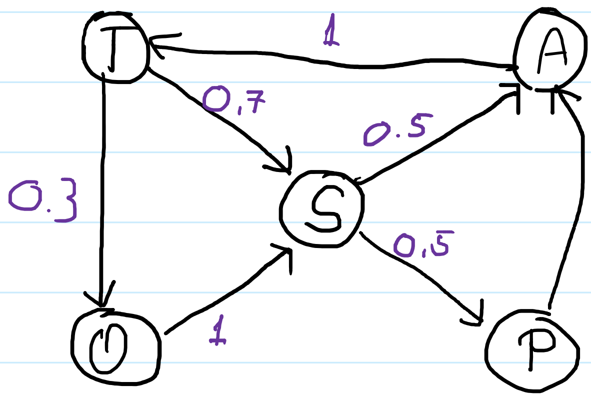
a)




T - Tagus
A - Alameda
O - Olaias
P - Praça Londres
S - Sete-Rios

$\mathcal{X} = \{T, A, O, P, S\}$

| | T | A | O | P | S |
|---|---|---|---|---|---|
| T | 0 | 0 | 0,3 | 0 | 0,7 |
| A | 1 | 0 | 0 | 0 | 0 |
| O | 0 | 0 | 0 | 0 | 1 |
| P | 0 | 1 | 0 | 0 | 0 |
| S | 0 | 0,5 | 0 | 0,5 | 0 |

b) $t=0 \rightarrow$ Tagus

$t=3 \rightarrow ?$

$$P = \begin{array}{c} \begin{array}{ccccc} T & A & O & P & S \end{array} \\ \begin{bmatrix} 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \end{bmatrix} \end{array} \begin{array}{c} T \\ A \\ O \\ P \\ S \end{array}$$

$$P^3 = \begin{bmatrix} 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0{,}35 & 0 & 0{,}35 & 0{,}3 \\ 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0{,}3 & 0 & 0{,}7 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0{,}5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.35 & 0.5 & 0 & 0.15 & 0 \\ 0 & 0.35 & 0 & 0.35 & 0.3 \\ 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.15 & 0 & 0.35 \end{bmatrix}$$

$P[x_3 = T \mid x_0 = t] = 0.35$

$P[x_3 = A \mid x_0 = t] = 0.5$

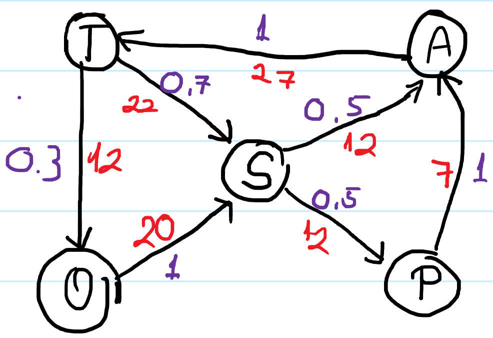
$P[x_3 = O \mid x_0 = t] = 0$

$P[x_3 = P \mid x_0 = t] = 0.15$

$P[x_3 = S \mid x_0 = t] = 0$

c) Seja $X$ a variável aleatória correspondente ao tempo que o shuttle demora a voltar a Oeiras tendo partido de lá, queremos calcular $E(x)$, ou seja, o Expeted Value de $X$

Seja $E(i)$ o tempo expectado que o shuttle demora a ir da estação $i$ até Oeiras, temos:




Nota: Foram somados 2 mins a todos os tempos de Viagem exceto ao caminho entre Oeiras e Sete-Rios. Estes 2 mins correspondem ao tempo que o shuttle fica em cada estação. No contexto do problema o shuttle sai logo de Oeiras, daí não ser adicionado esse tempo

$$E(T) = 0.3 \cdot 12 + 0.7\,(22 + E(S))$$

$$E(S) = 0.5 \cdot (12 + E(A)) + 0.5\,(12 + E(P))$$

$$E(P) = 7 + E(A)$$

$$E(A) = 27 + E(T)$$

$$\boxed{E(i) = \sum_{i=1}^{m} (x_i\, P(x = x_i))}$$

Resolvendo este sistema de equações temos:

Seja $E(T) = t$, $E(S) = s$, $E(P) = p$, $E(A) = a$

$$\begin{cases} t = 0.3 \cdot 12 + 0.7\,(22 + s) \\ s = 0.5 \cdot (12 + a) + 0.5\,(12 + p) \\ p = 7 + a \\ a = 27 + t \end{cases}$$

$$\Leftrightarrow \begin{cases} t - 0{,}7s = 19 \\ s - 0{,}5a - 0{,}5p = 12 \\ -a + p = 7 \\ -t + a = 27 \end{cases}$$

$$\begin{array}{c|c} & \begin{array}{cccc} t & s & p & a \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} & \left[\begin{array}{cccc|c} 1 & -0.7 & 0 & 0 & 19 \\ 0 & 1 & -0{,}5 & -0{,}5 & 12 \\ 0 & 0 & -1 & 1 & 7 \\ -1 & 0 & 1 & 0 & 27 \end{array}\right] \end{array}$$

$$\xrightarrow{R_4 + R_1} \left[\begin{array}{cccc|c} 1 & -0{,}7 & 0 & 0 & 19 \\ 0 & 1 & -0{,}5 & -0{,}5 & 12 \\ 0 & 0 & -1 & 1 & 7 \\ 0 & -0{,}7 & 1 & 0 & 46 \end{array}\right] \xrightarrow{R_4 + 0.7 R_2} \left[\begin{array}{cccc|c} 1 & -0{,}7 & 0 & 0 & 19 \\ 0 & 1 & -0{,}5 & -0{,}5 & 12 \\ 0 & 0 & -1 & 1 & 7 \\ 0 & 0 & 0{,}65 & -0{,}35 & 54{,}4 \end{array}\right]$$

$$\xrightarrow{R_4 - 0.65 \cdot R_3} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & -0,5 & -0,5 & 12 \\ 0 & 0 & -1 & 1 & 7 \\ 0 & 0 & 0 & 0,3 & 58,95 \end{array}\right] \xrightarrow{R_4 \cdot \frac{1}{0,3}} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & -0,5 & -0,5 & 12 \\ 0 & 0 & -1 & 1 & 7 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right]$$

$$\xrightarrow{R_3 - R_4} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & -0,5 & -0,5 & 12 \\ 0 & 0 & -1 & 0 & -189,5 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right] \xrightarrow{R_2 + \frac{1}{2} R_4} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & -0,5 & 0 & 110,25 \\ 0 & 0 & -1 & 0 & -189,5 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right]$$

$$\xrightarrow{-R_3} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & -0,5 & 0 & 110,25 \\ 0 & 0 & 1 & 0 & 189,5 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right] \xrightarrow{R_2 + \frac{1}{2} R_3} \left[\begin{array}{cccc|c} 1 & -0,7 & 0 & 0 & 19 \\ 0 & 1 & 0 & 0 & 205 \\ 0 & 0 & 1 & 0 & 189,5 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right]$$

$$\xrightarrow{R_1 + 0.7 R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 162,5 \\ 0 & 1 & 0 & 0 & 205 \\ 0 & 0 & 1 & 0 & 189,5 \\ 0 & 0 & 0 & 1 & 196,5 \end{array}\right] \Rightarrow \boxed{\begin{array}{l} t = 162,5 \\ s = 205 \\ p = 189,5 \\ a = 196,5 \end{array}}$$

$$\left.\begin{array}{l} E(T) = 162,5 \\ E(S) = 205 \\ E(P) = 196,5 \\ E(A) = 189.5 \end{array}\right\} E(X) = E(S) + 20$$

$$= 225 \text{ minutos}$$

O tempo espetável de espera são 225 minutos