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a) $A = \{U, D, L, R\}$

Up Down Left Right

$$X = \{(1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,1), (3,2), (4,0), (4,1), (4,2), (5,2), (6,2), (E,2)\}$$

(x, y)

↳ square we're on

↳ keys:

0 - Has No Key

1 - Has Blue Key

2 - Has Blue & Red Key

E ↳ Exit

b)

$$P_R = \begin{array}{cc}
\begin{array}{cccccccccccccc} (1,0) & (1,1) & (1,2) & (2,0) & (2,1) & (2,2) & (3,1) & (3,2) & (4,0) & (4,1) & (4,2) & (5,2) & (6,2) & (E,2) \end{array} & \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.8 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.8 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.8 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0.2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} &
\begin{array}{c} (1,0) \\ (1,1) \\ (1,2) \\ (2,0) \\ (2,1) \\ (2,2) \\ (3,1) \\ (3,2) \\ (4,0) \\ (4,1) \\ (4,2) \\ (5,2) \\ (6,2) \\ (E,2) \end{array}
\end{array}$$

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b cont.)

Taking into account that our agent's goal is to travel towards the **Exit** space, a valid, and simple, cost function for this problem would be:

c(x,a) = {x=E ? 1 : 0}

Or, in other words, **c(x,a)** is **1** if **x is not the E**, else its equal to **0**

As such, we have the following Cost Matrix:

$$C = \begin{array}{cc} \begin{matrix} U & D & L & R \end{matrix} & \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} (1,0) \\ (1,1) \\ (1,2) \\ (2,0) \\ (2,1) \\ (2,2) \\ (3,1) \\ (3,2) \\ (4,0) \\ (4,1) \\ (4,2) \\ (5,2) \\ (6,2) \\ (E,2) \end{matrix} \end{array}$$

c) Discount $\gamma = 0.9$

Action = R

Cost-to-Go ? $\quad \sum_{t=0}^{\infty} \gamma^t c_t$

$$J((E,2)) = \sum_{t=0}^{\infty} 0.9^t \cdot 0 = 0 + 0.9 \cdot 0 + \ldots = \underline{0}$$

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c cont.)

$$J((1,0)) = J((1,1)) = J((1,2)) = \sum_{t=0}^{\infty} 0.9^t \cdot 1 = 1 + 0.9 + \ldots = \frac{1}{1-0.9} = \frac{1}{0.1} = \underline{10}$$

$$J((3,1)) = J((3,2)) = \sum_{t=0}^{\infty} 0.9^t \cdot 1 = 1 + 0.9 + \ldots = \frac{1}{1-0.9} = \frac{1}{0.1} = \underline{10}$$

$$J((5,2)) = \sum_{t=0}^{\infty} 0.9^t \cdot 1 = 1 + 0.9 + \ldots = \frac{1}{1-0.9} = \frac{1}{0.1} = \underline{10}$$

$$J((2,0)) = 1 + \gamma J((1,0)) = 1 + 0.9 \cdot 10 = \underline{10}$$
$$J((2,1)) = 1 + \gamma J((1,1)) = 1 + 0.9 \cdot 10 = \underline{10}$$
$$J((2,2)) = 1 + \gamma J((1,2)) = 1 + 0.9 \cdot 10 = \underline{10}$$

$$J((4,0)) = 1 + \gamma J((2,0)) = 1 + 0.9 \cdot 10 = \underline{10}$$
$$J((4,1)) = 1 + \gamma J((2,1)) = 1 + 0.9 \cdot 10 = \underline{10}$$
$$J((4,2)) = 1 + \gamma J((2,2)) = 1 + 0.9 \cdot 10 = \underline{10}$$

$$J((6,2)) = 1 + \gamma J((4,2)) = 1 + 0.9 \cdot 10 = \underline{10}$$

As such, we have the following Matrix.

$$J = \begin{array}{cl} \left[\begin{matrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 0 \end{matrix}\right] & \begin{matrix} (1,0) \\ (1,1) \\ (1,2) \\ (2,0) \\ (2,1) \\ (2,2) \\ (3,1) \\ (3,2) \\ (4,0) \\ (4,1) \\ (4,2) \\ (5,2) \\ (6,2) \\ (E,2) \end{matrix} \end{array}$$