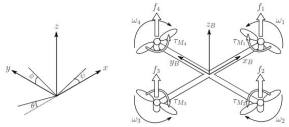
Real-time Nonlinear Model Predictive Control for Quadcopter Trajectory Tracking

Dynamics

Inertial and body frames:



The state vector consists of:

$$(x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}_B, \dot{\theta}_B, \dot{\psi}_B)^T$$

The control input consists of motor thrusts 1-4:

$$(T_1, T_2, T_3, T_4)^T$$

Let the control-affine continuous-time system be:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

The following dynamics are established from the Newton-Euler and Euler-Lagrange equations:

$$(\ddot{x}, \ddot{y}, \ddot{z})^T = R_{xyz}(0, 0, (T_1 + T_2 + T_3 + T_4)/m)^T$$

$$(\ddot{\phi}_B, \ddot{\theta}_B, \ddot{\psi}_B)^T = I^{-1}[(\tau_{Bx}, \tau_{By}, \tau_{Bz})^T - (\dot{\phi}_B, \dot{\theta}_B, \dot{\psi}_B)^T \times I(\dot{\phi}_B, \dot{\theta}_B, \dot{\psi}_B)^T]$$

Where:

Subscript **B** denotes body frame.

R_{xyz} is a rotation about x, y, z axes from body to inertial frame.

m is the mass of the quadcopter.

I is the diagonal inertial matrix consisting of Ixx, Iyy, Izz.

$(\tau_{Bx}, \tau_{By}, \tau_{Bz})$ represent the torques about x, y, z axes in body frame. τ_{Bx} and τ_{By} are dependent on the motor's distances from the center of mass, while τ_{Bz} is dependent on each individual motor's torque that is proportional to thrust.

Problem Formulation

Cost function:

$$\begin{aligned} J^*(x(t)) = \underset{\{u_k\}_{k=0}^{N-1}}{\text{minimize}} \quad & \sum_{k=0}^{N-1} l(x_k, u_k) \\ \text{subject to} \quad & x_0 = x(t) \\ & \text{for } k = 0, \ \ldots, \ N-1: \\ & \quad x_{k+1} = f(x_k, u_k) \\ & \quad x_k \in \mathcal{X} \\ & \quad u_k \in \mathcal{U} \end{aligned}$$

Using Acados's linear least-squares cost to formulate a quadratic cost:

The Lagrange cost term has the form

$$l(x, u, z) = \frac{1}{2} \left\| \underbrace{V_x x + V_u u + V_z z}_{y} - y_{\text{ref}} \right\|_W^2 \tag{25}$$

where matrices $V_x \in \mathbb{R}^{n_y \times n_x}$, $V_u \in \mathbb{R}^{n_y \times n_u}$ are $V_z \in \mathbb{R}^{n_y \times n_z}$ map x, u and z onto y, respectively and $W \in \mathbb{R}^{n_y \times n_y}$ is the weighing matrix. The vector $y_{\text{ref}} \in \mathbb{R}^{n_y}$ is the reference.

Where yref consists of x and u vertically concatenated.

The continuous-time dynamics are integrated via explicit Runge-Kutta and optimized via direct multiple shooting.

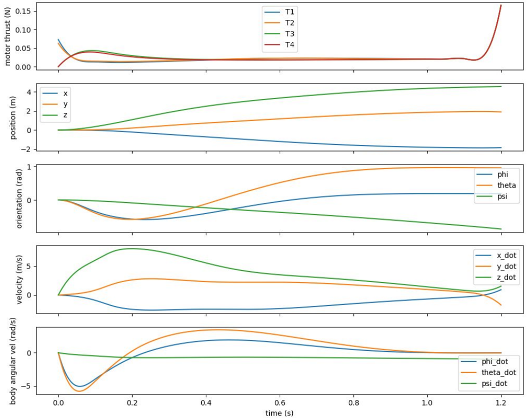
Implementation

The nonlinear continuous-time quadcopter dynamics are formulated in CasADI and converted into an Acados model.

The optimal control problem formulation is defined for the Acados solver, which consists of the predictive time horizon, number of shooting nodes, cost term, cost matrices, reference trajectory, and constraints.

The solver utilizes SQP methods to address OCP-structured NLP problems for efficient real-time performance. It is intended for embedded systems and is optimized for CPU usage.

Example MPC-calculated optimal trajectory from Crazyflie 2.0 parameters (system identification results from [this paper](#)):



MPC with time-step of 0.05s and 25 shooting nodes. Zero initial state and setpoint of x=-5, y=5, z=10 meters.