Formelsammlung Robotik

Vektoren

Skalare Multipl.	$\lambda \cdot \vec{a}$	$\begin{pmatrix} \lambda \cdot a_1 \\ \lambda \cdot a_2 \\ \lambda \cdot a_3 \end{pmatrix}$
Abstand P.-Urpsr. Betrag (Norm)	$\lVert\vec{a}\rVert$	$\lvert\vec{a}\rvert = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$
Skalarprodukt	$\vec{a} \cdot \vec{b}$	$a_1 \cdot b_1 + \ldots + a_n \cdot b_n = x$
Winkel		$\cos\alpha = \dfrac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert}$

Matrizen

Gleich	$A = B$	$(a_{ij}) = (b_{ij})$
Addition	$C = A + B$	$(c_{ij}) = (a_{ij}) + (b_{ij})$
Differenz	$C = A - B$	$(c_{ij}) = (a_{ij}) - (b_{ij})$
Multiplikation Skalar	$c \cdot A$	$cA \in R^{m \times n}$
Multiplikation Matrizen	$A \cdot B$	$AB = \sum_j a_{ij} b_{ij}$

Multiplikation

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

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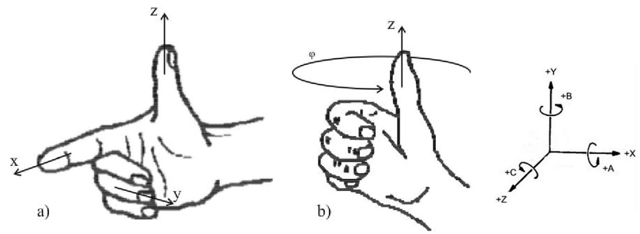
Rechte Hand Regel



Koordinatensysteme

Objekt in 3D (OKS)	$\vec{v} = (x, y, z, \alpha, \beta, \gamma)$	$\alpha, \beta, \gamma =$ Drehwinkel
Senkrechte		

Rotationsmatrizen

Rotation um X	$R_x(\alpha) = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Rotation um Y	$R_y(\alpha) = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Rotation um Z	$R_z(\alpha) = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Vormultipl. (Roll-pitch-yaw)	$H_{\text{neu}} = H_n \cdot H_{n-1} \cdot \ldots \cdot H_1$
Nachmultipl. (Euler-Winkel)	$H_{\text{neu}} = H_1 \cdot H_2 \cdot \ldots \cdot H_n$
Homogene 4×4-Matrix	$\begin{pmatrix} R_{3\times3} & u_{3\times1} \\ f_{1\times3} = 0 & 1 \times 1 \end{pmatrix} = \begin{pmatrix} n_{x\downarrow z} & o_{x\downarrow z} & a_{x\downarrow z} & u_{x\downarrow z} \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Invertierung 4×4	$T^{-1} = \begin{pmatrix} n_x & n_y & n_z & -n^T \cdot \vec{u} \\ o_x & o_y & o_z & -o^T \cdot \vec{u} \\ a_x & a_y & a_z & -a^T \cdot \vec{u} \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Translation um x, y, z	$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Verkettete Lagebeschr.	${}^{BKS}H_B = {}^{BKS}H_A \cdot {}^{A}H_B$ (H = Homo. Matr.)

Quaternionen

$$a, b, c, d \in \text{□Quaternion } q \Rightarrow q = a + b \cdot i + c \cdot *j + d \cdot k$$

$$a \in \mathbb{R}, u = (b, c, d)^T \in \mathbb{I}$$

$$q = (a, b, c, d)^T \text{ bzw. } q = (a, u)^T$$

$$\begin{aligned} i^2 = j^2 &= k^2 = ijk = -1 \\ ij = -ji &= k \\ jk = -kj &= i \\ ki = -ik &= j \end{aligned}$$

Gegeben	$q = (a_q, u_q), r = (a_r, u_r)$
Addition	$q + r = (a_q + a_r, u_q + u_r)$
Skalar	$q \cdot r = a_q a_r + < u_q, u_r >= a_q a_r + b_q b_r + c_q c_r + d_q d_r$
quatern. Multipl.	$q * r = (a_q + i \cdot b_q + j \cdot c_q + k \cdot d_q) \cdot (a_r + i \cdot b_r + j \cdot c_r + k \cdot d_r)$
konjugiert	$q^* = (a_q, -u_r)$
Norm	$\lvert q\rvert = \sqrt{qq^*} = \sqrt{q^* q} = \sqrt{a^2 + b^2 + c^2 + d^2}$
inverses Element	$q^{-1} = \frac{q^*}{\lvert q\rvert^2}$

Rotation mit Quaternionen

Vektor als Quaternion	$p = (x, y, z)^T \Leftrightarrow q_p = (0, p)^T$
Skalar als Quaternion	$s \Leftrightarrow q_s = (s, 0, 0, 0)^T$
Einheitsquaternion	$\lvert q\rvert = 1$
Rotation um θ, Achse u	$q_r = (\cos(\frac{\theta}{2}), u \cdot \sin(\frac{\theta}{2}))$
Rotation Punkt p	$q_{p'} = q_r q_p q_r^* = q_r q_p q_r^{-1}$

Sinus

a	0	30	45	60	90	120	135	150	180
$\sin a$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos a$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

Trigonometrie

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$