

## 1. Problem definition

Given a number  $n \in \mathbb{N}$ , find the minimal number  $k \in \mathbb{N}$  such that there are  $n$  sets  $A_1, \dots, A_n$  containing numbers in  $[k]$ , i.e  $A_i \subseteq \{1, \dots, k\}$  satisfying:

$$|A_i \cap A_j| = |i - j| \text{ for all } 1 \leq i < j \leq n$$

For example, for  $n = 4$ , the answer would be  $k = 5$ , with which we could pick the 4 sets as:

$$A_1 = \{1, 2, 3, 4\}$$

$$A_2 = \{1, 5\}$$

$$A_3 = \{1, 2\}$$

$$A_4 = \{1, 3, 4, 5\}$$

or a more visual alternative:

A <sub>1</sub> :	1	2	3	4	
A <sub>2</sub> :	1				5
A <sub>3</sub> :	1	2			
A <sub>4</sub> :	1		3	4	5