Set Intersection with Minimal Support
The SIMS-Problem

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1. Problem definition

Given a number $n \in \mathbb{N}$, find the minimal number $k \in \mathbb{N}$ such that there are n sets $A_1, ..., A_n$ containing numbers in $[k]$, i.e $A_i \subseteq \{1, ..., k\}$ satisfying:

$$|A_i \cap A_j| = |i - j| \text{ for all } 1 \leq i < j \leq n$$

For example, for $n = 4$, the answer would be $k = 5$, with which we could pick the 4 sets as:

$A_1 = \{1, 2, 3, 4\}$
$A_2 = \{1, 5\}$
$A_3 = \{1, 2\}$
$A_4 = \{1, 3, 4, 5\}$

or a more visual alternative:

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A₁:  1  2  3  4
A₂:  1              5
A₃:  1  2
A₄:  1     3  4  5
```

You can try to find sets which only use the numbers 1 to 4 but will hopefully be convinced that $k = 5$ is optimal.

2. Best known bounds

In the following table, we record our best known values for k.

n	**optimal value with combinatorial solver**	**optimal value with LP solver**
0	0	0
1	0	0
2	1	1
3	2	2
4	5	5
5	9	9
6	16	16
7	24	24
8		36
9		50
10		70
11		91
12		120
13		150
14		189
15		231
16		280
17		336
18		398
19		468
20		547
21		630
22		728
23		≤ 827
24		≤ 944
25		≤ 1064
26		≤ 1198
27		≤ 1341
28		≤ 1493
29		≤ 1661
30		≤ 1838
31		≤ 2027
32		≤ 2232
33		≤ 2442
34		≤ 2680
35		≤ 2918
36		≤ 3179

Our strategy in solving this problem combinatorically will be explained in Section 5. Our formulation of this problem as an (I-)LP will be explained in Section 6

3. Upper Bounds

4. Lower Bounds

5. Combinatorial approach

6. Linear Programming approach