Séance 1

Ex 4 $A = \{[n] \to [n] \text{ bijections}\}$ $\quad a_i \in [n-i+1]$

$B = [n] \times [n-1] \times \dots \times [1]$ $\quad$ 1 élém de B $(a_1, a_2, \dots, a_n)$ $\quad a_1 \in [n],\ a_2 \in [n-1],\ a_n \in [1]$

$= \{(a_1, a_2, \dots, a_n) \mid a_1 \in [n], a_2 \in [n-1], \dots, a_n \in [1]\}$

$\varphi : [n] \to [n]$ bijection $\qquad [n] = \{1, \dots, n\}$

ex : $n=4$

$1 \to 3$

$2 \to 1$

$3 \to 2$

$4 \to 4$

$\varphi \to (\cdot, \cdot, \cdot, \cdot)$

$(3, 1, 1, 1)$

$\{1,2,3,4\} \quad \{1,2,4\} \quad \{2,4\} \quad \{4\}$

Étant donné un code

$(4, 2, 1, 1) \rightsquigarrow$ bijection φ'

$1 \mapsto 4 \qquad \{1,2,3,4\}$

$2 \mapsto 2 \qquad \{1,2,3\}$

$3 \mapsto 1 \qquad \{1,3\}$

$4 \mapsto 3 \qquad \{3\}$

$[3] \times [2] = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

Séance 2

2.2. 4 cartes de 2 couleurs :

Soit 2×2 cartes de m̂ couleur $\to \binom{4}{2}\binom{10}{2}\binom{10}{2}$ (choisir 2 couleurs ; nb de choix de 2 cartes de m̂ couleur)

Soit 3 cartes couleur A + 1 carte couleur B

$4 \cdot \binom{10}{3} \cdot 3 \binom{10}{1}$ (choix couleur A ; cartes A ; choix B ; cartes B)

$\Rightarrow \binom{4}{2}\binom{10}{2}\binom{10}{2} + 4 \cdot 3 \binom{10}{3}\binom{10}{1}$

Séance 2. ex 6

$$\binom{n}{k} k = \frac{n!}{k!(n-k)!} \cdot k = \frac{n \cdot (n-1)!}{(k-1)!(n-k)!} = n \binom{n-1}{k-1}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$n(x+y)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} \cdot k\, x^{k-1} y^{n-k} = \sum_{k=1}^{n} \boxed{\binom{n}{k} \cdot k}\, x^{k-1} y^{n-k}$$

$$\left[= \sum_{k=1}^{n} n \binom{n-1}{k-1} x^{k-1} y^{n-k} = n \sum_{k=0}^{n-1} \binom{n-1}{k} x^k y^{n-1-k} \right]$$

Remplaçons $x = y = 1$ $\quad n(1+1)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} k$

$n 2^{n-1}$

$$n(x+y)^{n-1} = n\sum_{k=0}^{n-1}\binom{n-1}{k}x^k y^{n-1-k}$$

Si $x=y=1$: $$n\sum_{k=0}^{n-1}\binom{n-1}{k} = \sum_{k=0}^{n}\binom{n}{k}k$$ (page précédente)

10. $$\sum_{k=0}^{n}\frac{1}{k+1}\binom{n}{k} = \frac{1}{n+1}\sum_{k=0}^{n}\frac{n!\,(n+1)}{(k+1)!\,(n-k)!} = \frac{1}{n+1}\left[\overbrace{\sum_{k=0}^{n}\binom{n+1}{k+1}+\binom{n+1}{0}}^{2^{n+1}} - \overbrace{\binom{n+1}{0}}^{=1}\right]$$
$$= \frac{1}{n+1}\left(2^{n+1}-1\right)$$

Séance 3

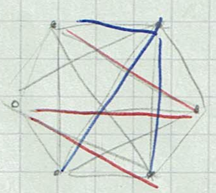
6. x_1 lettres 1
x_2 2
⋮
x_k k

$\rightarrow \binom{x_1+x_2+\dots+x_k}{x_1!\,x_2!\,\dots\,x_k!}$ mots possibles

8 Supposer pas $n+1$ identiques ni $n+1$ différents

$\rightarrow$ calculer max n^2 objets

10.



amis

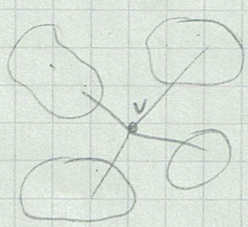
ennemis

$\rightarrow$ montrer $\exists\Delta$ bleu ou rouge

$\rightarrow$ Ramsey number $R_{3,3}$

Séance 4

5.



$c = \#$composantes connexes de G

k composantes auxquelles v est relié

$G' = G + v$ $\quad$ G' a $c-k+1$ composantes connexes

G' a au moins $m+k$ arêtes

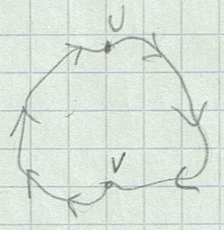
a $n+1 = n'$ sommets

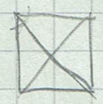
$$\Rightarrow c' = c-k+1 \geq n'-m'$$
$$= n+1-(m+k)$$
$$\Leftrightarrow c \geq n-m \quad \text{ok}$$

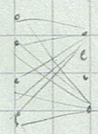
ex 8 $\quad$ A matrice d'adjacence :

$$(A^k)_{ij} = \begin{cases} \ell & \text{si } \exists \ell \text{ chemins de } i \text{ vers } j \text{ de longueur } k \\ 0 & \text{sinon} \end{cases}$$

Séance 5

Ex 3  G

Ex 2  K_4 K_2 



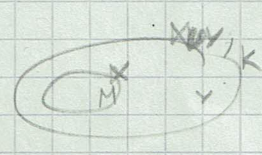
TP6.

9. $A_k = \{ (M, K) \mid |K| = k,\ M \subseteq K \subseteq N,\ |M| = m \}$

$$\left| \bigcup_k A_k \right| = \sum_{k=m}^{n} \binom{n}{k} \binom{k}{m}$$

$B = \{ (X, Y) \mid |X| = m,\ X \subseteq N,\ Y \subseteq N \setminus X,\ \forall Y \}$

$$|B| = \binom{n}{m} 2^{n-m}$$



$f : B \to \bigcup_k A_k : (X, Y) \mapsto (X, X \cup Y)$

$f^{-1} : \bigcup_k A_k \to B : (M, K) \mapsto (M, K \setminus M)$

$f^{-1} \circ f \{ B \to B$

$$(X,Y) \xmapsto{f} (X, X \cup Y) \xmapsto{f^{-1}} (X, (X \cup Y \setminus X)) = (X,Y)$$

$f \circ f^{-1} \{ \bigcup_k A_k \to \bigcup_k A_k$

$$(M,K) \xmapsto{f^{-1}} (M, K \setminus M) \xmapsto{f} (M, (K \setminus M \cup M)) = (M, K)$$

10. $x_1 = \#1$, $x_2 \# 2$, $\ldots$, $x_6 = \#6$ $\qquad x_i \geq 0 \in \mathbb{N}$

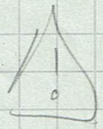
$x_1 + x_2 + x_3 + \ldots + x_6 = n$

$\sum (x + x^2 + x^3 + x^4 + x^5 + x^6)$

Séance 8

3. $$\alpha = \lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \lim_{n \to \infty} \frac{F_n + F_{n-1}}{F_n} = 1 + \lim_{n \to \infty} \frac{F_{n-1}}{F_n} = 1 + \lim_{n \to \infty} \frac{F_n}{F_{n+1}} = 1 + \frac{1}{\lim_{n \to \infty} \frac{F_{n+1}}{F_n}}$$

$$\alpha = 1 + \frac{1}{\alpha} \Rightarrow \ldots \alpha = \frac{1 \pm \sqrt{5}}{2} \Rightarrow \alpha = \varphi$$

 wolfram alpha