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Contents

1. Introduction

The equity risk premium has been a most prominent feature in both economic and financial literature for decades. Since the appearance of the defining paper "Portfolio Selection" (1952) by Harry Markowitz, marking the beginning of modern portfolio theory, the market's expected excess return or equity risk premium has been at the centre of many academic fields.[1] For instance, in corporate finance the market's equity risk premium is used for valuation purposes, whereas in portfolio construction it is an elementary building block.

In the financial literature, much has been written about the estimation of the equity risk premium. In doing so, the estimation of expected equity returns has been approached from many angles. Merton (1980), for instance, proposes a model based on the agent´s risk aversion and market variance. Others, like Hull et al. (2017) and Hull and Qiao (2017) use macro-economic data to foresee the magnitude of the equity risk premium in the short run. Graham and Harvey (2018), on the other hand, estimate the ten-year equity premium based on survey expectations expressed by chief financial officers. Campbell and Thompson (2008), among others, use fundamental valuation ratios to reach long-term, slow-moving, estimates of this measure. In "Common Risk Factors in the Return on Stocks and Bonds" (1993) Nobel prize laureate Eugen Fama and Kenneth French relate the equity risk premium to common risk factors, thereby laying the groundwork for factor investing. Many of these estimation methods are long-term and stable in nature.

Ian Martin (2017a) tackles the estimation from a completely different perspective. He provides volatile short-term equity risk premium proxies. In "What is the Expected

[1] I use the terms expected return, expected excess return, equity premium, and equity risk premium interchangeably in this thesis.

Return on the Market?” (2017a) Martin approaches the expected return on the market from the derivative pricing perspective. Based on the observation that both option prices and the equity risk premium theoretically depend on risk-neutral variance, he constructs a lower bound of the equity risk premium based on S&P 500 equity index options. He does so for the period between January 1996 and January 2012. In “What is the Expected Return on a Stock?” (2017b) Martin applies a similar analysis to single securities. The strength of Martin’s analysis is that it connects theoretical pricing measures to quantities that may be observed in today’s financial markets in real time. Other ground-laying results such as Hansen and Jagannathan’s (1991) derivation of the higher bound of the equity risk premium miss this feature of being measurable.

The aim of this paper is twofold. Firstly, the paper shows how to construct a proxy for the expected return on the market from equity index option data. Then, the results obtained by Martin (2017a) are verified. More importantly, the time frame of the analysis is extended to include the most recent data available. Secondly, the persistence of the generated time series is investigated with a decomposition of the data. This allows to filter the time series for persistent components and compare these across various time frames. Thereby, the contribution of this thesis lies both in the application of timely data to the results of Martin (2017a) and the further extension of these results to a new kind of analysis.

This thesis is structured as follows. Section 2 starts with a review of “What is the Expected Return on the Market?” (Martin, 2017a). This section establishes the crucial relationship between the expected equity premium and risk-neutral variance. Based on this result, the so called SVIX index is derived as a founding block of this thesis. Further, this section demonstrates how the theoretical results may be practically computed given price data of S&P 500 index options. Section 3 compares the SVIX index to its close relative, the VIX index and explains the relevance of the difference between these two indices. Section 4 explores summary statistics and attributes of the SVIX index. Additionally, this section empirically tests whether the scaled SVIX index may in fact be the equity risk premium. Section 5 demonstrates how the SVIX index relates to the rational investor assumption. Section 6 explores the behaviour of the term structure of the SVIX index before section 7 derives measures for the persistence of the SVIX based on a time series decomposition. Section 8 follows Martin (2017a) in constructing a contrarian market-timing strategy based on SVIX. Starting from the results derived in previous sections, section 9 develops the theory of a possible, central-bank induced regime change in financial markets and proposes an empirical test for this theory. Section 10 verifies Martin’s derivation of the probability of a substantial market drop and extends the analysis to the most recent period. Finally, section 11 concludes this paper.

2. The SVIX Index

Martin (2017a) approaches the estimation of the equity risk premium from the derivatives pricing perspective. Importantly, he introduces a theoretical relationship between the expected return and risk-neutral variance. Based on this result, he shows that it is possible to construct an index, the so called SVIX index, that provides a real-time proxy of the expected equity risk premium at various time horizons. In this section I follow Martin (2017a) in his derivation of this SVIX index.

Martin (2017a) and Tebaldi (2017) state that there are two possible ways to price time T cash flows X_T at an earlier time t. First, given the time T contingent claim X_T and the stochastic discount factor (SDF) M_T, one can price the present value of the future cash at time t:

(1) $$\text{price at } t = E_t(M_T X_T)$$

Second, one may price the same time T contingent claim X_T with the risk-neutral notation. Applying risk-neutral expectations and discounting at the risk-free rate at time t yields:

(2) $$\text{price at } t = \frac{1}{R_{f,t}} E_t^* X_T$$

where "*" denotes risk-neutral quantities. $R_{f,t}$ is the simple risk-free return plus one. The risk-neutral notation is generally used in derivative pricing under the no-arbitrage argument. Given this notation, the conditional risk-neutral variance of a return R_T between time t and T may be derived as follows:

(3) $$\mathrm{var}^*_t R_T = E^*_t R^2_T - (E^*_t R^2_T)^2 = R_{f,t} E_t (M_T R^2_T) - R^2_{f,t}$$

It is important to note that contrary to the conventional notation, R_T in this case is the simple return plus one. Therefore, it is never negative. Expected returns are closely connected to the risk-neutral variance via the following equation.

$$E_t R_T - R_{f,t} = [E_t(M_T R^2_T) - R_{f,t}] - [E_t(M_T R^2_T) - E_t R_T]$$

(4) $$= \frac{1}{R_{f,t}} \mathrm{var}^*_t R_T - \mathrm{cov}_t (M_T R_T, R_T)$$

One arrives at equation (4) by firstly adding and subtracting $E_t(M_T R^2_T)$. Then, equation (3) and the fact that $E_t M_T R_T = 1$ lead to the results of equation (4). This is a crucial relationship. Equation (4) demonstrates that the general asset risk premium may be decomposed into two parts.[2]

[2] From now on, I assume that the asset of consideration is the S&P 500 index.

Firstly, the risk premium is related to the risk-neutral variance of its return R_T, scaled by the risk-free interest rate. As I show later, one may calculate risk-neutral variance with S&P 500 index option price data. Secondly, the equity risk premium is negatively impacted by the covariance between the return R_T and $M_T R_T$. Martin (2017a) shows that under a measure of circumstances which are all sufficient but not necessary this covariance is negative. Therefore, equation (4) simplifies further.

(5) $$E_t R_T - R_{f,t} \geq \frac{1}{R_{f,t}} \operatorname{var}^*_t R_T$$

Interestingly, together with the relationship established by Hansen and Jagannathan (1991), a bounded region for the equity risk premium results.

$$\frac{1}{R_{f,t}} \operatorname{var}^*_t R_T \leq E_t R_T - R_{f,t} \leq R_{f,t} \cdot \sigma_t(M_T) \cdot \sigma_t(R_T)$$

where $\sigma_t(\cdot)$ denotes a standard deviation. This relationship establishes that the equity risk premium exceeds the risk-neutral variance scaled by the risk-free rate but is less than the bound established by Hansen and Jagannathan (1991). The advantage of the lower bound established by equation (5) is that it may be measured in the real world. The higher bound is more of a theoretical concept. Given option-pricing theory, the risk-neutral variance may be uniquely inferred from European option prices and therefore a lower bound of the equity risk premium may be constructed.

2.1 The derivation of the SVIX index

This section demonstrates that it is possible to estimate risk-neutral variance with European option prices. As before, I continue to follow the derivation of Martin (2017a). I first assume that option prices are available and observable for a continuous section of strike prices K. Importantly, both European call and put options are convex functions of the strike price K under no-arbitrage conditions.

As a first step, the forward price of the underlying from time t to time T, $F_{t,T}$, is derived. This measure may be inferred by means of the put-call parity. Denoting the price of the call option by C, the price of the put option by P, the discount factor by D, the forward by F and the strike price by K, the relation is expressed as follows:

$$C - P = D(F - K)$$

Thus, the prices of the call and put option are equal when the strike price K equals the forward price F. Martin (2017a) shows that this equally implies that the forward price occurs at the strike level where put and call curves cross. This relation between call and put prices and the forward price is used later in the construction of the SVIX index. Further, the forward price of the underlying asset is related to present risk-neutral expectations of the future price of the underlying asset at time T, S_T.

(6)
$$F_{t,T} = E^*_t \, S_T$$

As stated, the aim is to measure risk-neutral variance. Therefore, using the results of equation (3) and assuming dividends occurring between t and T to be known, one finds that:

(7)
$$\frac{1}{R_{f,t}} \operatorname{var}^*{}_t R_T = \frac{1}{S_t^2} \left[\frac{1}{R_{f,t}} E^*{}_t S^2{}_T - \frac{1}{R_{f,t}} (E^*{}_t S_T)^2\right]$$

The second term inside the brackets equals the squared forward contract scaled by the riskless rate, as may be seen from equation (6). The first term is more complex to interpret, however. Indeed, it is the risk-neutral expectation of the squared underlying at time T. Martin (2017a) answers the question as how to price this claim in Figure 1. Buying two call options starting at the strike price K = 0.5 and continuing for each subsequent strike (i.e. K = 1.5, K=2.5) results in a curve that is tangent to the payoff-curve of the squared contract. Therefore, the squared contract can be approximately priced as twice the sum of call prices at increasing strike prices. This sum, however, slightly underestimates the true payoff of the squared contract. Since a lower bound of the equity risk premium is derived, this does not constitute an issue though.

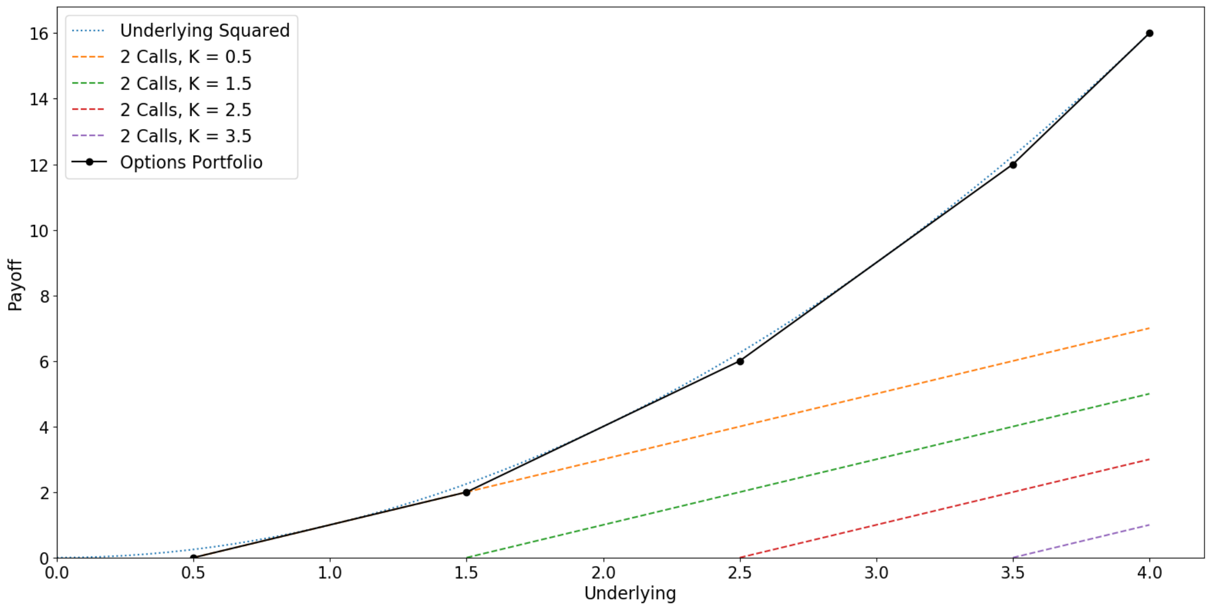

Figure 1. Replication of the payoff of the squared underlying. Adapted from "What is the Expected Return on the Market?", by I. Martin, 2017, Quarterly Journal of Economics, 132(1), p. 380.

(8) $$\frac{1}{R_{f,t}} E^*_t\, S^2_T \approx 2 \sum_{k=0.5,1.5...} \mathrm{call}_{t,T}(K)$$

More accurately, given the mathematical fact that $x^2 = 2 \int_0^\infty \max(0, x - K)\, dK$ for any $x \geq 0$, setting $x = S_T$, multiplying by $1/R_{f,t}$ and applying risk-neutral expectations, yields:

$$\frac{1}{R_{f,t}} E^*_t S^2_T = 2\int_0^\infty \frac{1}{R_{f,t}} E_t^* \max(0, S_T - K)\, dK$$

(9) $$= 2\int_0^\infty \mathrm{call}_{t,T}(K) dK$$

Equations (6), (7), and (9) allow to define the following relationship:

(10) $$\frac{1}{R_{f,t}} \mathrm{var}^*_t R_T = \frac{1}{S_t^2} [2 \int_0^\infty \mathrm{call}_{t,T} (K) dk - \frac{F_{t,T}^2}{R_{f,t}}]$$

Generally, deep in the money options are illiquid. Therefore, one may use the put-call parity to decompose the integral in equation (10) once again.

$$\int_0^\infty \text{call}_{t,T}(K)\,dk = \int_0^{F_{t,T}} \text{put}_{t,T}(K)dk + \frac{1}{R_{f,t}}(F_{t,T} - K)dK + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K)dk$$

$$= \int_0^{F_{t,T}} \text{put}_{t,T}(K)dk + \frac{F_{t,T}^2}{2R_{f,t}} + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K)dk$$

A substitution of this equation and equation (10) into equation (7) yields the final result.

(11) $$\frac{1}{R_{f,t}}\text{var}^*_t R_T = \frac{1}{S_t^2}\left[\int_0^{F_{t,T}} \text{put}_{t,T}(K)dk + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K)dk\right]$$

As Martin (2017a) points out, the right-hand side of equation (11) is very similar to the definition of the VIX index. I revisit the differences between the two measures later. However, instead of the VIX index Martin (2017a) derives the so called SVIX index.

(12) $$\text{SVIX}^2_{t\to T} = \frac{2}{(T-t)R_{f,t}S_t^2}\left[\int_0^{F_{t,T}} \text{put}_{t,T}(K)dk + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K)dk\right]$$

The $\text{SVIX}^2_{t\to T}$ index is calculated at time t based on option prices with a maturity of time T. It “measures the annualized risk-neutral variance of the realized excess return from t to T” (Martin, 2017a, p. 382).

This new construct may now be related to the previous discussion regarding a lower bound of the equity risk premium. Simply substituting equation (11) into equation (5) shows how the SVIX is related to the equity risk premium.

(13) $$E_tR_T - R_{f,t} \geq \frac{2}{S_t^2}\left[\int_0^{F_{t,T}} \text{put}_{t,T}(K)dk + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K)dk\right]$$

(14) $$\frac{1}{(T-t)}(E_tR_T - R_{f,t}) \geq R_{f,t} \cdot \text{SVIX}^2_{t\to T}$$

This establishes a fundamental result. The SVIX calculated at time t with prices of options maturing at time T, scaled by the risk-free rate may be interpreted as a lower bound of the equity risk premium. With SVIX as a proxy for the lower bound of the equity risk premium, econometric analysis of much interest may be applied.

This section introduces this new index theoretically. Nonetheless, as stated above, the striking result is that it may be measured in real time. The following section goes on to describe the empirical construction of the SVIX index before applying the results to an econometric analysis.

2.2 Construction and replication of the SVIX index

The construction and replication of the SVIX index is based both on the description of Martin (2017a) and the official CBOE white paper on the construction of the VIX index (2018). As noted earlier, both VIX and SVIX resemble each other very closely.

The objective is to construct the SVIX for maturities of one, two, three, six and twelve months. I obtain European option price data from OptionMetrics via the Wharton Research Data Services (WRDS).[3] The data sample starts on January 4, 1996 and ends on December 31, 2017. From now on, this is referred to as the complete period. Martin's original paper (2017a) is based on the period between January 4, 1996 and January 31, 2012. This time frame is referred to as the original period.

The daily option data retrieved includes the date of the observation, the expiration date of the option, the days until expiration, the strike price of the option, the highest closing bid across all exchanges for the option, the lowest closing ask across all exchanges for the option, and the mid price calculated as an arithmetic mean of the lowest closing ask and the highest closing bid.

[3] More information about the data may be found at: https://wrds-web.wharton.upenn.edu.

I start by deleting all data points with a highest closing bid of zero. As a further constraint, the days until expiration may not surpass 550 days and must be higher than seven days due to liquidity constraints. After cleaning the dataset, it consists of 9,449,402 data points.

For each day with available options price data the same procedure is applied. Given a specific SVIX maturity, the two closest existing maturities encompassing the desired maturity are chosen. The SVIX is calculated separately for these two distinct maturities. Then, the two SVIX measures are interpolated to arrive at the final result. At times, extrapolation is necessary due to data constraints. For instance, in the calculation of the June 30, 1999 SVIX with one-month maturity, the closest available maturities relate to option data with 17 and 80 days until maturity. The SVIX then is calculated for each of these maturities.

In the calculation of the SVIX, as a first step put and call option prices are sorted according to their strike prices from lowest to highest. Next, for each strike the absolute value of the difference between put and call option prices is calculated in order to find the approximate forward price of the underlying. This relation has been established in equation (6). Following the June 30, 1999 one-month maturity example, Table 1 and Table 2 depict a snapshot of the data for both the close and the far maturity options. Where the difference between the option prices (Delta) is minimal, the approximate forward price is found. For instance, given a minimal difference of 2.5, the 17 days implied forward is approximately 1370. In Table 1 and Table 2 the rows with the respective forward prices are highlighted with bold letters.

The CBOE recommends the following adjustment to arrive at the exact forward price:

$$F = K + e^{rT} * (\text{Call Price} - \text{Put Price})$$

where K is the strike price minimizing the price differential, r is the riskless rate and Call Price and Put Price are the corresponding option prices with strike price K. However, given the fact that the adjustment is miniscule in practice, it turns out that my SVIX algorithm is more efficient when applying the calculation of the approximate forward price.

Once the forward price has been established, the summation of out of the money option prices multiplied by Delta K is calculated. The sum includes put option prices up to but not including the forward strike. Further, call option prices from the forward strike to the last strike with available data are part of the sum. Before summing prices, they are multiplied by Delta K. This is the average difference between the current strike and its adjacent strikes. Thus, for the close maturity Delta K for a strike of K = 1370 quals (1375 - 1365)/2 = 5. Delta K for the lowest and highest strike (not included in Table 1 and Table 2) equals the difference to the closest strike price by default.

Strike K	Call	Put	Delta	Delta K
1360	27.5	14.75	12.25	5
1365	24.25	15.25	7.5	5
1370	**21.25**	**18.75**	**2.5**	**5**
1375	18.5	21	2.5	5

Table 1. Close maturity options (17 days until maturity).

Strike K	Call	Put	Delta	Delta K
1360	64	43	21	5
1365	60.675	44.75	16.125	7.5
1375	**54.75**	**48.75**	**6**	**17.5**
1400	41	59.25	18.25	15

Table 2. Far maturity options (80 days until maturity).

Given both call and put prices and Delta K, the sum of scaled put and call prices is calculated. The resulting number directly flows into the SVIX calculation. Indeed, it constitutes the term inside the squared brackets in equation (12). Further terms used for the SVIX calculation are the value of the underlying at time t squared, the time fraction between now and maturity T-t, and the appropriate risk-free rate. The time fraction T-t follows the convention that it equals the days until maturity as a fraction of a year (365 days) for the different maturities. For the underlying, S&P 500 index data from The Center of Research in Security Prices (CRSP), which is available via the Wharton Research Data Services, is used.[4] For the risk-free interest rate, I choose the zero-coupon yield curve obtained from OptionMetrics after verifying its validity. An outline about how interest rates for the specific maturity dates are obtained may be found in the appendix. Given the risk-free interest rate of the chosen SVIX maturity, a linear adjustment is applied to arrive at the interest rate for both close and far maturities. For instance, the one-month interest rate of 0.445% is multiplied by 17/30 to arrive at the appropriate approximate 17-day interest rate. Then, one is added to arrive at the original definition of $R_{f,t}$. Given all building blocks, in this example 17-day and 80-day SVIX may be calculated according to equation (12).

Finally, following the CBOE's manual, with 17-day and 80-day SVIX data the one-month SVIX is interpolated as follows:

$$\mathrm{SVIX}^2_{t\text{-}30} = \mathrm{SVIX}^2_{t\text{-}17}\frac{(80-30)}{(80-17)} + \mathrm{SVIX}^2_{t\text{-}30}\frac{(30-17)}{(80-17)}$$

In general, I apply the convention of a 30-day month for the SVIX construction. The steps outlined above are applied for the five different maturities on every trading day in the sample.

I use code written in the programming language Python to automatically perform data manipulation and SVIX calculation for every day in the data set.

[4] More information about the data may be found at: https://wrds-web.wharton.upenn.edu.

3. The SVIX and its Relative the VIX

As previously mentioned, the SVIX index shares striking similarities with the VIX index, which measures market-expectations of 30-day implied volatility (CBOE, 2018). This section briefly elaborates on the differences between these two measures. Further, it demonstrates the consequences of these differences.

The VIX index is calculated according to equation (15):

$$\text{(15)} \qquad \mathrm{VIX}^2_{t\to T} = \frac{2R_{f,t}}{(T-t)}\left[\int_0^{F_{t,T}} \frac{1}{K^2}\,\mathrm{put}_{\,t,T}(K)dk + \int_{F_{t,T}}^{\infty} \frac{1}{K^2}\,\mathrm{call}_{\,t,T}(K)dk\right]$$

This equation is strikingly similar to equation (12) which defines $\mathrm{SVIX}^2_{t\to T}$. There are some minor differences, however. Most notably $\mathrm{VIX}^2_{t\to T}$ weights option prices by their strike prices. Therefore, it puts more emphasize on out of the money put options. $\mathrm{SVIX}^2_{t\to T}$, one the other hand, equally weights option prices. Through the emphasis on put prices the left tail of the curve is given more weight in the VIX index.

A further difference relates to the exact measure of the two indices. The SVIX index is a measure of risk-neutral variance. Martin (2017a) states that the VIX index measures risk-neutral entropy of returns. Entropy is a measure of variability, as is variance. Entropy measures “the extent to which a concave function of an expectation of a random variable exceeds an expectation of a concave function of a random variable” (Martin, 2017a, p. 401). Further, De Prado (2018) defines Entropy as “the average amount of information […] produced by a stationary source of data” (p.263), which measures local unpredictability in the system (Filia, 2018). Therefore, SVIX and VIX offer different measures of variability. Further, both are sensitive to distinct areas of the return distribution. Due to its relative high weights in out of the money put options, the VIX index is more sensitive to the left tail, while the SVIX index is more sensitive to the right tail.

Figure 2 plots both ten-day moving averages of VIX and one-month SVIX while Figure 3 plots their respective difference. I retrieve data of the VIX index from the Wharton Research Data Services.[5] From Figure 2 one may observe that VIX and SVIX indeed are very similar. They almost seem to be identical. Upon closer observation, however, it becomes apparent that VIX generally is larger than SVIX. Figure 3 confirms this impression. Except for a few outliers, I generally can confirm the results obtained by Martin (2017a) which show that VIX exceeds SVIX on almost every day during the sample period. This is especially interesting because under log-normal return assumptions the opposite would be the case. In a world characterized by log-normal returns, SVIX always exceeds VIX. Especially in times of stress, such as during the subprime mortgage crisis, the spread between VIX and SVIX tends to blow out. Martin (2017a) concludes that this is strong evidence against the hypothesis of log-normal returns at the one-month time horizon.

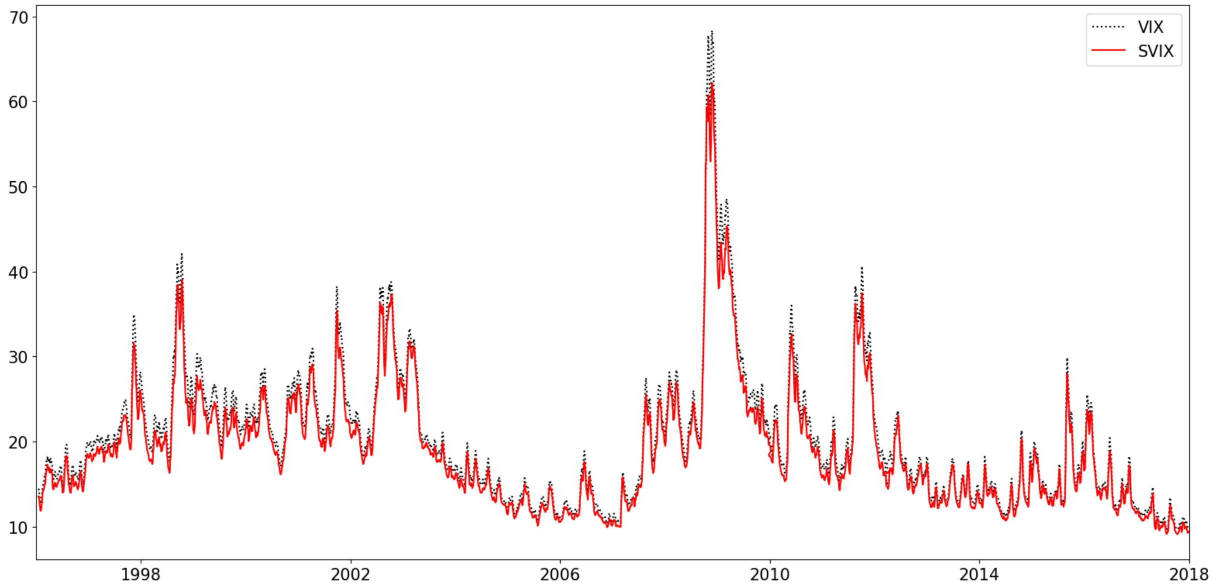

Figure 2. VIX and SVIX.

[5] More information about the data may be found at: https://wrds-web.wharton.upenn.edu.

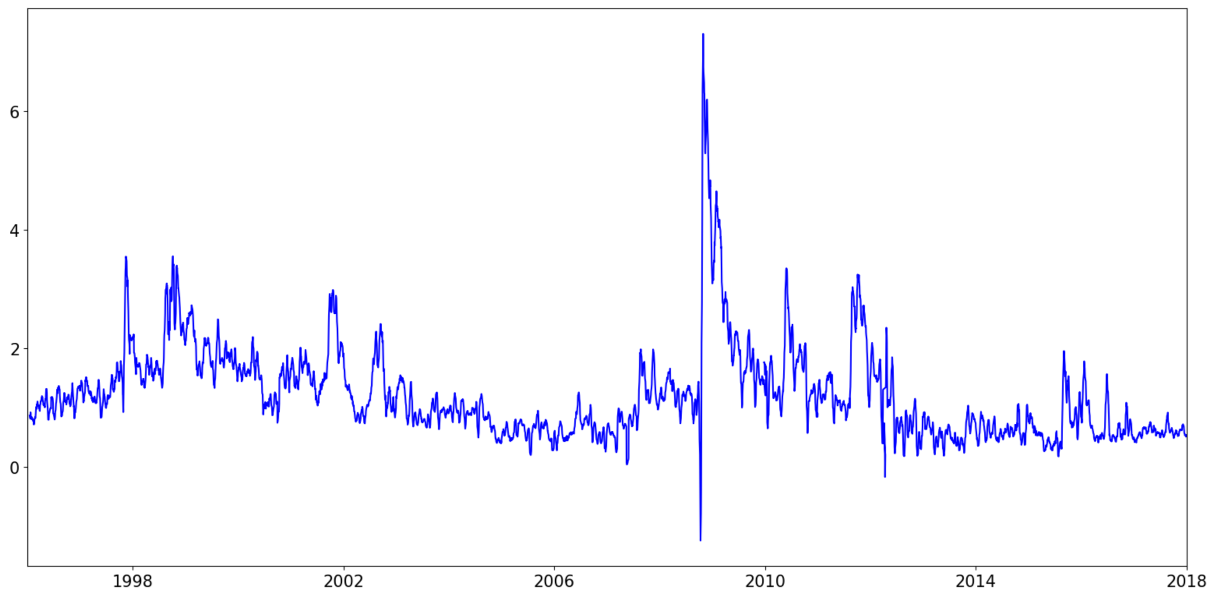

Figure 3. VIX minus SVIX.

To further investigate this puzzle, Table 3 lists the main distribution features of VIX, one-month SVIX, and their difference. Generally, the VIX is higher than SVIX. It has a larger mean, median, and maximum value. Further, the average and median value of VIX minus SVIX are both positive at 1.22 and 1.05, respectively. Even though the minimum value is negative at -1.24, negative values seem to concern only few data points. At 7.3 the maximum value is much larger in magnitude. These summary statistics further confirm the view that returns are not log-normal at the one-month horizon.

Data	Mean	Median	Std. dev.	Min	Max	Skewness	Kurtosis
VIX	20.14	18.63	8.06	9.61	68.33	1.93	6.3
SVIX	19.09	17.75	7.44	9.11	62.15	1.89	5.97
VIX-SVIX	1.22	1.05	0.78	-1.24	7.3	2.22	8.88

Table 3. Summary statistics of VIX, SVIX, and VIX minus SVIX.

After this brief comparison between the SVIX index and its closest relative, section 4 explains the main statistical features of the SVIX index.

4. SVIX Summary Statistics

After constructing and replicating $SVIX^2_{t\to T}$ with the written Python program and the available option price data, the aim is to compare whether the results have a close enough resemblance with the data provided by Ian Martin.[6] In fact, the values of the indices replicated by the program generally mirror Martin's data nicely. For the one-month, two-months, three-months, six-months, and twelve-months lower bound of the equity premium defined by equation (14), the correlation coefficients between the two datasets are 99.78%, 99.98%, 99.96%, 99.95%, and 99.77%, respectively.

Figure 4 depicts the annualized one-month lower bound of the equity risk premium. As becomes quite apparent, the expected return is far from stable. Rather, it is extremely volatile and inherently quick to adjust to market turmoil. In the past two decades, big spikes in $R_{f,t} \cdot SVIX^2_{t\to T}$ and therefore in the lower bound of the equity risk premium have occurred during the Asian financial crisis, the Long-Term Capital Management crisis, the dotcom crash, the European debt crisis, the United States downgrade, the Chinese market sell-off and the subprime mortgage crisis. In all cases, the SVIX adjusted very quickly. This is in sharp contrast to valuation-based equity risk premia estimates which almost did not change during many of these crises (Martin, 2017a). Further, it is interesting to note that SVIX indicated moderate to high expected returns during the dotcom boom of the 90s. At this time, valuation-based models predicted low or in some cases even negative expected returns due to excessive valuations.

[6] More information about the data provided by Ian Martin may be found at: http://personal.lse.ac.uk/martiniw.

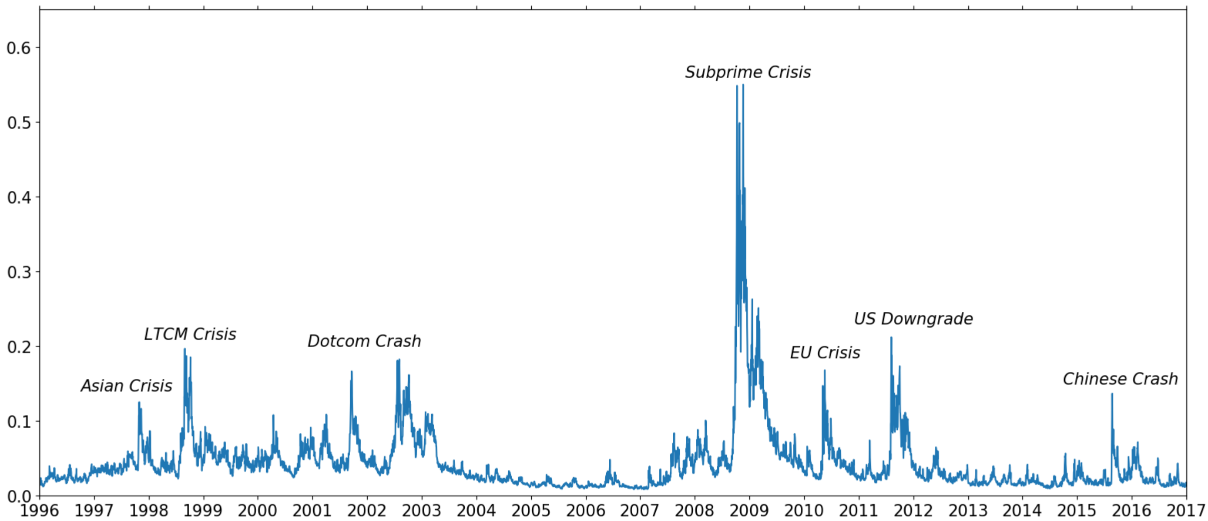

Figure 4: Annualized lower bound of the equity risk premium at the one-month horizon.

Figure 5 plots the one-month lower bound of the equity risk premium at the side of its two-months, six-months and twelve-months counterparts. All series are annualized. Notably, while the graphs generally resemble each other closely, the SVIX is even more volatile in the short run. Looking at the summary statistics in Table 4 strengthens this impression.

Table 4 displays the main summary statistics for the lower bound of the equity premium for the complete period. Generally, the results confirm the findings of Martin (2017a) for the original period. The lower bound of the equity risk premium is right-skewed and very volatile. Especially for short time frames large annualized equity risk premia are observable. Interestingly, for all maturities the mean equity risk premium closely resembles values that have been empirically estimated in the academic literature. Fama & French (2002) provide estimates of 3.83% and 4.78% for the equity risk premium. The mean of the lower bound of the equity risk premium is in this range. It is close to 4.2% for all maturities. This indeed may lead to the conclusion that the lower bound is tight (Martin, 2017a), i.e. that it equates the equity risk premium. After investigating the robustness of the SVIX construction in the next section, a subsequent part of this thesis explores the tightness of the lower bound.

Finally, Figure 6 plots the general distribution of the lower bounds of the equity risk premia for different maturities. As indicated by the summary statistics, the lower bounds are skewed and fat tailed, with the average exceeding the median for every time frame.

The following section examines the robustness of the construction of the SVIX index before this volatile index is applied in predictive regressions later.

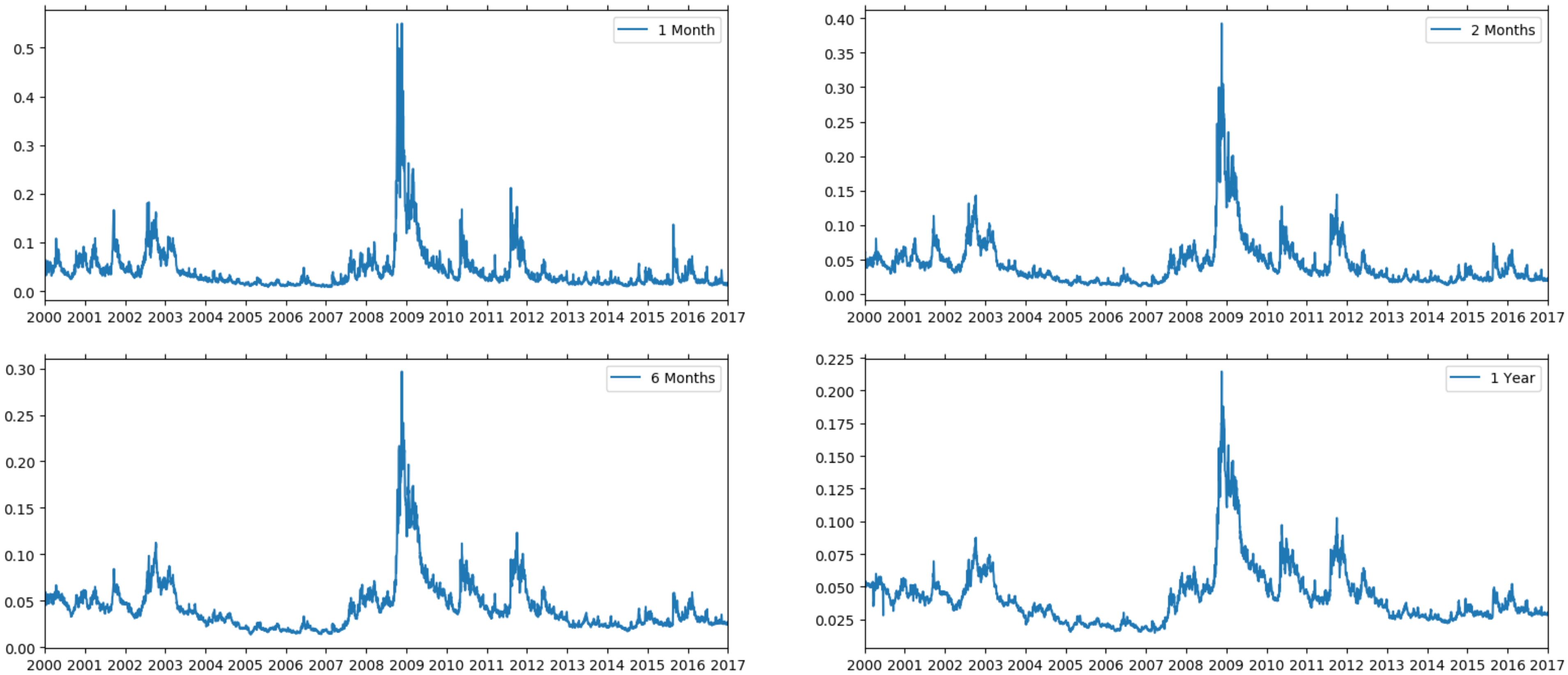

Figure 5. Annualized lower bound of the equity risk premium at different horizons.

Maturity	Mean	Std. dev.	Skew	Kurt	Min	1%	10%	25%	50%	75%	90%	99%	Max
1 Month	4.23	4.26	4.54	31.84	0.71	0.89	1.33	1.87	3.14	5.00	7.80	22.31	55
2 Months	4.29	3.69	3.70	21.35	1.00	1.11	1.54	2.06	3.34	5.19	7.62	19.96	46.4
3 Months	4.30	3.32	3.31	17.06	1.05	1.28	1.68	2.19	3.48	5.25	7.41	18.36	39.2
6 Months	4.32	2.76	2.73	11.90	1.29	1.53	1.90	2.44	3.70	5.29	7.13	15.93	29.7
12 Months	4.22	2.23	2.23	8.09	1.29	1.63	2.12	2.70	3.8	5.10	6.69	13.36	21.5

Table 4. Summary statistics of the lower bound of the equity risk premium at various horizons (annualized and measured in %).

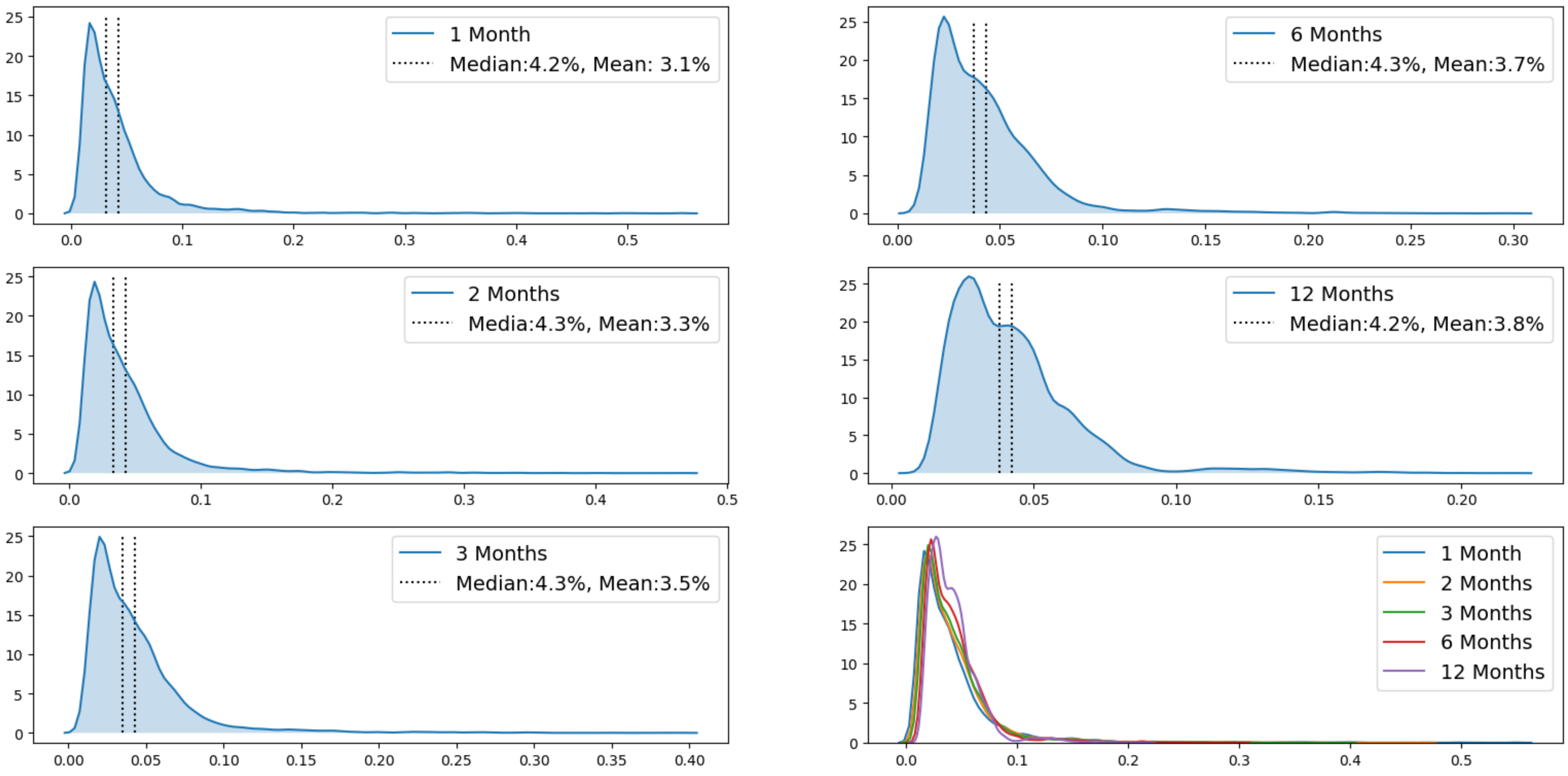

Figure 6. Distribution of the lower bound of the equity risk premium at different horizons.

4.1 The robustness of the results

Given the findings of the previous section, it seems appropriate to investigate whether the constructed lower bound is robust and closely resembles the equity risk premium. After all, use of various approximations has been made in the calculation of the SVIX index.

Firstly, as is mentioned previously, it is not possible to accurately price the payoff of the squared contract S^2_T. This results from the contract not being observable in financial markets. The approximation in Figure 1 fails to perfectly replicate the payoff. However, the approximated payoff is lower than the true payoff. Since $R_{f,t} \cdot SVIX^2_{t \to T}$ is claimed to be the lower bound of the equity risk premium, this does not jeopardize the results. Rather, it strengthens the conviction that this truly is a lower bound.

Secondly, mid prices of options are used in this calculation. Thus, theoretically it would be possible that mid prices distort the view of true liquidity if bid prices are virtually non-existent. However, it is possible to replicate the results with bid prices. Martin (2017a) shows that the results differ insignificantly when bid prices are used instead of mid prices.

Thirdly, the calculation assumes that option prices are observable for continuous strike prices. This does not resemble reality though. In markets, only discrete strike prices are traded. However, Martin (2017a) shows that this merely results in a slight underestimation of the SVIX. This further strengthens the assumption of approximating a lower bound of the equity risk premium.

4.2 Is the SVIX the equity risk premium?

The previous analysis has shown that the time series mean of the lower bound of the equity risk premium ranges between 4.22% and 4.32% for the complete observation period. These values closely resemble estimates for the long run equity risk premium estimated by Fama & French (2002). Given this observation, Martin (2017a) poses the question whether the measure of the lower bound of the equity risk premium is equal to the true expected return on the market. This is a very crucial and important question. If the SVIX-related construct indeed was the equity risk premium, it would be possible to have a real time index for this central economic measure.

To test this hypothesis, I follow the econometric analysis of Martin (2017a) and run the following regression for both the original and the complete period:

$$\frac{1}{(T-t)}(R_T - R_{f,t}) = \alpha + \beta \times R_{f,t} \cdot SVIX^2_{t \to T} + \varepsilon_T \tag{16}$$

As established, returns R_T are calculated as simple returns plus one. For the return calculation, I conventionally apply the assumption of 21, 42, 63, 126, and 252 trading days for the one-month, two-months, three-months, six-months, and twelve-months SVIX maturities. Given daily data of the riskless rate at the appropriate time frame, the excess return is obtained easily by deducting the riskless rate from the realized return. The term 1/(T-t) on the left side of equation (16) implies that the excess return is annualized. This is appropriate as SVIX is an annualized measure too. To annualize the excess return, I simply multiply it by one divided by its yearly time fraction. Therefore, for instance the one-month excess return is multiplied by 1/(1/12) = 12 to arrive at the annualized return.

To control for heteroskedasticity and autocorrelation, I estimate HAC standard errors which are consistent with these statistical features. Table 5 depicts the regression results of equation (16) for the original period tested by Martin (2017a).

Horizon	α	Std. error.	β	Std. error	R^2 (%)	R^2_{os} (%)
1 Month	-0.0028	0.051	0.6761	1.085	0.3	0.8
2 Months	-0.0167	0.055	0.9171	1.149	0.8	1.96
3 Months	-0.0188	0.067	0.9558	1.427	1	2.8
6 Months	-0.0708	0.054	2.0377	0.983	5.6	8.73
12 Months	0.0475	0.069	1.6715	1.022	4.4	9.88

Table 5: Results of regression (16) for January 4,1996 to January 31, 2012.

My regression estimates replicate Martin's results very well, with coefficients being in the same range. Especially two things are noteworthy. Firstly, the alpha of the regression generally is not statistically different from zero. Secondly, even though the value of beta fluctuates rather strongly among various time frames, I fail to reject the hypothesis that beta equals one at the 5% significance level. Rather, the 95% confidence interval for every beta includes the value one. Therefore, I fail to reject the hypothesis that the real time measure of the lower bound of the equity risk premium, $R_{f,t} \cdot SVIX^2_{t \to T}$, may indeed be the equity risk premium itself. Consequently, with α = 0 and β = 1, equation (16) simplifies to:

(17) $$\frac{1}{(T-t)}(R_T - R_{f,t}) = R_{f,t} \cdot SVIX^2_{t \to T}$$

An objection may be made that based on the low R-squared measures for virtually all regression results, the explanatory power of the regression seems to be limited. To explore this result further, I follow Goyal and Welch (2008) and Martin (2017a) in constructing an out of sample R-squared measure. The out of sample R-squared, R^2_{os}, is defined according to:

(18) $$R^2_{os} = 1 - \frac{\sum \varepsilon_t^2}{\sum v_t^2}$$

v_t is the error of using the rolling long-term equity risk premium as predictor variable for the realized return, while ε_t is the regression residual when the lower bound of the equity risk premium is used as a predictor variable.

For the calculation of the rolling long-term equity risk premium I use two data sets. First, I use monthly real return data on the S&P 500 from January 1871 until December 1926 from Robert Shiller´s website.[7] The mean is computed on a rolling basis and multiplied by twelve to annualize the monthly returns. Second, I apply previously obtained daily data for real one-month returns on the S&P 500 from The Center of Research in Security Prices from January 3, 1927 until December 29, 2017. The data is annualized by multiplying by twelve. However, when calculating the rolling mean from the two data sets, the data is weighted to correctly account for the occurrences of daily and monthly data.

A positive R^2_{os} indicates that the regression with the chosen predictor variable performs better than when predicting the equity risk premium with its simple long-term rolling mean. This is insofar noteworthy as Goyal and Welch (2008) state that most models predicting the equity risk premium are weak in nature and have performed poorly for many decades. Simply using the long-term rolling mean equity risk premium in predicting equity returns beats most of these models. Goyal and Welch (2008) test the predictive power of various fundamental variables for the period from 1871 to 2005. Generally, these predictor variables perform well in sample, but their predictive power deteriorates significantly out of sample. The book-to-market ratio, dividend-price ratio, dividend-yield, and earnings-price ratio generate negative out of sample R-squared values, indicating worse out of sample results. The R^2_{os} for regression equation (16) is positive for all maturities, however. Therefore, the lower bound of the equity premium seems to add predictive power.

On closer inspection, R^2_{os} is small in magnitude. This is especially true for close maturities. Campbell and Thompson (2008) explain that the magnitude of the out of

[7] More information about the data may be found at: http://www.econ.yale.edu/~shiller/.

sample R-squared should always be evaluated relative to the squared Sharpe Ratio of a given strategy. Even small out of sample R-squared readings may increase investment Sharpe Ratios substantially. Indeed, they continue to argue that regressions results "with large R^2 statistics would be too profitable to believe" (Campbell & Thompson, 2008, p. 14).

To further contribute to the discourse, I run regression (16) for the complete period.

Horizon	α	Std. error	β	Std. error	R^2 (%)	R^2_{os} (%)
1 Month	0.0369	0.039	0.4503	1.004	0.1	0.21
2 Months	0.0286	0.040	0.5710	1.023	0.3	0.66
3 Months	0.0299	0.049	0.5151	1.238	0.3	3.6
6 Months	-0.0056	0.039	1.3347	0.886	2.5	3.66
12 Months	0.0046	0.051	1.1264	0.922	2.0	4.07

Table 6: Result of Regression (16) for January 4, 1996 to December 31, 2017.

The change in time frame seems not to have impacted the regression outcome. I fail to reject the hypothesis that alpha equals zero and that beta equals one. Rather, the coefficients for farther maturities are even closer to the hypothesized value of one. Running regression equation (16) for the complete period provides further support for the notion that $R_{f,t} \cdot SVIX^2_{t \to T}$ indeed may be the equity risk premium.

The next section explores one final indicator that may lead to the conclusion of the SVIX measure being a good proxy for the equity risk premium.

5. The Rational Investor Hypothesis

As a social psychologist, I have long been amused by economists and their curiously delusional notion of the "rational man." Rational? Where do these folks live? Even 50 years ago, experimental studies were demonstrating that people stay with clearly wrong decisions rather than change them, throw good money after bad, justify failed predictions rather than admit they were wrong, and resist, distort or actively reject information that disputes their beliefs. (Richard Thaler, 2015)

Much has been written in the literature about rational investors and expectations of equity returns. Even though much economic and financial theory suppose investors to be rational, empirical findings often contradict this notion of rational and optimizing beings.

Pound and Shiller (1986) find that contagious enthusiasm among investors may overweight fundamental considerations in the investment process. Even professional investors are subject to this bias. Similarly, Ben-David, Graham, and Harvey (2013) conclude that “executives are severely miscalibrated” (p.19). Their confidence intervals regarding market expectations are excessively narrow and they tend to be overconfident in their predictions. Further, investors share significant home-country bias. Expectations of returns for the same market tend to vary strongly across the globe (Fumiko, Shiller & Yoshiro, 1991). Despite market participants’ general strong conviction in their ability to market-time equity markets (Shiller, 1987), they tend to be subject to availability bias and fare especially poorly in assigning probabilities to possible market crashes (Dasol, Goetzmann & Shiller, 2017). Even more significant, Greenwood and Shleifer (2014) analyse investor survey data and note that investor expectations share a strong negative correlation with subsequently realized returns. They conclude that their findings are inconsistent with rational expectations. Given these findings, one may almost suppose that a necessary condition for a proxy of the equity risk premium may be a negative correlation with investor expectations.

Following this notion and Martin (2017a), I test whether the SVIX as a proxy for the equity risk premium fulfils this condition.

I compare expected excess returns implied by SVIX to three different survey results. Following Martin’s analysis, I choose the American Association of Individual Investors

(AAII) survey[8] and Robert Shiller's investors survey[9] as market expectation proxies. Further, I select the monthly purchasing manager index (PMI) provided by the Institute of Supply Management (ISM).[10] The data of this time series is available on the financial data platform Quandl. The PMI tends to be a measure for general confidence in the economy (Koenig, 2002) and therefore seems to be appropriate. The AAII survey measures the percentage of investors who are bullish minus the percentage of investors who are bearish over the next six months. To be consistent with the time frames of the survey measures, I compare the AAII survey results to the expected return implied by six-months SVIX. Further, the index of Robert Shiller's survey measures the percentage of individual investors expecting the stock market to advance over the next twelve months. Consequently, expected returns implied by the twelve-months SVIX are chosen as comparison. Finally, the monthly PMI data is compared to the equity premia derived from the one-month SVIX.

To obtain expected equity returns, the corresponding risk-free yield is added to the equity risk premia. Further, all time series are normalized to zero-mean and unit-variance series. Figure 7 plots the normalized series. As may be observed from the charts, expectations tend to move in the opposite direction of the implied equity returns. The correlation between the PMI and the SVIX amounts to –0.589, showing a strong negative relationship. Indeed, the purchasing manager index bottoms in 2008 when SVIX reaches new highs. Similarly, the AAII survey results have a negative correlation of –0.165 with the corresponding implied equity returns, confirming the notion of rather irrational investors. Results from the Shiller survey analysis do not affirm these findings, however. Whereas the correlation with the corresponding equity premium is

[8] More information about the AAII survey results may be found at: https://www.aaii.com.
[9] More information about Robert Shiller's survey results may be found at: https://som.yale.edu.
[10] More information about the Purchasing Manager index may be found at: https://www.instituteforsupplymanagement.org.

strongly negative (–0.43) until February of 2012, confirming Martin's findings, the relationship changes subsequently. Indeed, the correlation turns positive (0.255) for the entire dataset. Yet, the last decade of bull-market may have attributed to this change in correlation. A later section explores this topic.[11] As may be observed in Table 7, skewness and kurtosis are generally low and close to zero for the chosen survey measures, whereas they are large in magnitude for the SVIX, as demonstrated earlier.

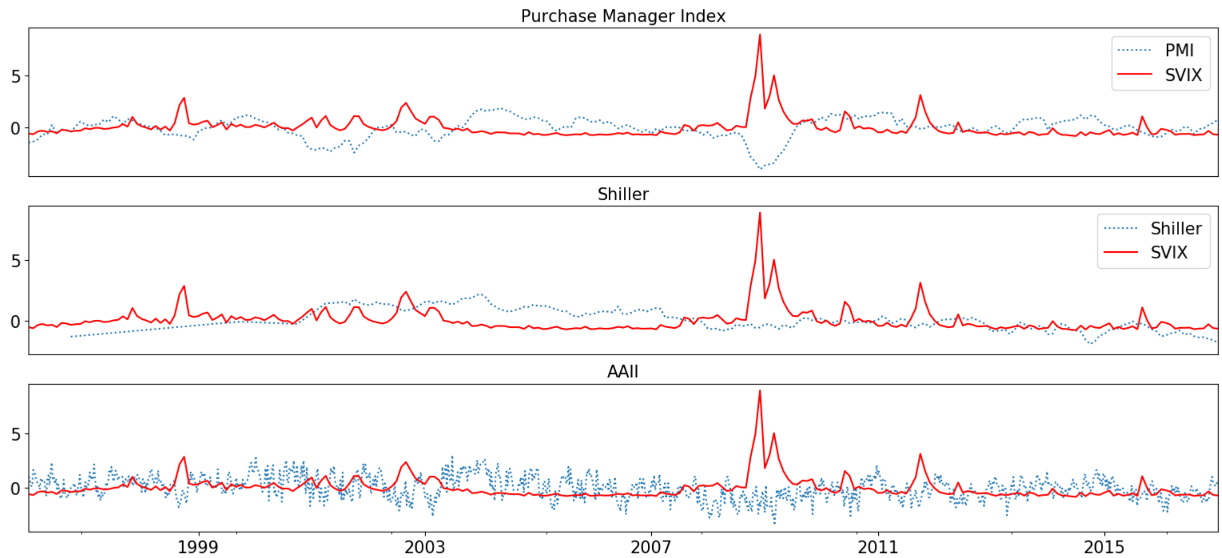

Figure 7. Survey measures of expected returns and SVIX-implied expected returns.

	Shiller	AAII	PMI
Skewness	–0.044	0.03	–1.023
Excess Kurtosis	–0.618	–0.104	1.859
Corr(Suvey,ER)	0.255	–0.165	–0.589

Table 7. Summary statistics of survey results and correlation to expected equity returns implied by SVIX.

This analysis supports some further evidence against the rational expectation hypothesis. Two of the three chosen expectation surveys correlate negatively with corresponding return measures. For the final survey the correlation only turns positive after including the past low volatility years into the dataset. Thus, there may be further evidence for the rejection of the rational investor hypothesis. Further, the SVIX fulfils the requirement of being negatively correlated with investor expectations, thereby

[11] See Section 9: The Possibility of a Regime Change.

strengthening the conclusion of $R_{f,t} \cdot SVIX^2_{t \to T}$ being the expected return on the market. Starting from this conclusion, the next section demonstrates how the equity risk premium fluctuates among maturities.

6. A Term Structure for the SVIX

As has already been observed earlier in the analysis, SVIX and consequently the lower bound of the equity risk premium are very volatile measures. This is especially true for close maturities. The maximum annualized SVIX print decreases the longer the maturity of the index is. Therefore, it may be informative to generalize how SVIX varies among maturities. In the following analysis, I replicate what Martin (2017a) calls the term structure of equity risk premia. As the previous sections have shown, the SVIX measure for the lower bound of the equity premium may indeed be the true expected equity return. Therefore, constructing a term structure of the SVIX allows to determine the term structure of equity risk premia. In this section, I show that the term structure of equity risk premia is very volatile. Especially in times of distress much of the equity risk premium is located at the short end of the curve.

6.1 The derivation of the term structure

I follow Martin (2017a) in the derivation of the term structure. Firstly, equation (19) defines the forward equity risk premium between T_1 and T_2. The time fraction $1/(T_2\text{-}T_1)$ shows that is measure is annualized.

(19) $$EP_{T1 \to T2} \equiv \frac{1}{T_2 - T_1}\left(\log\frac{E_t R_{t\to T_2}}{R_{f,t\to T_2}} - \log\frac{E_t R_{t\to T_1}}{R_{f,t\to T_1}}\right)$$

The annualized spot equity premium from time t to T is defined as:

(20) $$EP_{t \to T} \equiv \frac{1}{T-t}\log\frac{E_t R_{t\to T}}{R_{f,t\to T}}$$

This is where SVIX as proxy for the equity risk premium may be substituted. Combining equations (17) and (19) yields the following:

(21) $$EP_{T1\rightarrow T2} = \frac{1}{T_2 - T_1} \log \frac{1+SVIX^2_{t\rightarrow T_2}(T_2 - t)}{1+SVIX^2_{t\rightarrow T_1}(T_1 - t)}$$

(22) $$EP_{t\rightarrow T} = \frac{1}{T-T} \log(1+SVIX^2_{t\rightarrow T}(T-t))$$

This shows how forward and spot equity premia depend on the respective SVIX index. The long run equity risk premium $EP_{t\rightarrow TN}$ may be decomposed according to:

(23) $$EP_{t\rightarrow TN} = \frac{T_1 - t}{T_N - t} EP_{t\rightarrow T1} + \frac{T_2 - T_1}{T_N - t} EP_{T1\rightarrow T2} + \ldots + \frac{T_N - T_{N-1}}{T_N - t} EP_{TN-1\rightarrow TN}$$

Equation (23) demonstrates that the long run equity premium $EP_{t\rightarrow TN}$ is merely a weighted average of the corresponding forward equity risk premia. Consequently, I break the one-year equity premium into its various components. These components are the one-month spot premium and the corresponding forward premia from one to two, two to three, three to six and six to twelve months. The construction of the spot and forward premia is elaborated on in the appendix. Figure 8 depicts how these components, which are not annualized, make up the annual equity risk premium. The region between two lines on the graph depicts the contribution of the corresponding forward equity premium adjusted for its time frame.

Figure 8 confirms the prior impression about the volatility of the short-term equity premium. In relatively sanguine periods such as in the early 2000s, the term structure of the equity risk premia is steep. This means that in these times short-term equity premia add relatively little to the annual equity risk premium. On the contrary, this situation changes significantly in times of market turbulence such as during the subprime mortgage crisis. In this case equity premia of short duration make up an ever-larger proportion of the annual equity risk premium.

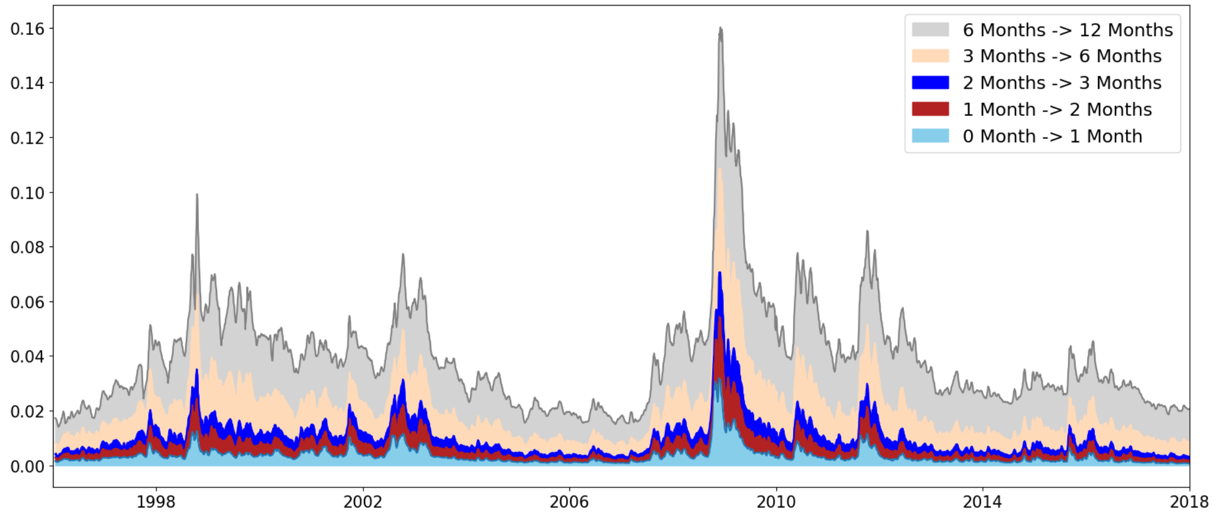

Figure 8. Ten-day moving average of the term structure of equity risk premia.

The following section explores a different approach of how to decompose the annual SVIX time series.

7. Persistence in the Time Series

A further approach of having a look at the decomposition of the twelve-months SVIX is to investigate the persistence of shocks within the time series. Ortu, Tamoni and Tebaldi (2013) decompose a time series of consumption growth into various components. Each component relates to a level of persistence. More specifically, each component is classified by its half-life and thereby captures persistence falling into a specific time frame. This allows the authors to differentiate between economic phenomena at different horizons. Through the decomposition they detect that there exist predictable patterns and components in the consumption growth data. Their approach is targeted at the wide statistical problem of the lack of statistical power of conventional tests in the case of testing high persistence components with low volatility. Standard statistical models generally fail to distinguish such a high persistence, low volatility component from white noise.

I apply the decomposition of the time series to twelve-months SVIX and compare the varying components at different time-scales to other SVIX measures for lower maturities. I find that some persistence does exist in the data. Interestingly, the SVIX at different horizons shares a correlation of close to 0.5 with the corresponding component for all chosen pairs. When I compare the index to the squared component, the correlation generally increases to close to 0.6

7.1 Decomposing the time series

In this section, I follow Ortu et. al (2013) in the derivation of the decomposition of a time series. As a first step, starting from the time series $\{g_t\}_{t\in Z}$, I construct moving averages $\pi_t^{(j)}$ which have size 2^j.

(24) $$\pi_t^{(j)} = \frac{1}{2^j}\sum_{p=0}^{2^j-1} g_{t-p}$$

By definition, $\pi_t^{(0)} \equiv g_t$. Firstly, one may observe that the following conditions hold.

(25) $$\pi_t^{(j)} = \frac{\pi^{(j-1)}{}_t + \pi^{(j-1)}{}_{t-2^{j-1}}}{2^j}$$

(26) $$g_t^{(j)} = \pi_t^{(j-1)} - \pi_t^{(j)}$$

$g_t^{(j)}$, the difference between the moving averages of size 2^{j-1} and 2^j, captures an important measure. It "captures fluctuations that survive to averaging over 2^{j-1} terms but disappear when the averaging involves 2^j terms, that is, fluctuations with half-life in the interval $[2^{j-1},2^j)$" (Ortu et. al, 2013, p. 6). Consequently, the moving average $\pi_t^{(j)}$ incorporates all fluctuations with a half-life greater than 2^j. As the authors conclude, the series $\{g_t^{(j)}\}_{t\in z}$ includes the components of series $\{g_t\}_{t\in Z}$ with persistence j.

Interestingly, these results allow us to decompose the original series $\{g_t\}_{t \in Z}$ into two separate parts.

(27) $$g_t = \sum_{j=1}^{J} g_t^{(j)} + \pi_t^{(J)}$$

This splits the time series into the sum of persistent shocks at persistence level j, $g_t^{(j)}$, each with a specific half-life, and a moving average of size 2^J, $\pi_t^{(J)}$. Equation (27) may still pose the problem of serial correlation since the moving averages which make up $g_t^{(j)}$ according to equation (26) may overlap. The result in turn may be a biased estimation of the persistence. To eliminate any possible serial correlation, the authors introduce decimated components (Ortu et. al, 2013) with persistence level j.

(28) $$\{g_t^{(j)}, t = 2k^j, k \in Z\}$$

(29) $$\{\pi_t^{(j)}, t = 2k^j, k \in Z\}$$

The decimation crucially eliminates from the two components $g_t^{(j)}$ and $\pi_t^{(j)}$ the information that is not relevant when reconstructing the initial time series (Ortu et. al, 2013). The authors further show that "for any J ≥ 1 one can find a linear, invertible operator $T^{(J)}$ that maps the decimated components […] into the time series" (Ortu et. al, 2013, p. 6). Indeed, they show that it is possible to construct $T^{(J)}$ in a way satisfying the following:

(30) $$c = T^{(J)} \{g_t\}_{t \in Z}$$

(31) $$\{g_t\}_{t \in Z} = (T^{(J)})^{-1} c$$

where $\{g_t\}_{t \in Z}$ is a vector of the original time series as introduced before, $T^{(J)}$ is the linear, invertible operator that maps the decimated components into the time series, and c is a vector consisting of the moving average of size 2^j and the persistent components up to persistence level j. Equation (30) shows how the original time series may be deconstructed with help of the operator. Since $T^{(J)}$ is invertible, equation (30) can be inverted to obtain (31). The time series may be reconstructed from the components.

7.2 The decomposition of twelve-months SVIX

In this section, it is my aim to decompose the time series of twelve-months SVIX into its various persistent components via the approach introduced in the previous section. Further, it is of interest to observe how the components relate to the different SVIX indices. More specifically, I compare each of the five SVIX indices to the component with a half-life most closely matching its maturity. Following the approach of Ortu et al. (2013) I use a Python program to construct the invertible operator $T^{(J)}$. The program then yields a result like the vector c in equation (30). More specifically, given a time series and a chosen persistence level j, the code returns a matrix with j+1 columns. Each of the first j columns depicts a time series of the j-th component. Column j+1 belongs to a time series of the moving average of size 2^j. Appropriately, the summation of the columns exactly reconstructs the original time series $\{g_t\}_{t \in Z}$ according to equation (27).

Given the daily frequency of the twelve-months SVIX data, the half-life of the components is presented in Table 8. Here, twelve components are chosen to closely approximate the depth of the time series.

Component	Daily Frequency Resolution
$g_t^{(1)}$	1-2 days
$g_t^{(2)}$	2-4 days
$g_t^{(3)}$	4-8 days
$g_t^{(4)}$	8-16 days
$g_t^{(5)}$	16-32 days
$g_t^{(6)}$	32-64 days
$g_t^{(7)}$	64-128 days
$g_t^{(8)}$	128-256 days
$g_t^{(9)}$	256-512 days
$g_t^{(10)}$	512-1024 days
$g_t^{(11)}$	1024-2048 days
$g_t^{(12)}$	2048-4096 days

Table 8. Daily frequency of the j-th component $g_t^{(j)}$. The original twelve-months SVIX time series is of daily observation.

I continue to compare specific components to the various SVIX measures to investigate both the correlation among various time series and the persistence in the data.

The fifth component of the twelve-months SVIX time series, $g_t^{(5)}$, has a half-life between 16 and 32 days. As may be inferred from equation (26), $g_t^{(5)}$ captures the fluctuations surviving into this time frame. Therefore, I use this component as proxy for the one-month SVIX. Figure 9 plots both the chosen one-month SVIX index on the left y-axis and $g_t^{(5)}$ on the right y-axis for the complete data sample observed.

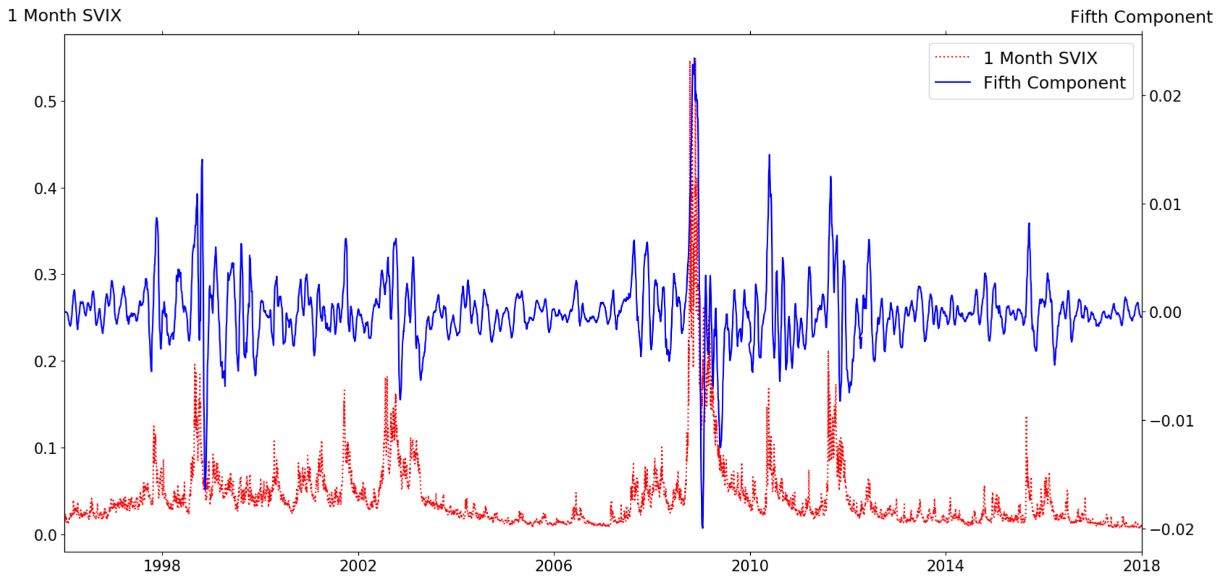

Figure 9. One-month SVIX and the fifth component of the decomposition.

Over the entire time frame the correlation between the two series amounts to 0.419. As may be observed closely, a correlation value of around 0.4 – 0.5 between the chosen SVIX and the component is shared for virtually all pairs examined in this analysis. I go one step further and plot one-month SVIX against the squared fifth component in Figure 10.

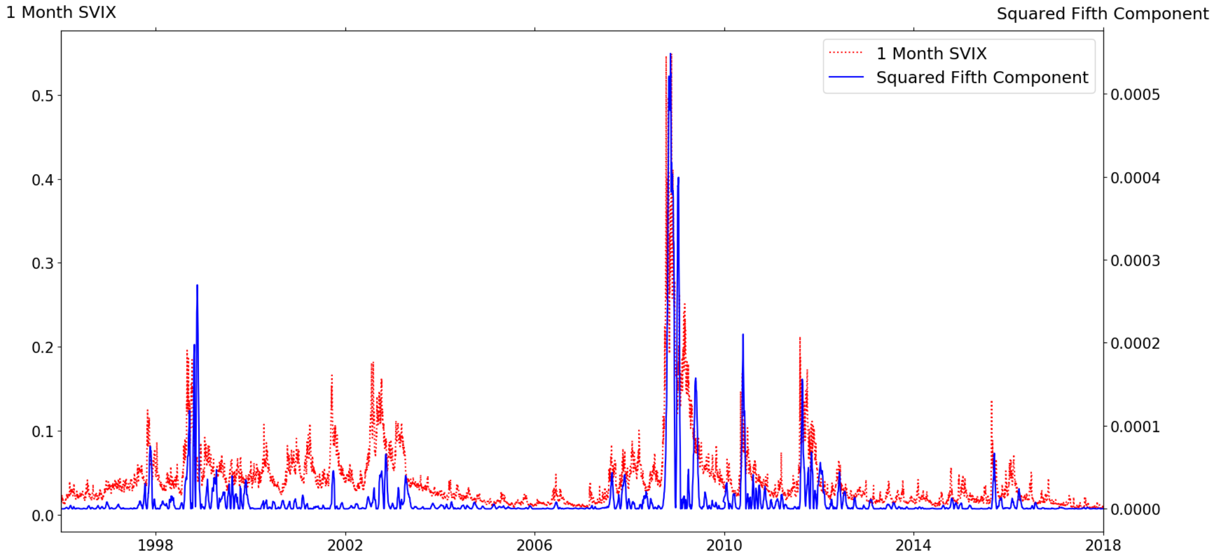

Figure 10. One-month SVIX and the squared fifth component of the decomposition.

The correlation of the two series increases to 0.664. Especially during periods of distress such as in the late 90s, 2008, and early 2010s, the squared fifth component is very responsive to the one-month SVIX index and spikes in tandem with it.

The sixth component, $g_t^{(6)}$, has a half-life between 32 and 64 days. Accordingly, I use it as a proxy for two-months SVIX. Figure 11 and Figure 12 plot the two-months SVIX against the sixth component and its squared alternative, respectively.

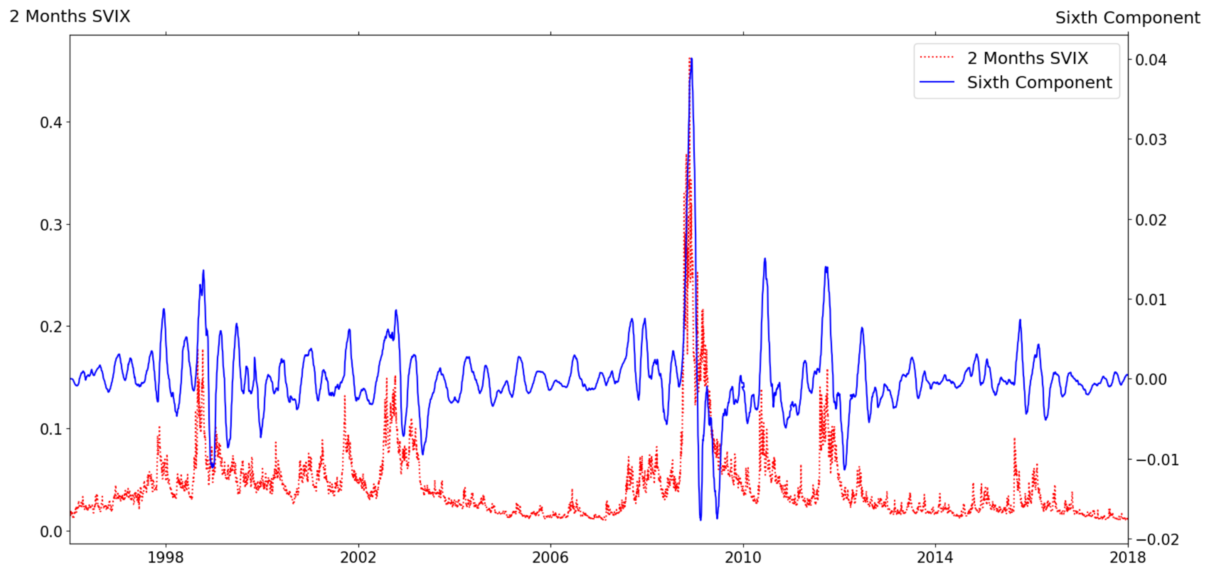

Figure 11. Two-months SVIX and the sixth component of the decomposition.

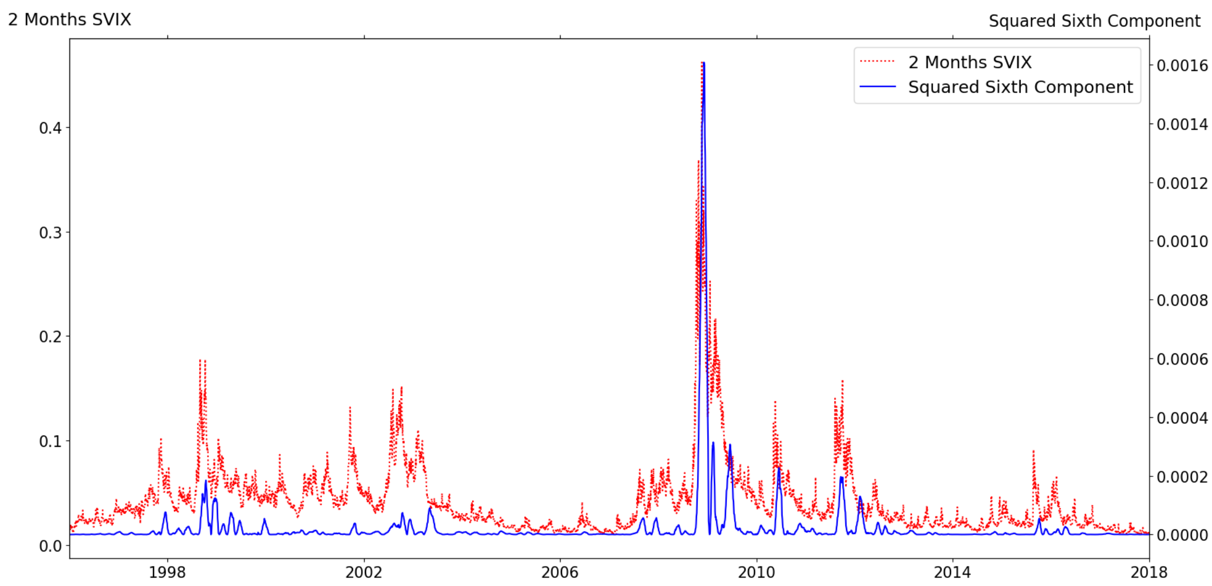

Figure 12. Two-months SVIX and the squared sixth component of the decomposition.

The relationship between the two series is like the one observed between one-month SVIX and the fifth component. Figure 11 represents a loose connection between two-months SVIX and the sixth component. There seem to be merely a few periods in time in which the correlation is strong. These occur especially during times of volatility. The correlation between the two time series is 0.457. Plotting the squared sixth component in Figure 12 instead, considerably increases the fit and correlation rises to 0.637.

Because of the convention of 63 trading days for three months, the sixth component also encompasses the time frame of three-months SVIX. Figure 13 and Figure 14 plot the three-months SVIX against the sixth component and the squared sixth component, respectively.

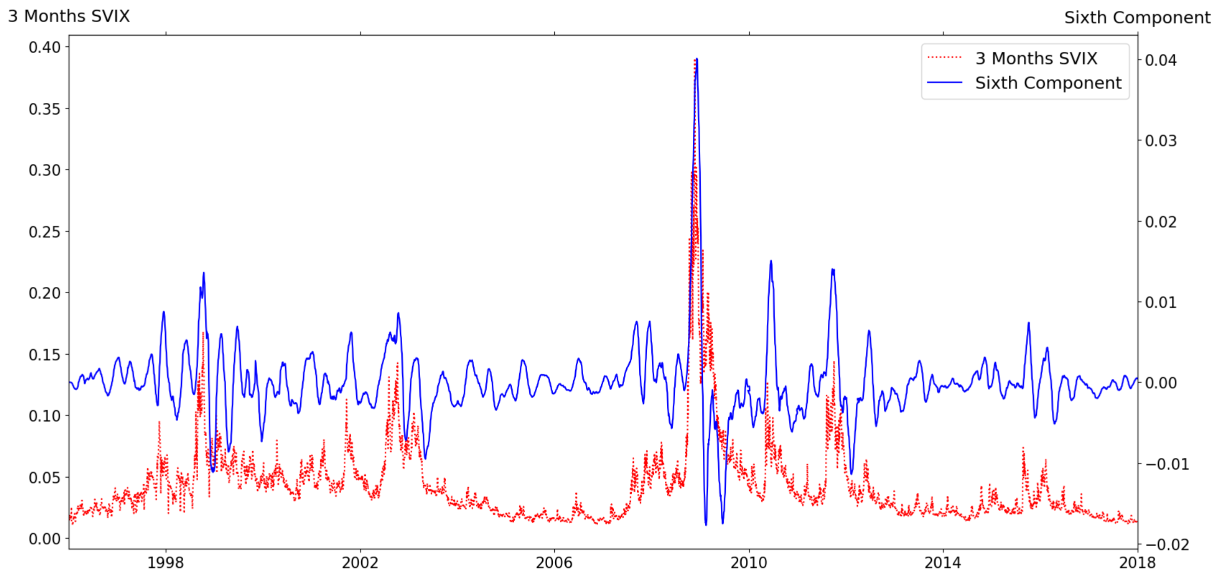

Figure 13. Three-months SVIX and the sixth component of the decomposition.

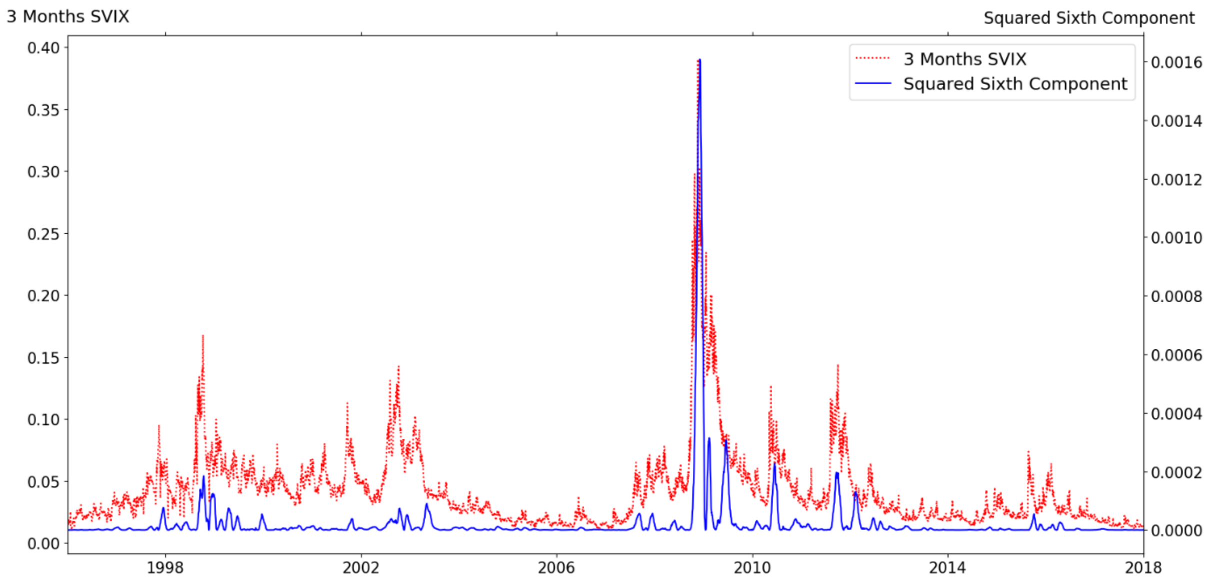

Figure 14. Three-months SVIX and the squared sixth component of the decomposition.

The previous impression is confirmed. Figure 13 represents less correlation. Most notably during the financial crisis of 2008, both three-months SVIX and the sixth component spike together. The correlation for these two series amounts to 0.437.

Using the squared component as a comparison tool instead, once more increases the correlation. This measure ascends to 0.645 when comparing three-months SVIX to the squared sixth component.

The seventh component, $g_t^{(7)}$, itself has a half-life between 64 and 128 days, thereby encompassing the six-months time frame. Figure 15 and Figure 16 depict the six-months SVIX against the seventh component and against the squared seventh component.

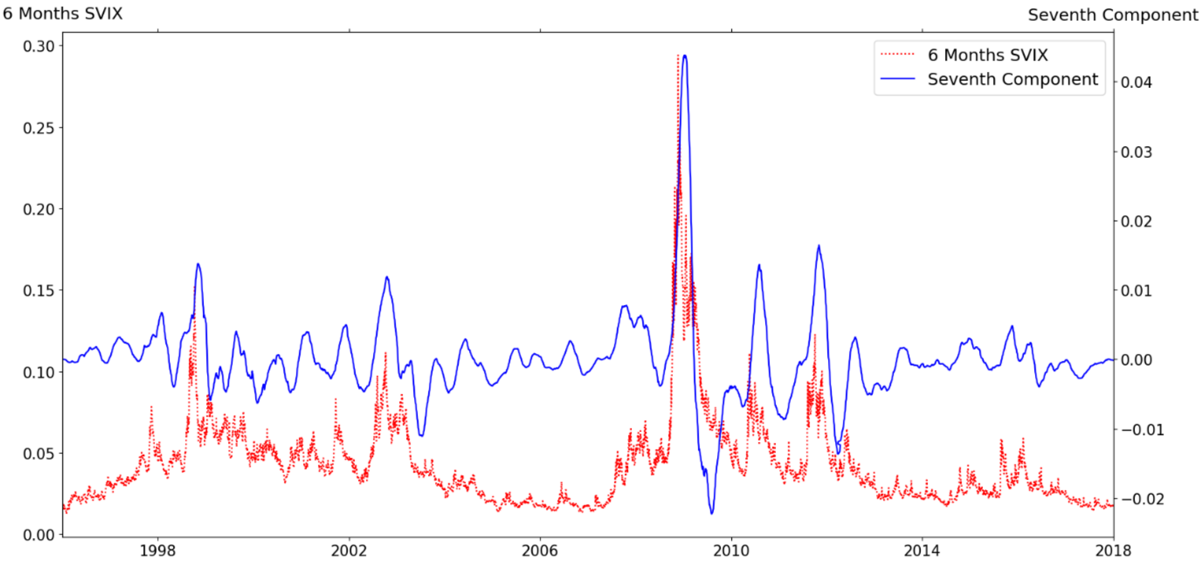

Figure 15. Six-months SVIX and the seventh component of the decomposition.

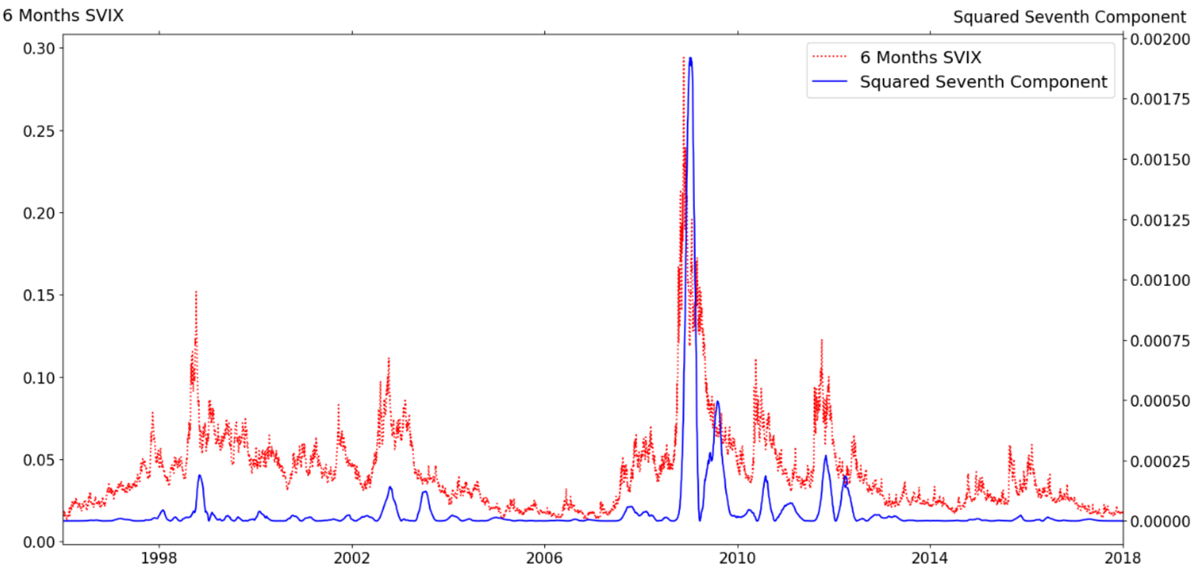

Figure 16. Six-months SVIX and the squared seventh component of the decomposition.

For this longer time frame, the correlation declines slightly. Six-months SVIX has a correlation of 0.453 with the seventh component. This correlation increases to 0.569 when the squared seventh component is chosen. As seen before, the most significant correlation seems to occur in times of market turbulence. Due to the longer time frame however, the squared component lags the time series slightly more than its more short-term alternatives as may be seen in Figure 16.

Finally, I compare the twelve-months SVIX against the eighth component of its own time series. This component has a half-life between 128 and 256 days. Therefore, its half-life includes the twelve-months time frame. Figure 17 and Figure 18 plot the twelve-months SVIX against this component and its squared alternative, respectively.

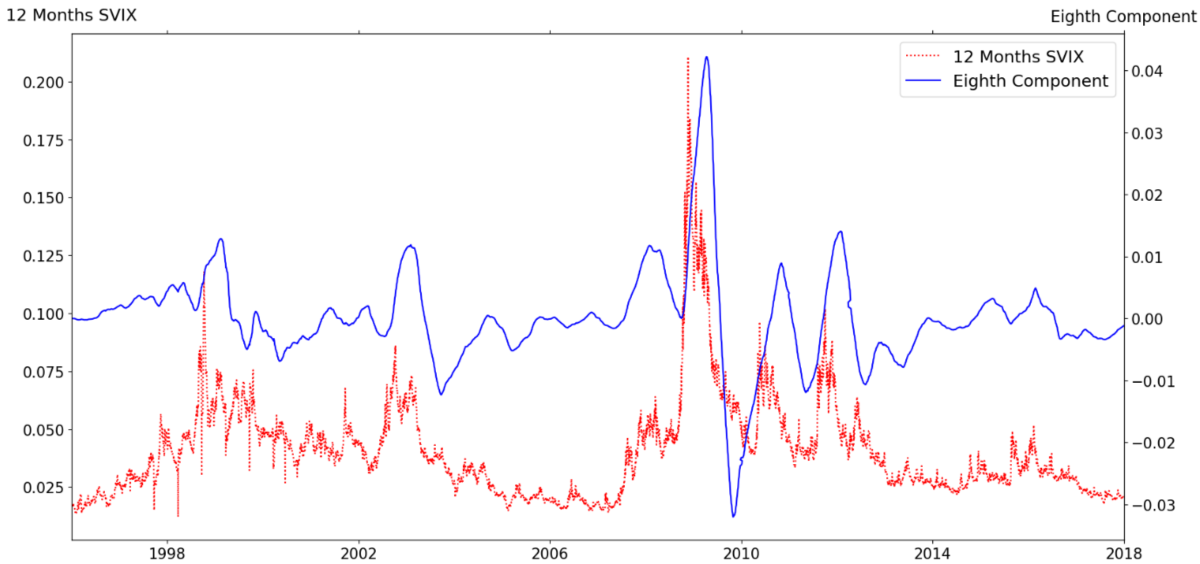

Figure 17. Twelve-months SVIX and the eighth component of the decomposition.

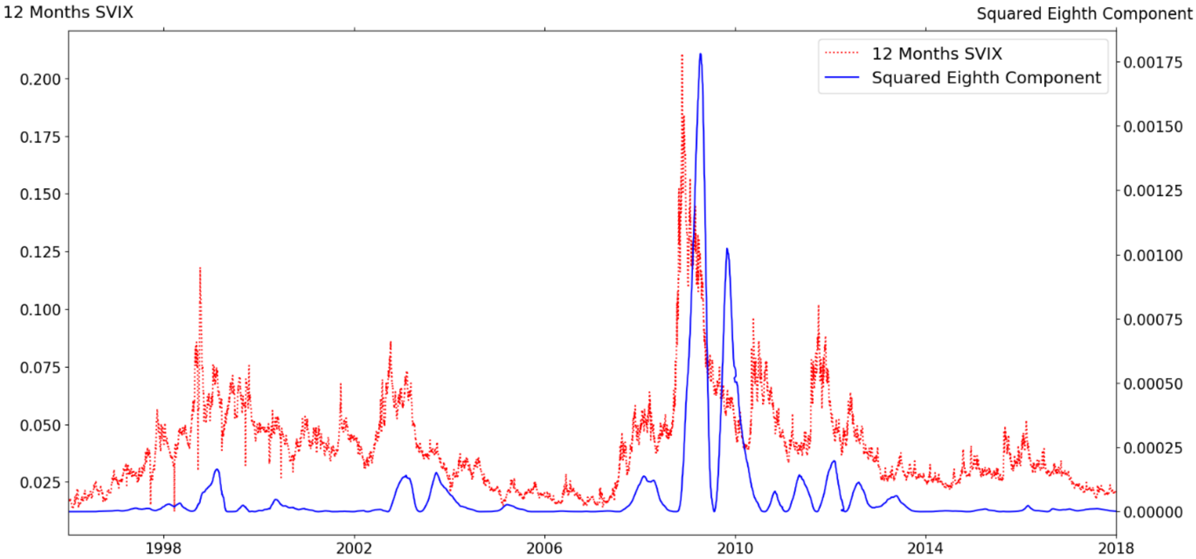

Figure 18. Twelve-months SVIX and the squared eighth component of the decomposition.

Even though, the long-run eighth component with a half-life between 128 and 256 days seems to fit the big waves of the twelve-months SVIX quite well in Figure 17, correlation once again is close to the pre-determined value of 0.5. More specifically, the correlation between the two series is 0.426. Comparing the index to the squared eighth component instead, once more increases the correlation to 0.525 which is close to 0.5-0.6 as observed beforehand.

In conclusion, it seems curious that one may decompose twelve-months SVIX into components with different levels of persistence which are at least partly correlated to the appropriate lower maturity SVIX indices. It may be interjected that the correlation may be the mere result of a strong correlation among the five SVIX indices. This may be true. However, it turns out that the correlation between the chosen persistence component and the SVIX index is the strongest and the most striking when the correct time frame is chosen. It is interesting to note that the correlation between the various SVIX indices and the chosen component proxy is in the range of $0.4-0.5$ for virtually all cases. Similarly, when using the squared component as a reference point instead, correlation usually increases to $0.5-0.6$. However, one may conclude that there is persistence in the twelve-months SVIX time series data. For both the mere component and especially for its squared alternative this persistence becomes especially apparent during market turmoil.

8. A Market-Timing Strategy

Given the regression results of regression (16), I recall the conclusion that $R_{f,t} \cdot SVIX^2_{t \to T}$ is the equity risk premium. Further, the previous two sections have demonstrated how volatile this equity risk premium is in the short run. A real time measure of the equity risk premium can be used in market-timing trading strategies, benefitting from the equity premium's volatility.

Martin (2017a) indeed proposes a contrarian market-timing strategy based on the SVIX index. This strategy consists of investing a fraction, α_t, of one's portfolio in the equity markets, exemplified by the S&P 500 in this case. The remaining fraction, $1-\alpha_t$, is invested in cash, exemplified by the risk-free one-month rate in this case. α_t is chosen proportionally to the one-month SVIX index. This strategy is contrarian by nature. When market turmoil increases and expected returns rise, the fraction invested in equity markets increases. When expected returns decrease, the portfolio is adjusted in a more conservative manner. Then, the proportion in equity markets is lowered and the cash proportion is increased. The constant of the proportionality factor may be chosen freely. Martin (2017a) selects it in a way to have a mean portfolio weight in equities of 35% while mean cash holdings are 65%. These weights closely resemble the well-known 60% bond, 40% equity portfolio. However, in the market-timing strategy the weights are dynamic. The median equity holdings of the portfolio are 27% with a median holding of 73% in cash.

For the original period equity markets experience a daily Sharpe Ratio of 1.35%. This contrasts with a daily Sharpe Ratio of the market-timing strategy of 1.97%, found both by Martin (2017a) and in my analysis. This constitutes an almost 46% increase in the daily Sharpe Ratio. Figure 19 plots the cumulative return of $1 invested on January 4, 1996 in either the S&P 500 (SPX), cash, or the market-timing strategy. As may be seen, the market-timing strategy above all generates stable returns until the start of the subprime mortgage crisis when it loses most of its capital. However, given its contrarian investment nature, the depths of this crisis is exactly the time when equity exposure is increased. Hence, by 2012 the market-timing strategy recoups most of its losses and handsomely outperforms broad equity markets.

The analysis does not end here, however. I continue to calculate total returns for the three investment strategies for the complete time period. From 2012 onwards, the market-timing strategy continues to perform well. It steadily follows a stable uptrend. The daily Sharpe Ratio declines to just under 1.93% though. At the same time, equity markets rally strongly, closing the gap to the market-timing strategy and finally surpassing it. The daily S&P 500 Sharpe Ratio increases to 2.21%.

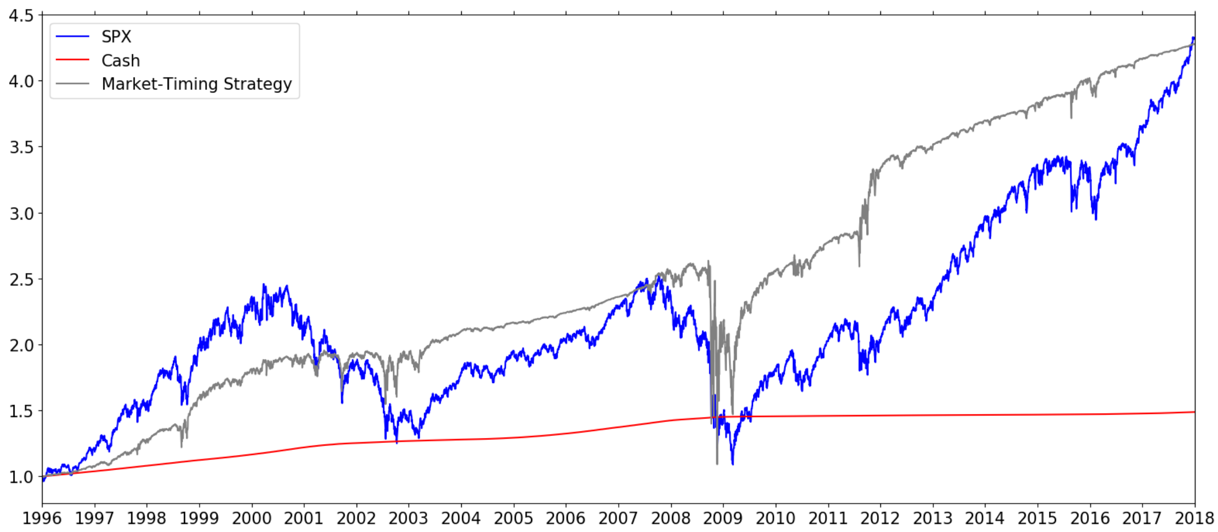

Figure 19. Cumulative returns of $1 invested on January 4,1996 for the S&P 500 (SPX), cash, and the market-timing strategy.

Does this indicate that the market-timing strategy has become obsolete and that passive equity exposure offers a better risk-adjusted return? It may be interesting to examine if market conditions have been changing for the past decade, rendering a SVIX-based contrarian market-timing strategy less effective. This is what section 9 investigates.

9. The Possibility of a Regime Change

In this section I explore possible evidence suggesting that market forces may have been changing within the last decade. More specifically, I want to examine if for this time frame the relationship between implied volatility, and therefore SVIX, and realized returns has shifted. First, however, it is of importance to mention that the original sample period between 1996 and 2012 is not entirely comparable to the ensuring period afterwards. This is because the original period encompasses at least one and most likely two entire business cycles with the bull-market of the late 90s, the burst of the dotcom bubble, and the posterior ensuring build-up and burst of the housing bubble. On the other hand, between 2012 and 2017 US equities experienced merely a strong bull-market. Indeed, according to the National Bureau of Economic Research (2018) since 2009 one of the longest expansions in history has taken place. Given the fact that the market-timing strategy is contrarian by nature, smaller returns must be expected during a low volatility bull-market. A similar low volatility development as today may be observed in Figure 19 during the recovery after the dotcom bubble and the following build-up of the housing boom. In this period between 2002 and 2007 the market-timing strategy underperformed and equity returns surpassed it. It is in times of distress and recoveries, when expected excess returns are high, that this contrarian approach outperforms.

Nevertheless, this bull-market seems to be exceptional by many standards. From the lows in early 2009, a simple investment strategy in the S&P 500 has resulted in an almost fourfold increase of the initial capital invested. This has happened in less than one decade. Surprisingly, the Sharpe Ratio of simply holding the S&P 500 in 2017 was an astounding 3.73 (Bloomberg, 2018). Realized volatility, implied volatility, SVIX, and consequently the lower bound of the equity risk premium have been depressed with cross-asset volatility at the lowest level in over three decades (Cole, 2015). In 2017 realized volatility, fixed-income volatility, and implied equity volatility measured by the VIX index reached new generational lows (Cole, 2017).

One crucial factor during this bull-market has been central bank intervention. Since the great financial crisis central banks have used more aggressive monetary tools to stimulate financial markets, asset prices, and economies. When the zero-lower bound for interest rates was reached, more creative approaches such as quantitative easing and even negative interest rates were undertaken. Between 2008 and 2015 alone, central banks globally cut rates more than 600 times while expanding balance sheets by over $15 trillion (Cole, 2015). In terms of quantitative easing, the Federal Reserve (FED) started relatively early with quantitative easing 1, which lasted from December 2008 to December 2010. Quantitative easing 2 and 3 were announced in November 2010 and September 2012, respectively. The European Central Bank (ECB), on the contrary, only started quantitative easing in 2015. The effect of quantitative easing programs may be observed on the balance sheets of the central banks. Since the depths of the subprime mortgage crisis the balance sheet of the Federal Reserve has increased from below $1 trillion in 2008 to above $ 4.5 trillion in 2015 (Federal Reserve, 2018). Similarly, the European Central Bank has grown its balance sheet from €1.5 trillion at the end of 2007 to almost €4.5 trillion one decade later (European Central

Bank, 2018). Figure 20 and Figure 21 plot the balance sheets of FED and ECB, respectively.

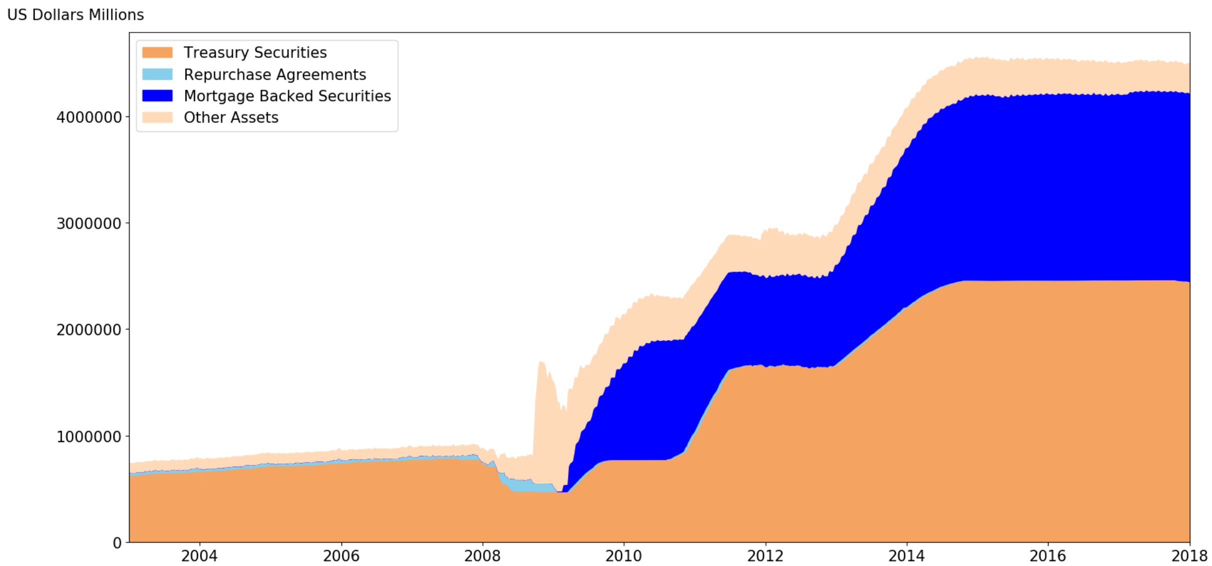

Figure 20. Federal Reserve balance sheet in millions of US Dollars. Source: Federal Reserve.[12]

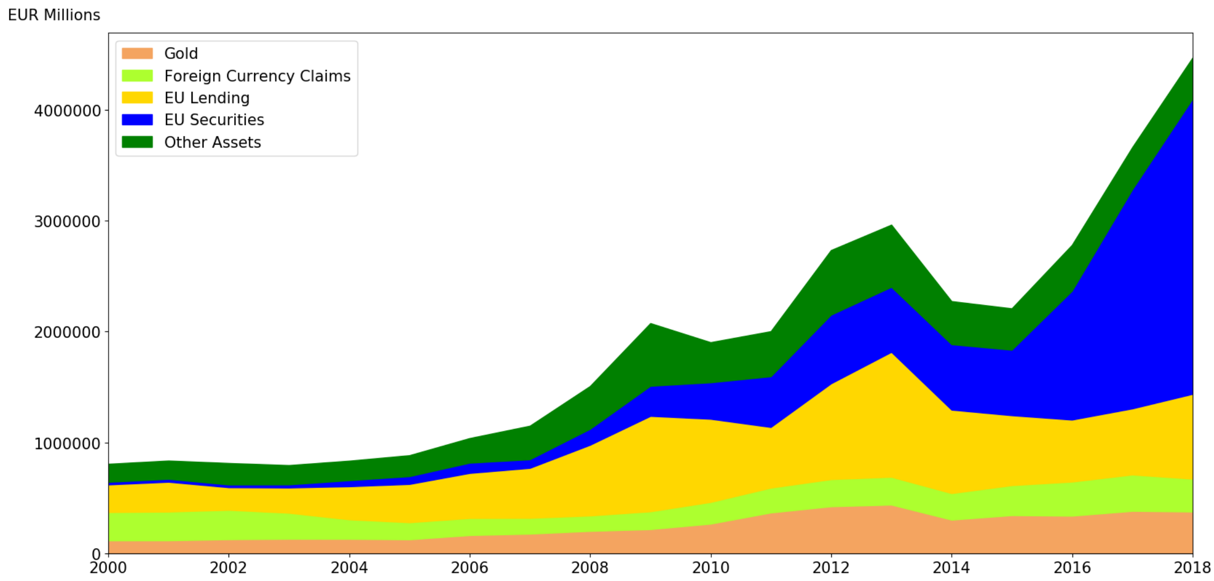

Figure 21. European Central Bank balance sheet in millions of Euros. Source: European Central Bank.[13]

[12] More information about the balance sheet data can be found at: https://www.federalreserve.gov.

[13] More information about the balance sheet data can be found at: https://www.ecb.europa.eu.

This balance sheet intervention has had various consequences for financial markets. As famous investor Stanley Druckenmiller has noted, it is not earnings but the central banks that ultimately move financial markets (Felder, 2018).

Firstly, central bank buying of government and even corporate bonds has considerably increased demand for these financial assets. As a consequence, bond yields have fallen across the curve.

Secondly, facing lower yields on fixed income, investors have been pushed out the risk curve (Cole, 2017). Muir (2018) notes that there has been a surprisingly strong outperformance of the equity markets on days when the Federal Reserve was actively buying during the so called *Permanent Open Market Operations*, the action of intervening in financial markets by purchasing or selling financial assets (Federal Reserve Bank of New York, 2018). The liquidity provided by central banks purchasing bonds and mortgage backed securities hence directly flows into other asset classes such as equities.

Thirdly, a low interest-rate, low-yield environment has suppressed volatility both implicitly and explicitly. In search for yield many individual and institutional investors have become sellers of volatility as an income generating strategy (Cole, 2017). This may be explicitly through short VIX exchange traded funds (ETFs) and exchange traded notes (ETNs) or option writing strategies and implicitly through volatility-targeting and mean-reverting investment strategies (Financial Times, 2018). For the last years entropy, another measure of variability which is expressed by the VIX index as noted earlier, has been decaying on an average annual rate of 4.5% (Balata & Filia, 2018). Given this explicit and implicit suppression of volatility, which SVIX is inherently related to, $R_{f,t} \cdot SVIX^2_{t \to T}$ may have failed to be a valid proxy for the equity risk premium in the most recent era. This is a crucial hypothesis that may be tested.

With this purpose in mind, I run regression (16) another time, splitting my dataset into two new periods. The regression is estimated for the period of January 4, 1996 until September 12, 2012. This period closely reflects the original period and ends just before the announcement of quantitative easing 3 by the Federal Reserve on September 13, 2012. I choose this date because of two reasons. Firstly, it dates closely to Mario Draghi's famous "whatever it takes" statement which he made on July 26, 2012. This may be interpreted as the European Central Bank's and thereby global central banks' commitment to intervene (McCrum, 2017). Secondly, in 2012 the economy was already well into its recovery from the great financial crisis. This makes it easier to distinguish subsequent returns from usually very strong cyclical recovery data. Regression (16) is also evaluated for the remaining data starting on September 13, 2012 and ending on December 31, 2017.

Table 10 mildly strengthens the assumption of a dislocation between $R_{f,t} \cdot SVIX^2_{t \to T}$ and the equity risk premium. Regression coefficients for this most recent period range from a minimum of above 2.3 for the six-months SVIX maturity to a high of above 7.3 for the one-month SVIX maturity. This means that low volatility has generally occurred together with relatively high returns. This is in stark contrast to the beta coefficients observed in Table 9, corresponding to the regression coefficients for the first period. Every beta coefficient in this table is lower than the corresponding result in Table 10. Only the six-months coefficient is similar in magnitude in both tables. However, it must not go unnoticed that the statistical significance for the coefficients in Table 10 is low. Due to large standard errors the confidence intervals of the coefficients are generally very wide. Only for the two-months maturity I reject the hypothesis that the beta equals one at the 5% significance level. For the other maturities, I fail to reject this hypothesis despite the magnitude of the coefficients.

Horizon	α	Std. error	β	Std. error	R^2 (%)	R^2_{os} (%)
1 Month	0.0017	0.055	0.6586	1.110	0.3	0.741
2 Months	-0.0145	0.055	0.9164	1.111	0.8	1.899
3 Months	-0.0180	0.059	0.9677	1.244	1	2.774
6 Months	-0.0661	0.057	2.0141	0.867	5.5	8.35
12 Months	-0.0432	0.067	1.693	0.985	4.5	8.99

Table 9. Results of regression (16) for January 4,1996 to September 12, 2012.

Horizon	α	Std. error	β	Std. error	R^2 (%)	R^2_{os} (%)
1 Month	-0.0240	0.065	7.3049	3.557	5.6	7.54
2 Months	-0.0212	0.054	6.5332	2.295	7.6	11.91
3 Months	0.0293	0.050	3.9621	1.819	4.2	12.13
6 Months	0.0540	0.042	2.3667	1.796	1.8	12.57
12 Months	0.0070	0.048	3.6007	1.642	5	23.45

Table 10. Results of regression (16) for September 13, 2012 to December 31, 2017.

For a visual understanding, Figure 22 displays the results in a scatterplot. I choose to plot the twelve-months data in this case because it relates to realized annual returns and is the least volatile. Data points of the more recent period are coloured more lightly in the chart. It is easy to note that these more recent data points relate to relatively low values of the lower bound proxy and therefore to low implied volatility. Realized returns, on the other hand, are relatively high. Further, regression lines based on the regressions of the complete period and the two sub-periods are plotted. This picture confirms that there is a much steeper relationship between the SVIX measure applied and subsequent equity returns for the most recent period. More specifically, data points for this period are characterized by generally low SVIX measure values and high realized subsequent returns.

Reiterating previous comments, one must be cautious to compare regression results based on this most recent bull-market data with the previous period encompassing various market cycles. Further, given very large standard errors of the estimated coefficients, in all but one case I fail to reject the hypothesis that the beta coefficient equals one. Nevertheless, this analysis theoretically introduces how returns and market volatility may have been influenced by central banks in recent years, resulting in low volatility markets with high returns, especially relative to SVIX. It is to be determined with more data if SVIX remains a valid proxy for the equity risk premium in this environment.

In the last section, I finally demonstrate how it is possible to derive market probabilities from equity index options.

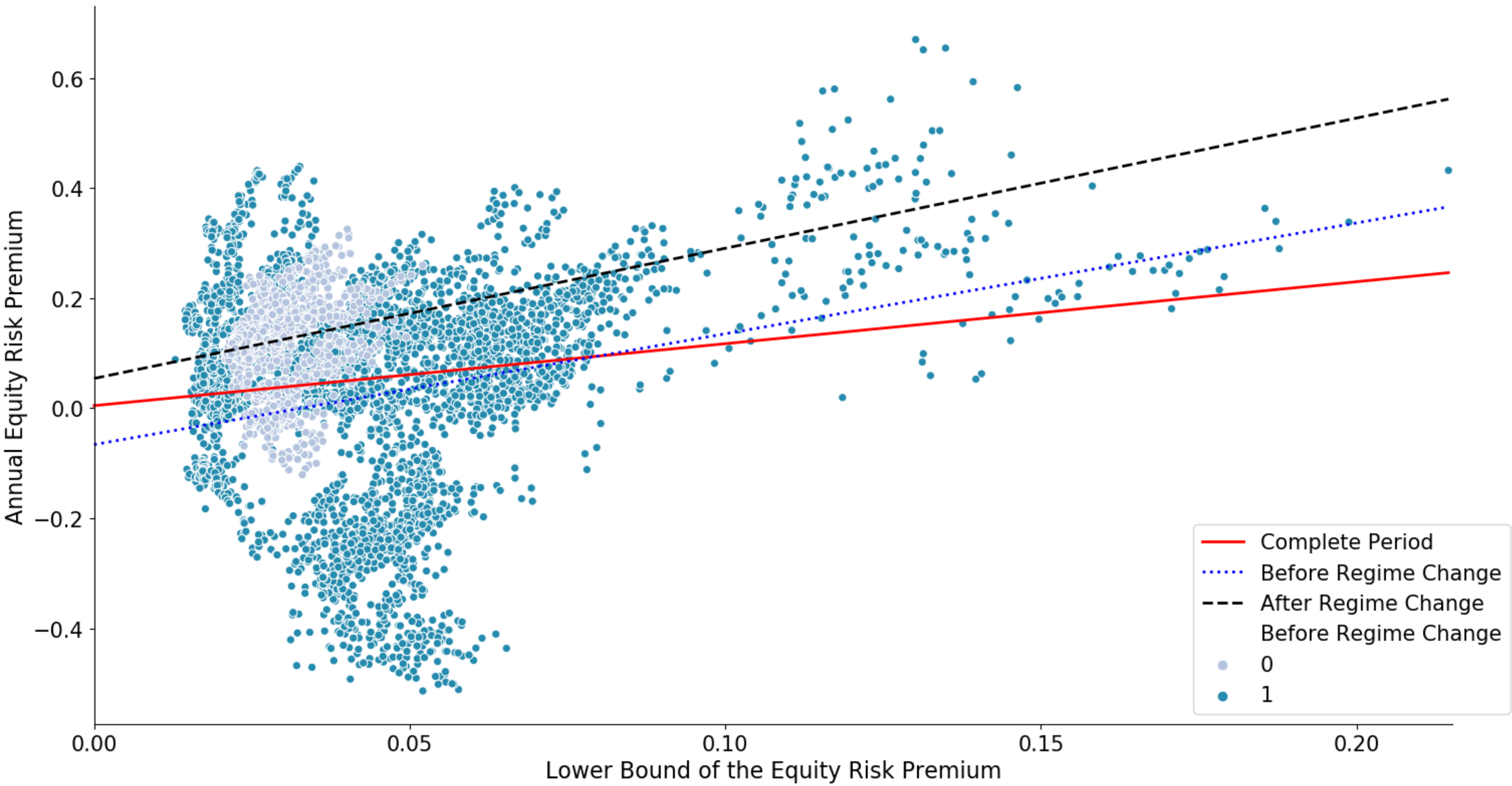

Figure 22. The lower bound of the equity risk premium at the twelve-months horizon and subsequent annual returns. The data points before the supposed regime change are indicated with a 1 in the legend.

10. Investors' Perceived Probability of a 20% Market Drop

Derivatives pricing is an interesting field of study insofar as it allows market participants to reconstruct market expectations from the quoted prices of derivatives. Besides the construction of the SVIX, even more may be inferred from index option prices. More specifically, from the viewpoint of a log-utility investor with full market investment it is possible to calculate the expectations of a given market drop from the option-price data at any given point in time. The following section outlines how to calculate this probability following the analysis of Martin (2017a).

Result 1. Let X_T be some random variable of interest whose value becomes known at time T, and suppose that we can price a claim $X_T R_T$ delivered at time T. Then we can compute the expected value of X_T by pricing an asset:

$\tilde{E}_t X_T$ = time t price of a claim to the time T payoff $X_T R_T$. (Martin, 2017a, p. 397)

where "~" expresses the subject's expectations in this section. Accordingly, one may calculate the probability of a given market drop. I assume that no dividends occur between time t and time T. Accordingly, one may simplify $R_T = S_T/S_t$. Then, the investor's perceived probability of a market return inferior to α may be calculated as follows:

(32) $$\tilde{P}(R_T < \alpha) = \alpha\left[\, put'_{t,T}(\alpha S_t) - \frac{put_{t,T}(\alpha S_t)}{\alpha S_t}\right]$$

This result may be derived from the simple fact that the perceived probability justifies $\tilde{P}(R_T < \alpha) = \tilde{E}(1_{\{RT < \alpha\}})$. Thus, the payoff $R_T 1_{\{RT < \alpha\}}$ is to be priced following Result 1.

$$R_T 1_{\{RT < \alpha\}} = \frac{S_T}{S_t} 1_{\{RT < \alpha\}}$$

$$= \alpha\left[1_{\{ST < \alpha St\}} - \frac{1}{\alpha St} \max(0, \alpha S_t - ST)\right]$$

The first term within the squared brackets is a digital option with strike price equal to αS_t. The second term is the payoff of a put with the same strike. The price of the digital option with strike αS_t is equal to $put'_{t,T}(\alpha S_t)$. This is the slope of the put price curve at the strike price αS_t. Inserting these results into the first term of the squared brackets yields equation (32). Equation (32) is very valuable because it decomposes the aforementioned probability into observable quantities. α is the given return expectation the probability is based on. $put'_{t,T}(\cdot)$ is defined as the "slope of the put option price curve when plotted as a function of strike" (Martin, 2017a, p. 398). Thus, the slope of the put option price curve at a strike of α times the current price of the equity index S_t is used in the calculation. This expression is especially interesting as it relates to the convexity of the put option price curve. Consequently, ceteris paribus the probability is high when convexity is strongly expressed in the curve. The final term simply represents the price of a put option with strike price equal to αS_t divided by the given strike.

Figure 23 depicts this probability of a market drop of at least 20% across various time frames. The construction of the probabilities may be observed in the appendix. Panel (A), (B), and (C) represent this probability over the ensuring one-month, six-months, and twelve-months maturities, respectively. Additionally, mean and maximum values of the probabilities are depicted. For the one-month probability, the mean is 0.75% with a maximum value of 7.6%. The average and maximum values increase to 5.77% and 15.34% for the six-months probability and finally to 8.78% and 19.1% for the twelve-months probability. Panel (D) plots these probabilities for the period around the subprime mortgage crises. The series are smoothed with a 20-day moving average. Interestingly, both six-months and twelve-months crash probabilities where rising during 2007 and 2008, even before the outburst of the financial crises. Contrary to this development prior to the housing burst, one-month, six-months, and twelve-months probabilities of a market drop of at least 20% were falling throughout 2017. Therefore, they failed to reflect the increased market volatility of 2018. As I have elaborated on earlier in the section on a possible regime change, the prices of options and thereby the validity of the market's perceived crash probability may have been affected in recent years.[14]

This section has shown how it is possible to infer market probabilities from option prices. Further, some interesting characteristics of these probabilities may be observed. It may be of considerable interest to calculate these probabilities with more timely data in future research.

[14] Unfortunately, at the time of writing no option data for the financial year 2018 that would allow a depiction of SVIX or the perceived market crash probabilities is available.

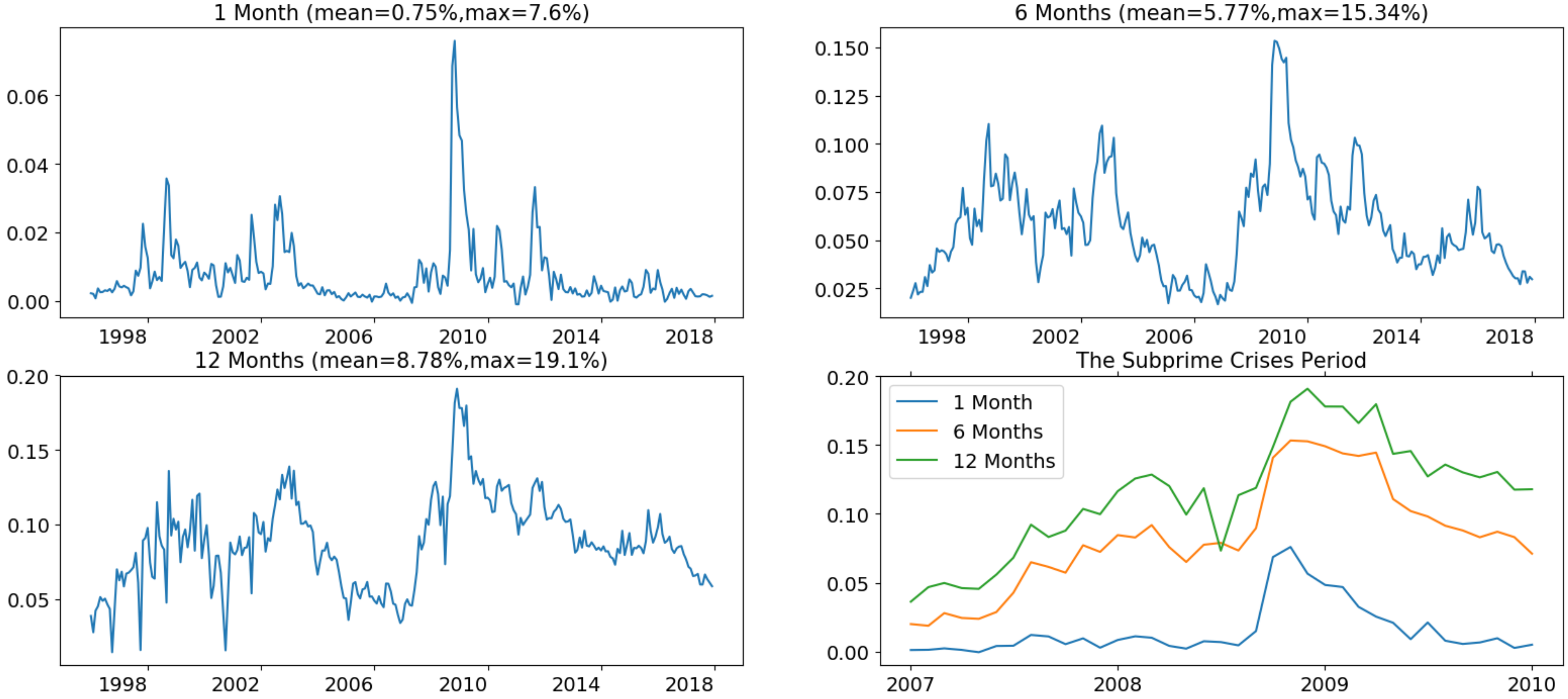

Figure 23. Inferred probability of a 20% market drop at different horizons.

11. Conclusion

This thesis bases its results on the contribution of “What is the Expected Return on the Market” (Martin, 2017a). Martin shows that the expected return on the market is closely related to risk-neutral variance and therefore index options. He constructs the so called SVIX index which is reminiscent of the famous VIX index provided by the CBOE. This paper verifies the results obtained by Martin (2017a) and aims to add to the existing research in two areas. Firstly, the existing analysis is replicated with more timely data. This allows to make a judgement about the validity of the results in the most recent period. Secondly, a connection between the SVIX time series and the analysis of time series decomposition is made.

Aiming at doing so, I start by making a connection between the equity risk premium and risk-neutral variance according to Martin (2017a), deriving the SVIX index. I show that theoretically $R_{f,t} \cdot SVIX^2_{t \to T}$ is the lower bound of the equity risk premium. I continue to describe the exact procedure of how to construct the SVIX index with the help of programming tools and compare the index to its close relative the VIX index. The time series of the lower bound of the equity risk premium at different maturities display interesting statistics. Firstly, the indices are right-skewed. Secondly, they are extremely volatile on an annualized basis, with volatility inversely related to the maturity of the index. This is in stark contrast to valuation-based estimation methods which yield more stable estimates. Thirdly, the mean of the series is close to values estimated to be long-run equity risk premia by the academic literature. This gives raise to the claim that the lower bound indeed may be the equity risk premium. Based on this observation, I run predictive regressions for the original and complete period and fail to reject the hypothesis that the SVIX measure is the equity risk premium. Rather, the out of sample R-squared based on Goyal and Welch (2008) indicates that the SVIX measure adds predictive power when estimating the expected excess return. This result may allow for the assumption that $R_{f,t} \cdot SVIX^2_{t \to T}$ is indeed the equity risk premium.

Academic research has repeatedly demonstrated a negative relationship between investor expectations and subsequent equity returns. To test whether the SVIX fulfils this condition, I compare it to three market survey measures. The correlation between these measures and the lower bound of the equity risk premium is mostly negative. Consequently, further evidence for SVIX being the equity risk premium is presented.

Across the five maturities, the SVIX index and therefore the expected excess equity return is very volatile. This is especially true at the short end of the curve. I examine this volatility by constructing a term structure of equity risk premia. This term structure shows how the annual equity risk premium may be decomposed into its lower time frame components. Further, the analysis demonstrates that the term structure is time dependent. While it is steep in sanguine times, it inverts in times of turbulence due to the volatility of the close maturities. Related to this decomposition of twelve-months expected equity returns, I decide to add a new approach to the analysis of Martin (2017a). Based on research by Ortu et. al (2013), I decompose the time series of twelve-months SVIX into various components. This allows to filter the time series for persistence with half-life close to the various SVIX maturities. The analysis establishes that there is persistence at each chosen time frame which correlates with the corresponding SVIX maturity.

Based on the SVIX measure being the equity risk premium, I verify a contrarian market-timing strategy proposed by Martin (2017a). This strategy invests a fraction of the portfolio which is proportional to one-month SVIX in equities and the remainder in cash. For the original period, I confirm impressive findings for this strategy with an almost 50% increase in the daily Sharpe Ratio compared to passive equity investments. More interestingly, however, this strategy underperforms equity markets in the last decade during which its Sharpe Ratio falls below that of passive equity exposure. Hence, I pose the question whether a low volatility regime change in the markets may have occurred within the last decade. I find that during this time frame volatility is low relative to returns and test the equality between $R_{f,t} \cdot SVIX^2_{t\rightarrow T}$ and the equity premium empirically. However, because of very large standard errors the analysis fails to deliver non-ambiguous results.

Finally, the derivatives pricing approach allows to extract more statistics than merely the SVIX index from the option index price data. More specifically, I calculate the inferred probability of a 20% market drop from the perspective of a log-utility investor for various maturities. The time series of probabilities is rather volatile. Most interestingly, both six-months and twelve-months probabilities trend upwards well in advance of the start of the subprime mortgage crisis but fail to predict the volatility experienced in 2018.

This thesis verifies the conclusions obtained by Martin (2017a). Further, more timely data is applied to the analysis, a new form of decomposition of the time series is introduced, and a discussion about a regime change is proposed. Future research may apply even more recent data to the analysis to test the robustness of the results and to conclude on the assumptions of a possible regime change.

12. Appendix

12.1 The interpolation of risk-free rates

For the interpolation of risk-free rates, I obtain continuously compounded zero-coupon interest rates from OptionMetrics. I use a Python program to interpolate the interest rates. For each date, every given interest rate quoted has a specific “Days Until Maturity” value. The first step is to adjust the quoted interest rate for the remaining days until maturity as a fraction of a year. Therefore:

$$1 + \mathrm{R}_{\mathrm{f,t\ adjusted}} = \mathrm{e}^{\mathrm{R}_{\mathrm{f,t}}\frac{\text{Days Until Maturity}}{365}}$$

Next, for each date in the dataset, interest rates are sorted by “Days Until Maturity” in an increasing order. Then, for each day a Python function that interpolates interest rates is run for 30, 60, 91, 182, and 365-day time frames. This function returns interpolations for respective risk-free rates with a time frame of one, two, three, six, and twelve months. Further, the function detects the two interest rates with days to maturities adjacent to the one chosen for the interpolation. Given a far-maturity risk-free rate, a close-maturity risk-free rate, their respective days until maturity, and the chosen days to maturity of the interest rate to interpolate, the interest rate is then interpolated in a way similar to the calculation of SVIX.

$$\mathrm{R}_{\mathrm{f,t}} = \mathrm{R}_{\mathrm{f,T1}}\frac{(\mathrm{T2}-\mathrm{t})}{(\mathrm{T2}-\mathrm{T1})} + \mathrm{R}_{\mathrm{f,T2}}\frac{(\mathrm{t}-\mathrm{T1})}{(\mathrm{T2}-\mathrm{T1})}$$

where t, T1, and T2 are the chosen time frame, the close days until maturity, and the far days until maturity, respectively.

This interpolation is done for every of the five chosen time frames on every date of the data sample.

12.2 The construction of the term structure of equity risk premia

I use a Python program to calculate the term structure of the equity risk premia. The data of interest is the time series of SVIX for the varying maturities generated earlier in the analysis. The data, which is sorted by date, and the appropriate fractions of time are fed into the function. This function calculates spot equity premium according to equation (22) and the forward equity premium according to equation (21). With the chosen time-fractions annualized spot and forward equity premia are then converted to measures that are not annualized. Finally, the equity premia may be stacked on top of one another according to equation (23). This data series is finally plotted in Figure 8.

12.3 The construction of the crash probabilities

I use put option data from OptionMetrics together with the previously introduced data of the S&P 500 index for the calculation of the crash probabilities. The central equation for the crash probabilities is equation (32). Thus, the main concern is to find both the slope of the put option price curve and the price of a put option at the specific strike αS_t. Since I calculate the probability of a market drop of at least 20%, α equals 0.8 to represent this discount. However, since option curves are not continuously available for every strike, some sort of interpolation is necessary for both the slope of the put option price curve and the option price itself.

I use a further Python program to calculate the crash probability. For every day, the data is sorted by the strike price of the options. Then, the option data along with the corresponding level of the S&P 500 S_t, the chosen time frame of the probability, and the α-value is fed into a function to calculate the daily probability.

I scan through the options data and delete any entry with less than 7 data points of the option curve. I do so as I experience very volatile and nonsensical variations in slopes and put prices with less data points.

As a first step, the Python function determines if the lowest strike price in the data is larger than αS_t. A lower strike is generally required for the slope of the curve. If indeed there is no strike inferior to this level, I make a simplifying assumption. I add one additional data point of a put option with zero strike and zero price. I find that my results generally are robust when making this adjustment. Further, the data point makes intuitive sense and is backed up by the put-call parity. After this adjustment, the program chooses the first three put options, their mid prices, and their strike prices for the calculation of the slope. If on the other hand the lowest strike available in the data is lower than αS_t, the function continues by searching for the closest adjacent strike prices. The corresponding options, mid prices and strike prices of the four options closest to this αS_t level are then chosen.

As a next step, to define a continuous set of options, I first generate a grid of 10,000 strike prices between the minimum and maximum strike prices of the option data. To translate these data points into put option prices I first fit the available option data. I do so by fitting the strike prices of the data points to the respective option prices with a polynomial regression of degree 30. The resulting coefficients allow to translate the grid of 10,000 strike prices into an almost continuous grid of corresponding 10,000 put prices.

With the help of this step, the required data can be read from the options curve. The price of the put option with strike αS_t is readily available on the curve that has just been fit. The slope of the option price curve is simply calculated according to (y2-y1)/(x2-x1), where y and x are put prices and strike prices of the put options in the newly generated grid that are closest to the option with strike price αS_t. Finally, I calculate the probability of a market drop of at least 20% by plugging in the generated values into equation (32).

The series of one-month, six-month and twelve-month probabilities of a market drop of at least 20% share a correlation with the probabilities provided by Martin of 98.97%, 96.51%, and 95.98%, respectively.

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