EE571 Fall 2025 HW 1

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Problem 1: A system's transfer function is $(s+1)/(s^2+0.5)$. The input is $r(t)$ and the output is $c(t)$. Give the differential equation of this system? All the donations/variables must be correct as given by the problem.

$$\begin{aligned}
G(s) &= \frac{s+1}{s^2+0.5} \\
G(s) &= \frac{C(s)}{R(s)} \\
&= \frac{b_{m-1}s + b_m}{a_{n-2}s^2 + a_{n-1}s + a_n} \\
&= \frac{s+1}{s^2+0.5} \\
\ddot{y} + 0.5y &= \dot{x} + x
\end{aligned}$$

Problem 2: Give the transfer function for the differential equation $3\frac{dy(t)}{dt} + 5y(t) = 0.5\frac{du(t)}{dt} + 0.3u(t)$ with y as the output. Is the transfer function of system $3\frac{dx(t)}{dt} + 5x(t) = 0.5\frac{du(t)}{dt} + 0.3u(t)$ with x as the output the same as that for $3\frac{dy(t)}{dt} + 5y(t) = 0.5\frac{du(t)}{dt} + 0.3u(t)$?

$$\begin{aligned}
3\frac{dy(t)}{dt} + 5y(t) &= 0.5\frac{du(t)}{dt} + 0.4u(t) \\
3\dot{(y)} + 5y &= 0.5\dot{u} + 0.3u \\
G(s) &= \frac{0.5s + 0.3}{3s+5}
\end{aligned}$$

for

$$\begin{aligned}
3\frac{dx(t)}{dt} + 5x(t) &= 0.5\frac{du(t)}{dt} + 0.4u(t) \\
3\dot{(x)} + 5x &= 0.5\dot{u} + 0.3u \\
G(s) &= \frac{0.5s + 0.3}{3s+5}
\end{aligned}$$

The transfer functions of the two system are equal.

Problem 3: A system's impulse response $g(t) = 0.5^t$ $(t \geq 0)$ $(g(t) = 0$ $(t < 0))$. The input is $x(t) = 1$ $(t \geq 0)$ $(x(t) = 0$ $(t < 0))$. Give the output $c(t)$ generated by this input in the form of convolutional integral – just the equation based on the definition, without the need to actually find the integral; solution using Laplace transform is not acceptable.

$$
\begin{aligned}
g(t) &= \begin{cases} 0.5^t & t \geq 0 \\ 0 & t < 0 \end{cases} \\
x(t) &= \begin{cases} 1 & t \geq 0 \\ 0, t < 0 \end{cases} \\
c(t) &= g(t) \cdot x(t) = \int_0^t x(\tau) g(t-\tau) d\tau \\
x(t) &= \delta(t) \\
c(t) &= \int_0^t \delta(\tau) g(t-\tau) d\tau \\
c(t) &= \int_0^t (1)(0.5^{t-\tau}) d\tau \\
c(t) &= \int_0^t 0.5^{t-\tau} d\tau
\end{aligned}
$$

Problem 4: (1) Give the equation for the Laplace transform of $g(t) = 0.5^t$ $(t \geq 0)$ $(g(t) = 0$ $(t < 0))$ per the definition of Laplace transform; just the definition and no simplification and further calculation are required. (2) This Laplace transform is the differential equation of the above system, right? (3) If not, what is it?

(1)

$$
\begin{aligned}
g(t) &= \begin{cases} 0.5^t & t \geq 0 \\ 0 & t < 0 \end{cases} \\
\mathcal{L}(g(t)) = G(s) &= \int_0^{\inf} g(t) e^{-st} dt \\
&= \int_0^{\inf} 0.5^t e^{-st} dt
\end{aligned}
$$

(2)

No, it is not the differential equationi of the above system.

(3)

It is the transfer function.

Problem 5: A closed-loop control system has its feedforward transfer function $G(s) = \frac{1}{s+0.5}$ and its open-loop transfer function $G(s)H(s) = 0.5s + 0.5$. Draw the diagram of the system with each block filled with actual transfer function rather than just notation.

$$
\begin{aligned}
G(s) &= \frac{1}{s+0.5} \\
G(s)H(s) &= \frac{0.5}{s+0.5} \\
\frac{1}{s+5} H(s) &= \frac{0.5}{s+0.5} \\
H(s) &= 0.5
\end{aligned}
$$

Add the Diagrams

Problem 6: Calculate the closed-loop system transfer function for the above system by hand.

$$
\begin{aligned}
C(s) &= G(s)E(s) \\
E(s) &= R(s) - B(s) \\
&= R(s) - H(s)C(s) \\
C(s) &= G(s)[R(s) - H(s)C(s)] \\
C(s) &= G(s)R(s) - G(s)H(s)C(s) \\
C(s)(1 + G(s)H(s)) &= G(s)R(s) \\
\frac{C(s)}{R(s)} &= \frac{G(s)}{1 + G(s)H(s)} \\
&= \frac{\frac{1}{s+0.5}}{1 + \frac{0.5}{s+0.5}} \\
&= \frac{1}{s + 0.5 + 0.5} \\
\frac{C(s)}{R(s)} &= \frac{1}{s+1}
\end{aligned}
$$

Problem 7: Calculate the closed-loop transfer function using Matlab.

```
%% Problem 7 - Feedback System Analysis
cleanup_workspace();
% Define forward transfer function G(s) = 1/(s+0.5)
G_num = [1];
G_dem = [1 0.5];

% Define feedback transfer function H(s) = 0.5
H_num = [0.5];
H_dem = [1];

% Calculate closed-loop transfer function
[num, dem] = feedback(G_num,G_dem, H_num, G_dem);

% Display result
printsys(num, dem)
```

```
num/den =

       s + 0.5
   ---------------
   s^2 +  s + 0.75
```

Problem 8: $G_1(s) = \frac{1}{s+1}$ and $G_2(s) = \frac{1}{s+3}$. Calculate the transfer function for the case they are in series and the case they are parallel using Matlab.

```
%% Problem 8 - Series and Parallel Combinations
cleanup_workspace();

% Define first transfer function G1(s) = 1/(s+1)
G1_num = [1];
G1_dem = [1 1];
```

```
% Define second transfer function G2(s) = 1/(s+3)
G2_num = [1];
G2_dem = [1 3];

% Calculate series combination G1*G2
[Series_num, Series_dem] = series(G1_num, G1_dem, G2_num, G2_dem);

% Calculate parallel combination G1+G2
[Parallel_num, Parallel_dem] = parallel(G1_num, G1_dem, G2_num, G2_dem);

% Display results
printsys(Series_num, Series_dem)
printsys(Parallel_num, Parallel_dem)
```

```
num/den =

        1
   -------------
   s^2 + 4 s + 3

num/den =

     2 s + 4
   -------------
   s^2 + 4 s + 3
```

Problem 9: In a PID controller, the controller output $u(t)$ is a function of the error signal $e(t)$. What is this function? (Give its equation)

check the I equation

$$u(t) = k + pe(t) + \frac{k_p}{T_i}\int_0^t e(t)dt + k_p T_d \frac{de(t)}{dt}$$

Problem 10: Derive the transfer function for the PID controller.

$$u(t) = k + pe(t) + \frac{k_p}{T_i}\int_0^t e(t)dt + k_p T_d \frac{de(t)}{dt}$$

$$U(s) = k_p E(s) + \frac{k_p}{T_i}\frac{E(s)}{s} + k_p T_d s E(s)$$

$$\frac{U(s)}{E(s)} = k_p[1 + \frac{1}{T_i s}|T_d s]$$

Appendix A

MATLAB Code for HW1

```
% EE571 HW 1
% Hersch Nathan
% Last Modified Augest 31, 2025

%%
cleanup_workspace();

%% Problem 7 - Feedback System Analysis
cleanup_workspace();
% Define forward transfer function G(s) = 1/(s+0.5)
G_num = [1];
G_dem = [1 0.5];

% Define feedback transfer function H(s) = 0.5
H_num = [0.5];
H_dem = [1];

% Calculate closed-loop transfer function
[num, dem] = feedback(G_num,G_dem, H_num, G_dem);

% Display result
printsys(num, dem)

%% Problem 8 - Series and Parallel Combinations
cleanup_workspace();

% Define first transfer function G1(s) = 1/(s+1)
G1_num = [1];
G1_dem = [1 1];

% Define second transfer function G2(s) = 1/(s+3)
G2_num = [1];
G2_dem = [1 3];

% Calculate series combination G1*G2
[Series_num, Series_dem] = series(G1_num, G1_dem, G2_num, G2_dem);

% Calculate parallel combination G1+G2
[Parallel_num, Parallel_dem] = parallel(G1_num, G1_dem, G2_num, G2_dem);

% Display results
printsys(Series_num, Series_dem)
printsys(Parallel_num, Parallel_dem)
```

Appendix B

MATLAB Helper Functions

```matlab
function cleanup_workspace()
% CLEANUP_WORKSPACE Clears command window, workspace variables, and closes all figures
%
% This function performs the standard MATLAB cleanup operations:
% - clc: Clear command window
% - clear: Clear all variables from workspace
% - close all: Close all figure windows
%
% Usage:
%   cleanup_workspace()
%
% Author: Hersch Nathan
% Date: August 31, 2025

    clc;        % Clear command window
    clear;      % Clear all variables from workspace
    close all;  % Close all figure windows

end
```