Certifying Lemons

- *Hersh Chopra*
(Northwestern University)

Introduction

- Adverse selection environment where a sender with private information (high or low ability) tries to convince a receiver of having higher ability.

- Without commitment or costly signaling, market failure can occur. Certification intermediaries reduce these frictions by enabling signaling through hard information.

- How does a monopolist certifier effect the market efficiency?
  - Reduced price competition enables costly signaling by the sender.
  - Monopoly power incentivizes provision of less information.

Timing

1. The monopolist certifier posts a menu of experiments and prices observable to the sender and receiver.

2. The sender privately observes his type. (h or l)

3. The sender privately purchases an option from the menu offered, or decides to not get certified.

4. The sender and receiver observe the realization of the purchased experiment. If no experiment was purchased the receiver observes that the sender is not certified.

5. The receiver chooses an action in $A = \{a_h, a_l\}$

Model

- Binary state $\theta \in \{h, l\}$, known to sender
- Receiver chooses action a_h if his belief about the state being high is greater than π^*, and a_l otherwise
- Certifier and receiver have common prior probability $\mu \in (0, \pi^*)$
- The senderhas state independent utility $1 = u(a_h) \geq u(a_l) = 0$
- Let $\sigma : \{l, h\} \to \Delta([0, \infty])$, be a statistical experiment.
- The certifier posts a menu of prices and experiments $m = \{(\rho_\theta, \sigma_\theta)_{\theta \in \{h,l\}}, (\Phi, 0)\}$, set of all menus is $\mathcal{M}$

Equilibrium

PBE : 1) Certifier rationality, 2) Sender rationality,
3) Bayes rule where possible and 4) Receiver rationality.

A typical equilibrium strategy profile is $((E_m)_{m \in \mathcal{M}}, \gamma^*, m^*)$ which represents the receiver's, sender's and certifier's strategy respectively.

Equilibria are **outcome equivalent** if they have same payoffs for all players **and** have the same distribution over messages conditional on acceptance.

Revelation principle: Truthful but not necessarily direct mechanism of the certifier are enough to induce all equilibrium outcomes.

Refinment

The game admits many equilibria, pessimistic off-path beliefs can support almost any menu in equilibrium.

Consistent Evaluation: Receiver does not change his acceptance set unless there is a compelling reason to do so.

$$\mathcal{E}_r := \{((E_m)_{m\in\mathcal{M}}, m^*) \in \mathcal{E} \mid \forall\, m \in \mathcal{M},\ (E_{m^*}, m) \in \mathcal{E} \implies E_m = E_{m^*}\}$$

- After observing the monopolist's deviation to a menu $m \in \mathcal{M}_{E_{m^*}}$ the receiver doesn't change his strategy believing the sender is using strategy $\gamma^*(m, E_{m^*})$.
- The receiver anticipates this and optimally uses $\gamma^*(m, E_{m^*})$.
- Receiver has "inertia" in deviating from equilibrium action.

Revenue Maximization

Let $\mathcal{M}_E^r$ be the set of all menus that maximize the seller's revenue among all menus in $\mathcal{M}_E$ given the receiver's strategy E

$\mathcal{M}_E$ is the set of all obedient menus, where receiver accepts after seeing a message in E

The refinement selects for equilibrium with on-path menu in $\mathcal{M}_E^r$ where E is the set of messages that leads to acceptance on-path.

Separating Equilibrium (Receiver First Best)

An equilibrium is **separating** whenever the high type sender is accepted with probability 1 and the low type sender is never accepted. Let $e^* := \inf(E_{m^*})$

Theorem 1: If an equilibrium in $\mathcal{E}_r$ is separating then $e^* \geq 1/\mu$ Moreover, if for an equilibrium in $\mathcal{E}_r$ it holds that $e^* > 1/\mu$, then the equilibrium is separating.

- The certifier excludes the low type for the revenue-maximizing motive.
- First best under **imperfect certification**.
- Higher informativeness then leads to the exclusion of the low type as including the low type would require leaving large rents to the high type.

Naive Strategy

A **naïve receiver** is described by the strategy $E_m = [1/l(\mu), \infty]$ for all m. The set of all equilibrium in which the receiver is naïve is given by $\mathcal{E}_n$.

Behaviorally a naïve receiver updates his beliefs at face value, disregarding selection effect from offering different menus.

Proposition1: The set $\mathcal{E}_n \subset \mathcal{E}_r$. Moreover, all equilibria in $\mathcal{E}_n$ are separating (if and) only if $\mu(<) \leq 2 - 1/\pi^*$.

By adopting a naïve approach to evaluating evidence, the receiver can guarantee himself first best outcome when he is sufficiently pessimistic about the sender's prior ability.

Sender Optimal

Theorem 2 : An equilibrium in $\mathcal{E}_r$ achieves the highest ex-ante expected payoff among all equilibria in $\mathcal{E}_r$ only if $\min(E_{m^*}) = 1/\mu$

- For large (small) enough prior the experiment offered in the sender optimal equilibrium is more (less) informative than the experiment in the Baysian persuasion.

- When the receiver is sufficiently confident about the sender's ability, the optimal sender equilibrium leads to an inefficient trade surplus.

- The inefficiency in trade surplus is due to the increased informativeness of equilibrium experiment; the greater informativeness helps high type sender to earn a greater share of the surplus but decreases the total surplus.

Structure of $\mathcal{M}_E^r$ (Proposition 2)

Fixing some $E \subset (0, \infty]$

$m^* \in \mathcal{M}_E^r$ if and only if m^* is such that

$$\rho_h^{m^*} = \int_E \left[d\sigma_h^{m^*}(e|h) - \left(1 - \frac{1}{e}\right) d\sigma_l(e|h) \right] \text{ and } \rho_l^{m^*} = \int_E d\sigma_l^{m^*}(e|l)$$

and solves the following optimization problem

$$\max_{\sigma_h, \sigma_l} \; \mu \int_E d\sigma_h^m(e|h) + \int_E \left(\frac{1}{e} - \mu\right) d\sigma_l^m(e|h)$$

$$\text{subject to}$$

$$\int_E \left(1 - \frac{1}{e}\right) d\sigma_h^m(e|h) \geq \int_E \left(1 - \frac{1}{e}\right) d\sigma_l^m(e|h) \geq 0$$

$$d\sigma_h^m(e|h) \geq \frac{1}{el(\mu)} d\sigma_l^m(e|h) \text{ for all } e \in E$$

- **No rent to low type**
- **High type values either menu option more than the low type**

Equilibrium Characterization (Theorem 3)

$$\mathcal{T}(E) := \{m \in \mathcal{M}^r_E |\; |supp(\sigma^m_h(.|h)) \cap E| \leq 3, \;\; |supp(\sigma^m_l(.|h)) \cap E| \leq 2\}$$

Theorem 3: For any equilibrium in $\mathcal{E}_r$ such that E_{m^*} is countable, then there exists an outcome equivalent equilibrium such that the on-path menu $m' \in \mathbf{cvx}(\mathcal{T}(E_{m^*}))$

- The equilibrium outcomes that survive the refinement are generated by a simple class of menus.
- Characterizes the implementable rent distribution.
- Proof follows from simplifying the point wise constraint for the optimization problem in proposition 2 and then using a geometric arguments to get the optima.

(Un) informative Equilibria

Certifier Optimal

- Certifier has incentive to provide as little information as possible.
- Certifier offers experiments that are state independent garbling of the sender's reported type. **(Soft information implementation)**

Optimistic Receiver

When $\mu \geq \pi^*$ there are only two equilibrium outcomes:

1. No type gets certified, receiver accepts under no certification.

2. Both types get certified and are accepted; certified extracts all surplus.

Literature

Akerlof (1970),

Viscusi (1978) ,

Lizzeri (1999),

Faure-Grimaud et al. (2009),

Ali et al. (2022),

Corrao (2023),

Weksler and Zik (2023)

END