Machine Learning in Business
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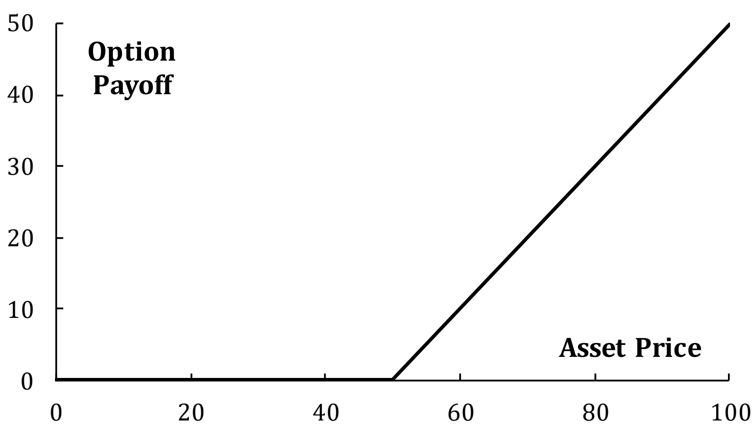
Chapter 10

Applications in Finance

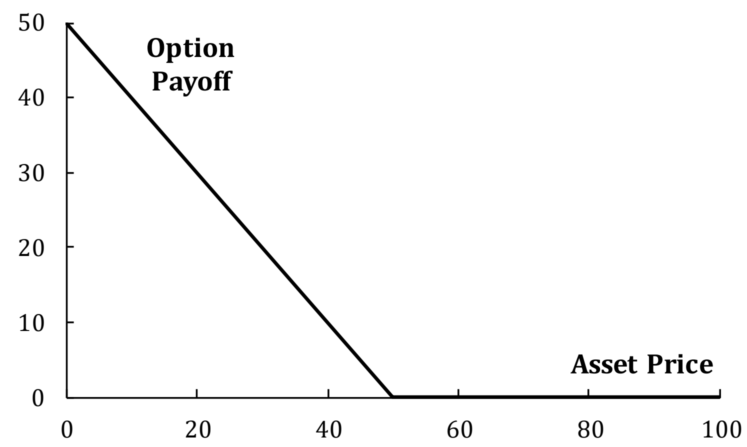
Call and Put Options

- Call option is an option to buy an asset on a certain future date (the maturity) for a certain price (the strike price)
- Put option is an option to sell an asset on a certain future date (the maturity) for a certain price (the strike price)

The Payoffs



Call Option



Put Option

Importance of volatility

- The asymmetry in the payoff means that volatility, σ, is important in determining option prices
- As volatility increases, the price of a call or put option increases

Moneyness

- Moneyness is a measure of the extent to which an option is likely to be exercised
- Popular definitions (S = asset price, K = strike price)
  - At-the money: $S = K$
  - In-the money: $S > K$ for call options and $S < K$ for put options
  - Out-of-the-money: $S < K$ for call options and $S > K$ for put options

Delta

- The delta of a portfolio is the sensitivity of the portfolio to the underlying asset price
- A delta-neutral portfolio is not sensitive to small changes in the underlying asset price
- If

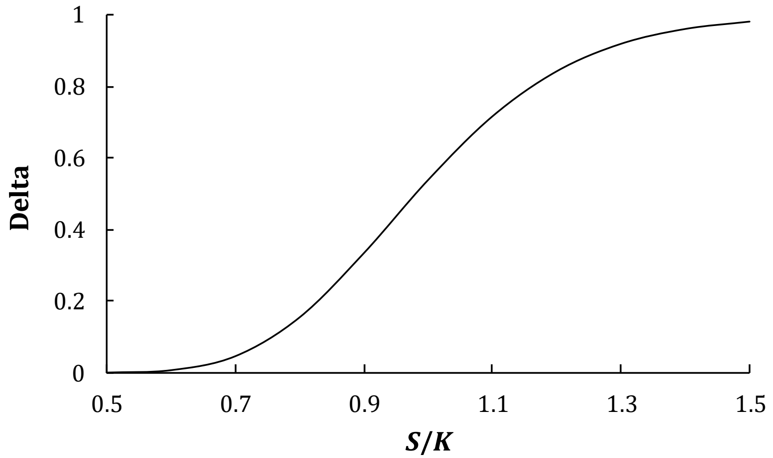
$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

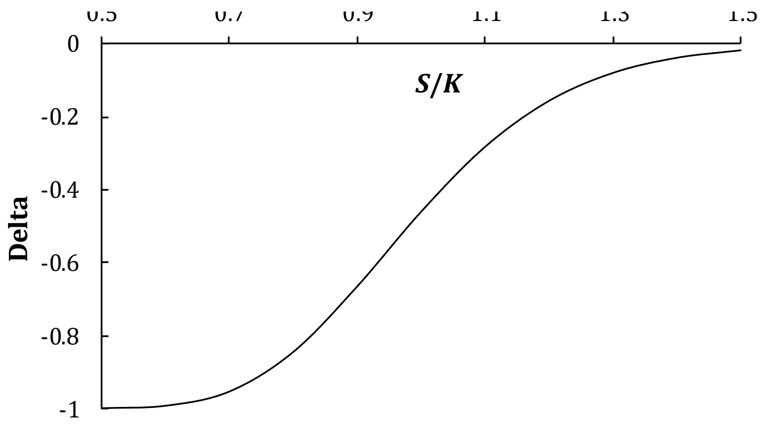
$$\text{Delta (call)} = e^{-qT} N(d_1)$$

$$\text{Delta (put)} = e^{-qT} [N(d_1) - 1]$$

where T is time to maturity, r is risk-free rate, and q is dividend yield

Delta continued





Call Option

Put Option

Delta continued

- Traders can make themselves delta-neutral by trading the underlying asset
- They also use delta as a measure of moneyness for call and put options
  - Call is at-the-money when delta =0.5
  - Call is in-the-money when delta > 0.5
  - Call is out-of-the-money when delta < 0.5
  - Put is at-the-money when delta = –0.5
  - Put is in-the-money when delta < –0.5
  - Put is out-of-the-money when delta > –0.5

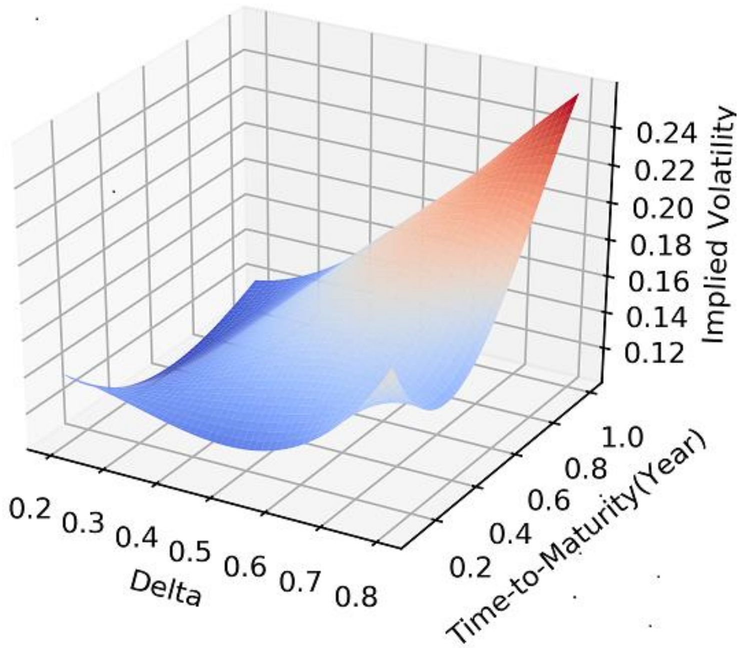
Implied volatilities

- The Black-Scholes-Merton price of an option depends on S, K, r, q, σ, and T
- S, K, r, and T are known for an option
- q can be estimated from futures or forward contracts
- This means that σ is the only unknown.
- There is a one-to-one correspondence between option price and σ
- The implied volatility of an option is the volatility that when substituted into Black-Scholes-Merton formula gives the price of the option in the market

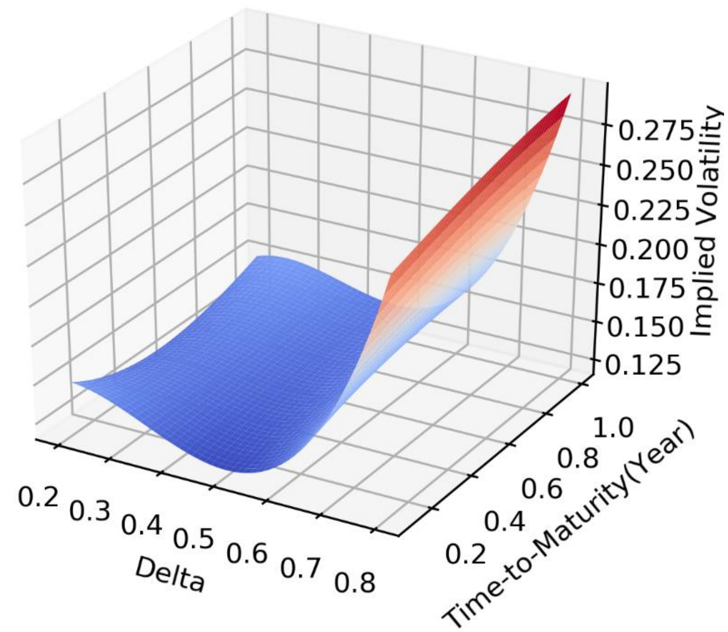
Implied volatilities continued

- If Black-Scholes-Merton was used for pricing, all options on an asset would have the same implied volatility
- In fact, there is quite a variation in implied volatilities
- Nevertheless implied volatilities are used to communicate prices and it is therefore important for traders to monitor implied volatilities
- The volatility surface shows implied volatilities as a function of
  - Moneyness (measured by delta)
  - Time to maturity, T

Volatility Surfaces for S&P 500



Jan 31, 2019



June 25, 2019

Volatility Surface Movements

- When the price of the underlying asset increases (decreases), implied volatilities tend to decrease (increase)
- However not all implied volatilities change by the same amount
- This explains the variation in volatility surfaces

Understanding Volatility Surface Movements

- To understand volatility surface movements we used data on S&P 500 call options to construct a neural network
- Input layer:
  - Daily asset price return
  - Moneyness (measured by delta)
  - Time to maturity
- Output layer:
  - Change in implied volatility

Details

- 3 hidden layers
- 20 neurons per layer
- Observations from 2014-2019
- Randomly sampled 100 options per day
- 125,700 options in total
- 60% for training set
- 20% for validation set
- 20% for test set
- Z-score scaling

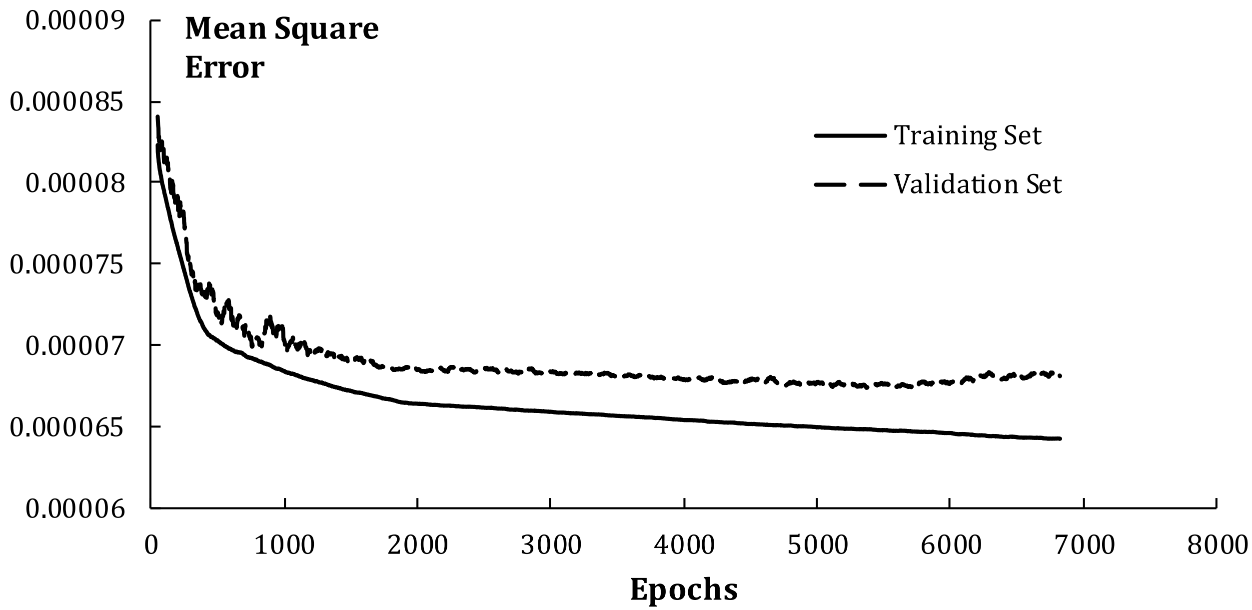
Sample of raw data

Date	*Return (%) on S&P 500*	*Maturity (years)*	*Delta*	*Change in implied volatility (bps)*
Jan 28, 2015	0.95	0.608	0.198	−31.1
Sept 8, 2017	1.08	0.080	0.752	−54.9
Jan 24, 2018	0.06	0.956	0.580	−1.6
Jun 24, 2019	−0.95	2.896	0.828	40.1

After scaling:

Date	*Return (%) on S&P 500*	*Maturity (years)*	*Delta*	*Change in implied volatility (bps)*
Jan 28, 2015	1.091	−0.102	−1.539	−31.1
Sept 8, 2017	1.247	−0.695	0.445	−54.9
Jan 24, 2018	0.027	0.289	−0.171	−1.6
Jun 24, 2019	−1.176	2.468	0.716	40.1

Mean Squared Error (Training stopped after 5,826 epochs)



Results

- Test set were a modest 14% improvement over a simple analytic model proposed by Hull and White in 2017
- However, when the VIX index on Day *t* was used as a feature to predict changes between Day *t* and Day *t*+1 there was a considerable improvement
- The behavior of the volatility surface is different in high and low volatility environments

Second Application: Hedging

- Suppose we do not know how to calculate delta
- Can reinforcement hedging find a good hedging strategy?

The set up

- We generated 3 million stock price paths for training using the process assumed by Black-Scholes-Merton
- We assumed that a trader wishes to hedge a short position in 10 call options
- The options last 10 days
- Probability of exploration starts at 1 and has a decay factor of 0.999999
- Each day the trader can change her position so that 0, 1, 2,…, or 10 shares are held
- Hedging cost on day *i* is

$$H_i = N_i(S_{i+1} - S_i) - 10(c_{i+1} - c_i)$$

where N_i is the number of shares held, S_i is the share price at the beginning of Day i and c_i is the BSM call option price at the beginning of Day i

Test Set Results

- The objective was to minimize the variance of total hedging costs
- Our results were close to those given by delta hedging
- The mean absolute difference between positions taken by the algorithm and those that would be taken with delta hedging when averaged across all hedging days was 0.33

Extensions

- Results can be extended to the situation where there are transactions costs (non-negligible bid-offer spreads) so that (a) delta hedging is not optimal and (b) the problem is a genuine multi-period one
- It is then necessary to trade off the mean cost of hedging with the variance of the cost of hedging
- A similar approach can be used to hedge volatility where transaction costs are high
- A mixture of processes can be used when the hedger is uncertain about the true process. (This is a device to ensure that the hedging strategy works reasonably well for all processes.)

Other Finance Applications

- Investing
- Private equity
- Hiring decisions
- Lending
- Identifying fraud
- Order execution
- Collateral management
- Fast derivative valuation