Derivatives Calculation Worksheet

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Contents

Notes

This worksheet covers the calculation of derivatives, which will be used for several weeks. This worksheet is based on many previous worksheets used in this course.

1 Basic Derivative Rules

1. Write down the following rules:

   (a) The Power Rule

   (b) The Sum Rule

   (c) The Constant Multiple Rule

   (d) The Product Rule

   (e) The Quotient Rule

2. Find the first and second derivative of

   (a)
   $$f(x) = 3x^2$$

   (b)
   $$f(t) = 2e^t - 5$$

3. True or False:

(a) If f, g are differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)\right) = f'(x)g'(x)$$

(b) $y = e^2$, then $y' = 2e$.

(c) The derivative of a polynomial is a polynomial.

4. Find derivative:

   (a)
   $$f(x) = \frac{x^2 + x + 1}{x}$$

   (b)
   $$f(p) = 3p - \sqrt{p}$$

   (c)
   $$B(a) = 5e^a + \sqrt{a} + 6a^2$$

5. Find the limit

   (a)
   $$\lim_{x \to 0} \frac{\sin 4x}{\sin 6x}$$

   (b)
   $$\lim_{x \to 0} \frac{\sin(x^2)}{x}$$

2 Relatively Complicated Calculations of Derivative

1. Find for what values of x the function $f(x) = x|x|$ is differentiable and find a formula for f'.

2. Find all points on the curve
   $$u = \frac{7}{3}x^3 + \frac{1}{4}x^4 + 6x^2$$
   where the tangent is horizontal. Graph the function.

3. Find the derivatives:

   (a)
   $$f(x) = \ln(\ln(\ln(x)))$$

   (b)
   $$f(x) = e^x + 34\pi e + x^e + 4x^\pi + \frac{x+3}{\sqrt{x}}$$

2

(c)
$$y = \frac{x^3}{1 - x^2}$$

(d)
$$g(x) = e^x \sqrt{x^2 - 1}$$

(e)
$$z = \frac{t^2}{(3t - 2)^3}$$

(f)
$$f(t) = \frac{t - t^{1/3}}{\sqrt{t}}$$

(g)
$$f(x) = \frac{y - y^{2/7}}{e^y}$$

(h)
$$y = (x^2 + e^{2x-1})^2$$

4. Find the derivative

(a)
$$y = 2 \sec x - 2 \csc x$$

(b)
$$f(\theta) = \sin(\theta) \cos(\theta)$$

(c)
$$y = \frac{1 - \sec x}{\tan x}$$

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(d)
$$f(\theta) = \sin(\cos(\tan(\theta)))$$

3 Implicit Differentiation

1. Write down the derivatives of

(a)
$$\arcsin x$$

(b)
$$\arccos x$$

(c)
$$\arctan x$$

(d)
$$\cot^{-1}(x)$$

2. Find the derivatives

(a)
$$x^2 + y^2 = 1$$

without solving for y.

(b)
$$x^3 + y^3 = 6xy$$

(c)
$$xe^y = x - y$$

(d)
$$\tan(x - y) = \frac{y}{1 + x^2}$$

(e)
$$f(x) = \arcsin\left(x^3 + 2\right)$$

3. Find an equation of the tangent line to the curve
$$\sin(x + y) = 2x - 2y$$

at the point (π, π)

4. Find the derivative:

(a)
$$y = x^x$$

(b)
$$y = \frac{x^{5/3}\sqrt{x^4 + 2}}{(4x + 1)^5}$$

(c)
$$y = (\sqrt{x})^x$$

(d)
$$y = \sin x^{\ln x}$$