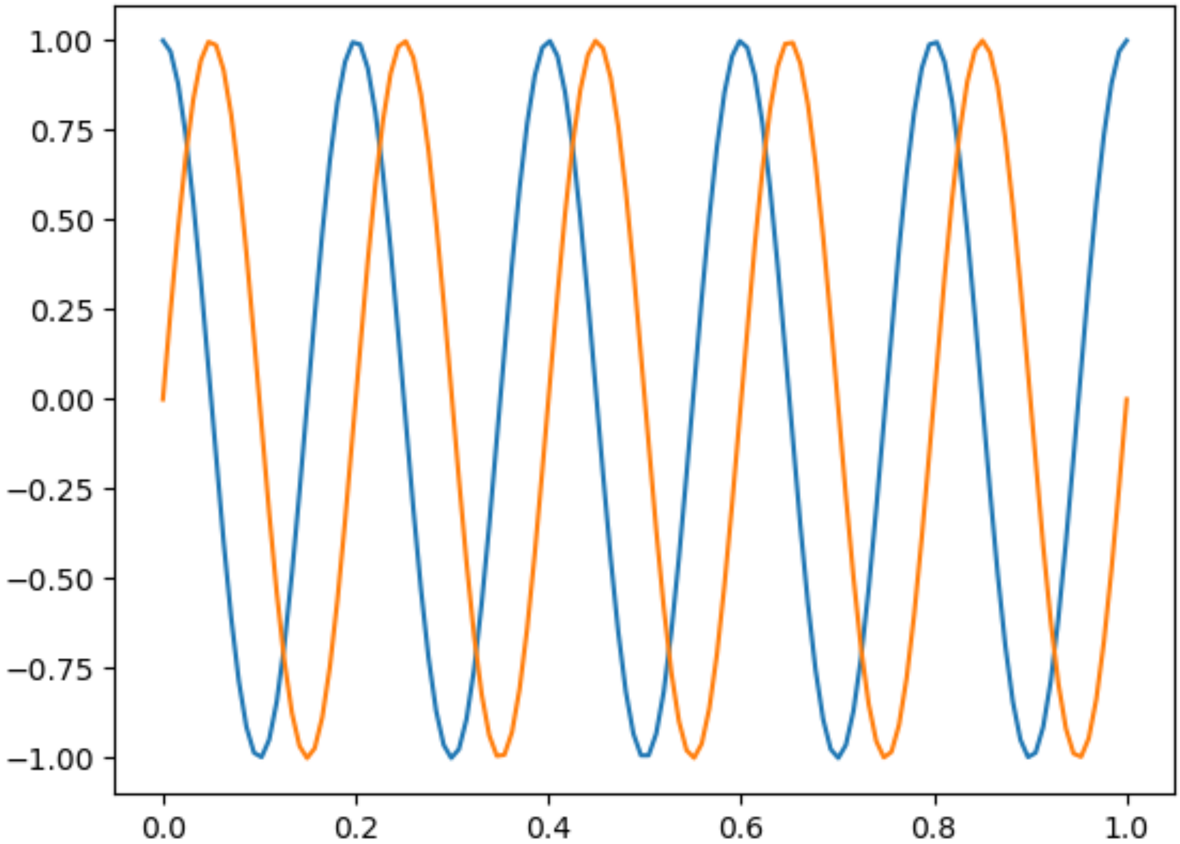
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
from scipy.integrate import odeint
import scipy.constants as cste

import time
```

TD 3: Signal Processing

DFT Discrete Fourier transform

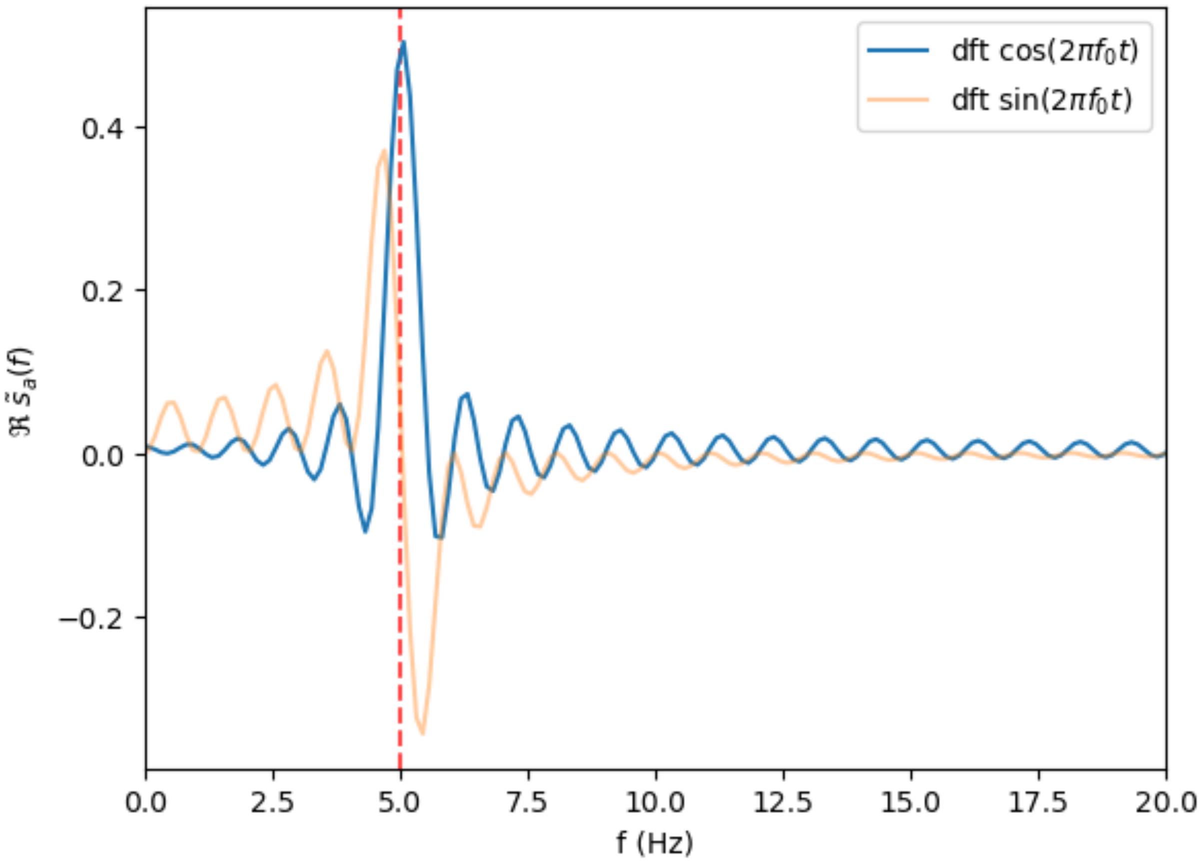
In [ ]:
```
T_a = 1
f_0 = 5
N_p = T_a*f_0

print(f"{N_p=}")

N_e = 128

f_e = N_e/T_a # sampling frequency

t_tab = np.linspace(0, T_a, N_e)

signal_1 = np.cos(2*np.pi*f_0*t_tab)
signal_2 = np.sin(2*np.pi*f_0*t_tab)
plt.plot(t_tab, signal_1)
plt.plot(t_tab, signal_2)
plt.show()
```

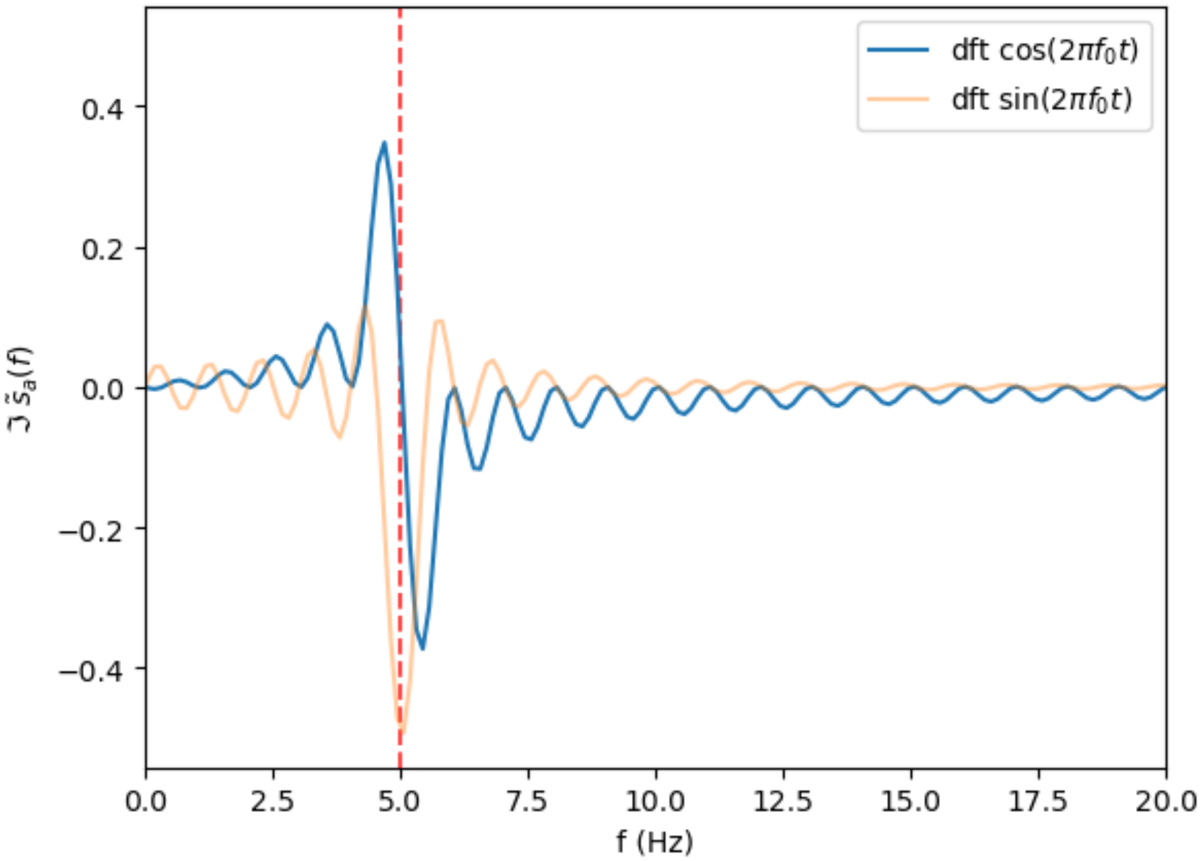
```
N_p=5
```



In [ ]:
```
f_tab = np.linspace(-f_e/2, f_e/2, 1024)

def DFT_mano(s_a, f, f_e):
    T_e = 1/f_e

    dft = np.zeros(len(f))*1j
    for n in range(len(s_a)):
        dft += s_a[n]*np.exp(-1j*2*np.pi*f*n*T_e)*T_e


    return dft
```

In [ ]:
```
sig_1_dft = DFT_mano(signal_1, f_tab, f_e)
sig_2_dft = DFT_mano(signal_2, f_tab, f_e)


plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(f_tab, sig_1_dft.real, label="dft $\cos(2\pi f_0 t)$")
plt.plot(f_tab, sig_2_dft.real, label="dft $\sin(2\pi f_0 t)$", alpha=0.4
plt.xlim((0, 20))
plt.legend()

plt.xlabel("f (Hz)")
plt.ylabel(r"$\Re$ $\tilde s_a(f)$")
plt.show()

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(f_tab, sig_1_dft.imag, label="dft $\cos(2\pi f_0 t)$")
plt.plot(f_tab, sig_2_dft.imag, label="dft $\sin(2\pi f_0 t)$", alpha=0.4
plt.xlim((0, 20))
plt.legend()

plt.xlabel("f (Hz)")
plt.ylabel(r"$\Im$ $\tilde s_a(f)$")
plt.show()
```



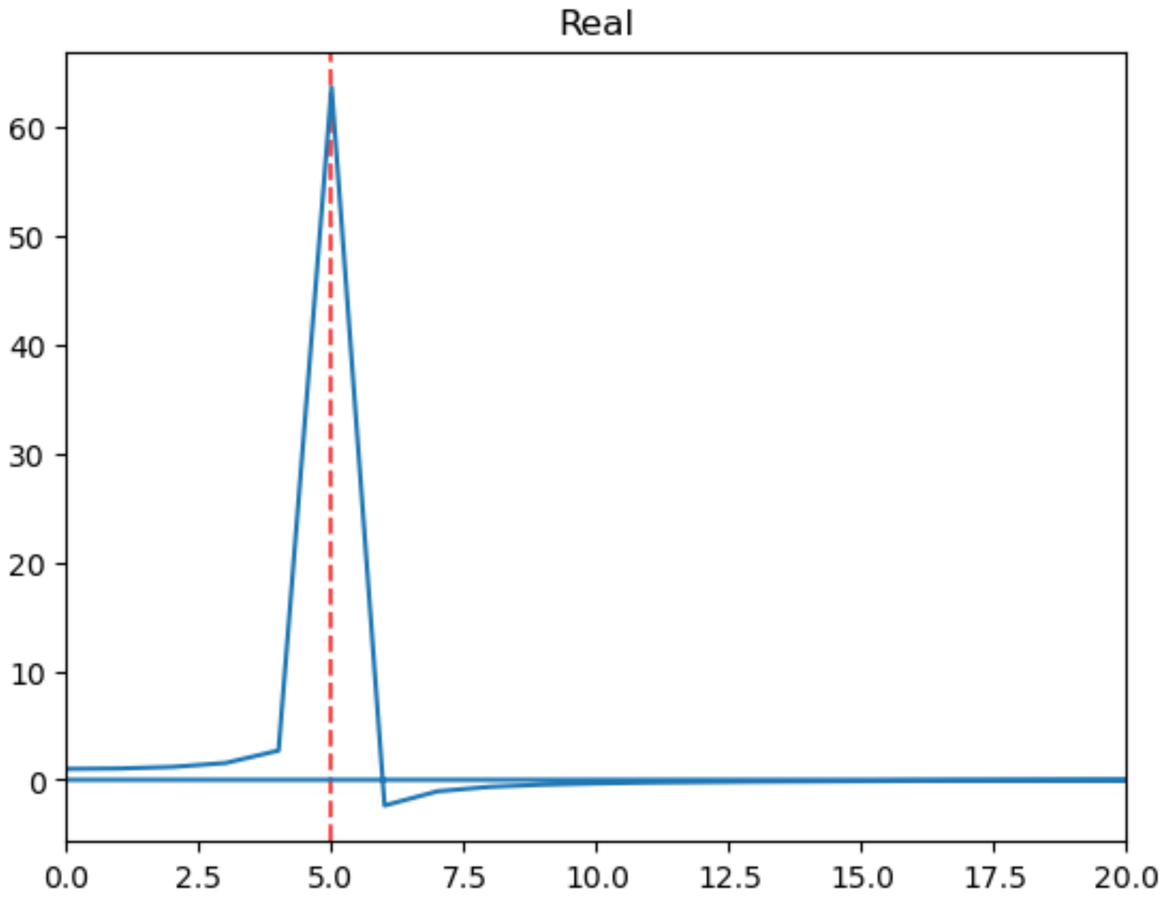


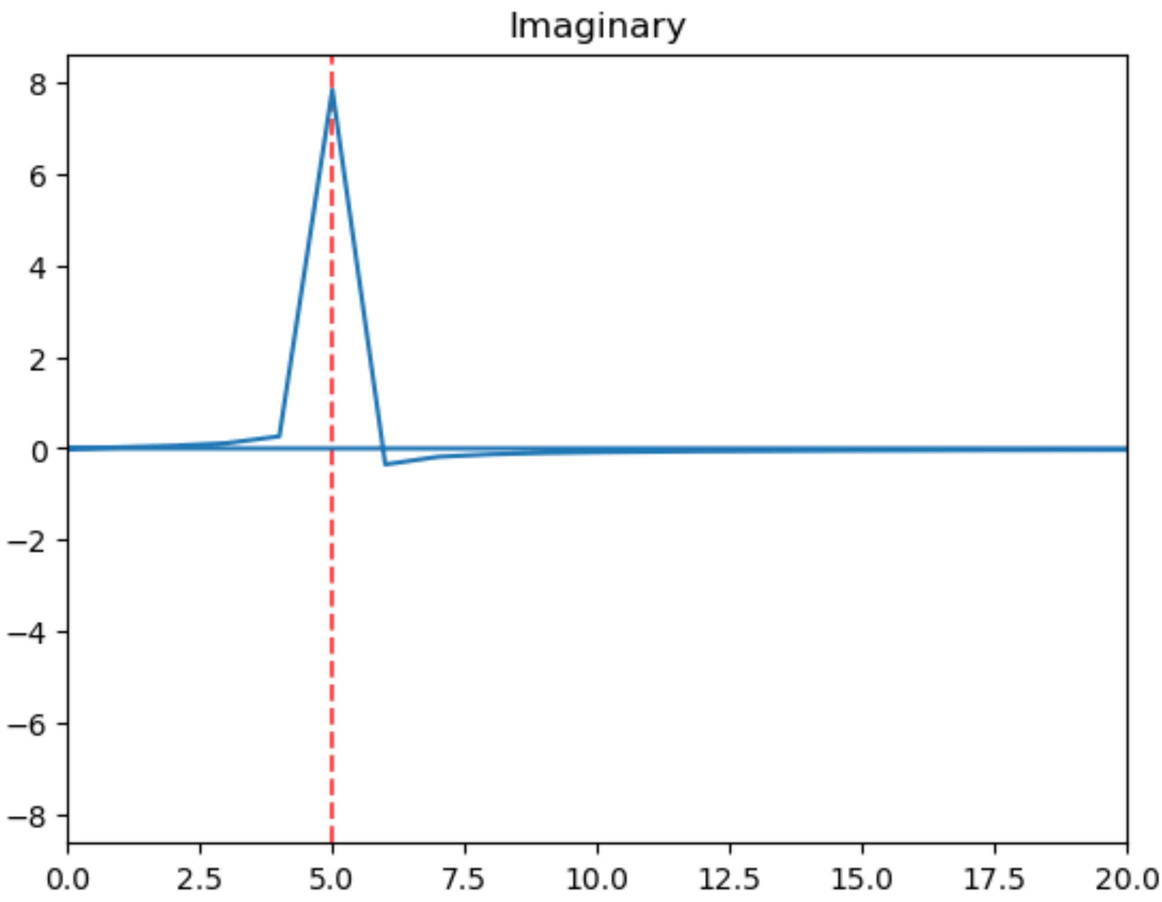
in theory, since it is a pure sinusoidal, the exact FT would be a real delta function centred at f_0

FFT Fast Fourier Transform

In [ ]:

```
sig_1_fft = np.fft.fft(signal_1)
freq_domain = np.fft.fftfreq(N_e, 1/f_e)

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, sig_1_fft.real)
plt.title("Real")
plt.xlim((0, 20))
plt.show()

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, sig_1_fft.imag)
plt.title("Imaginary")
plt.xlim((0, 20))
plt.show()
```

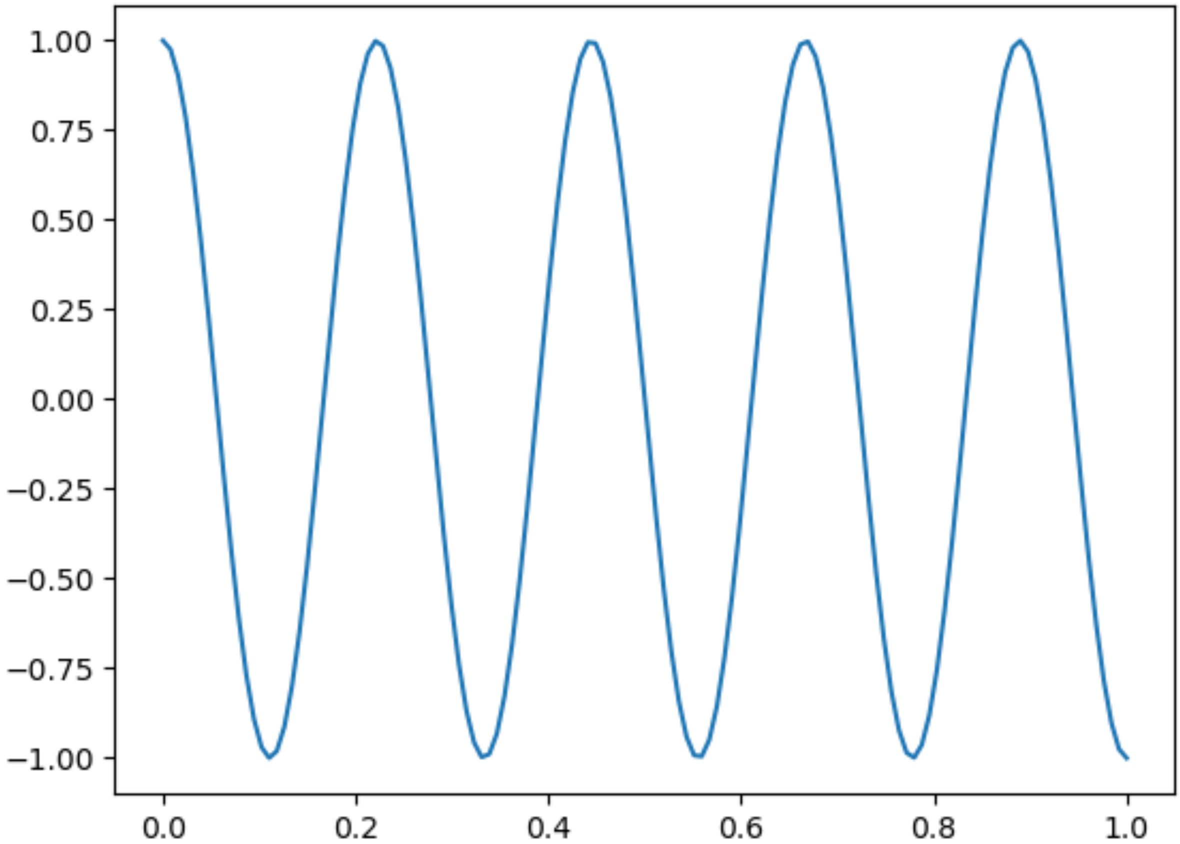


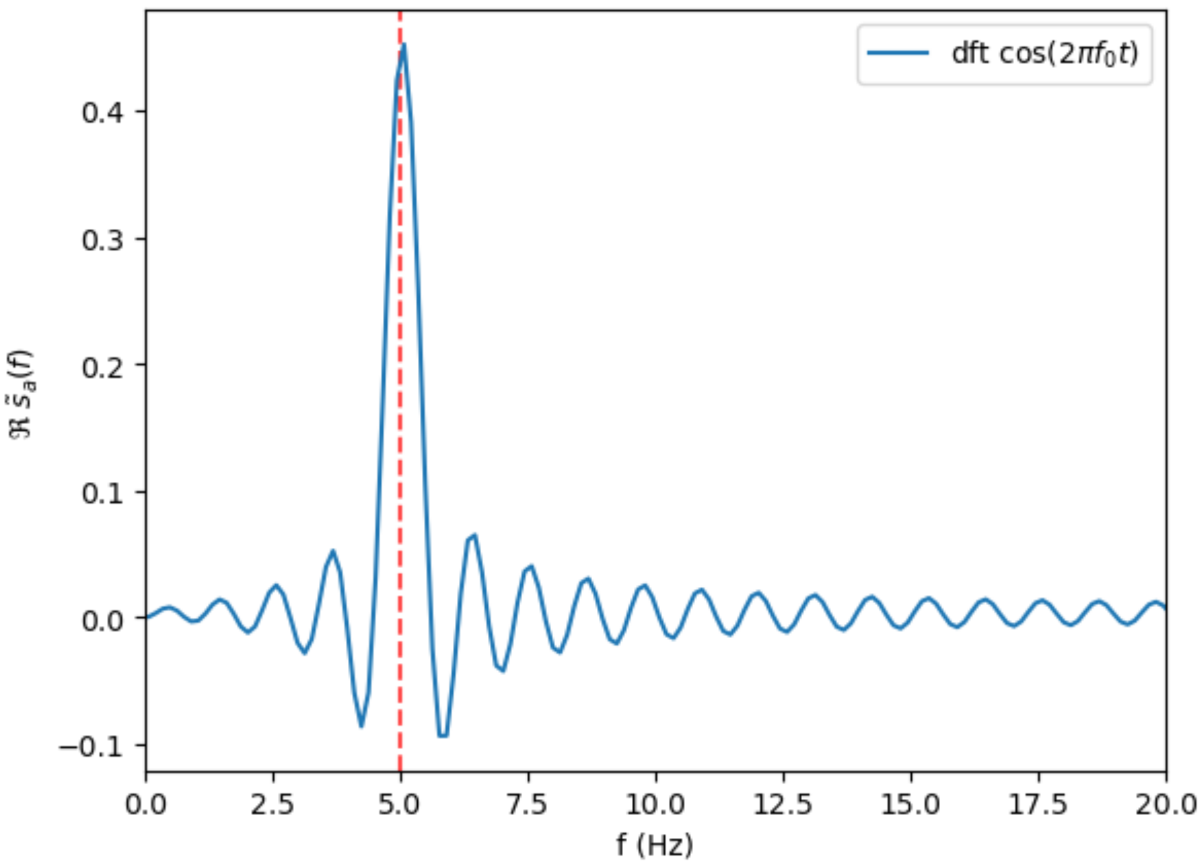


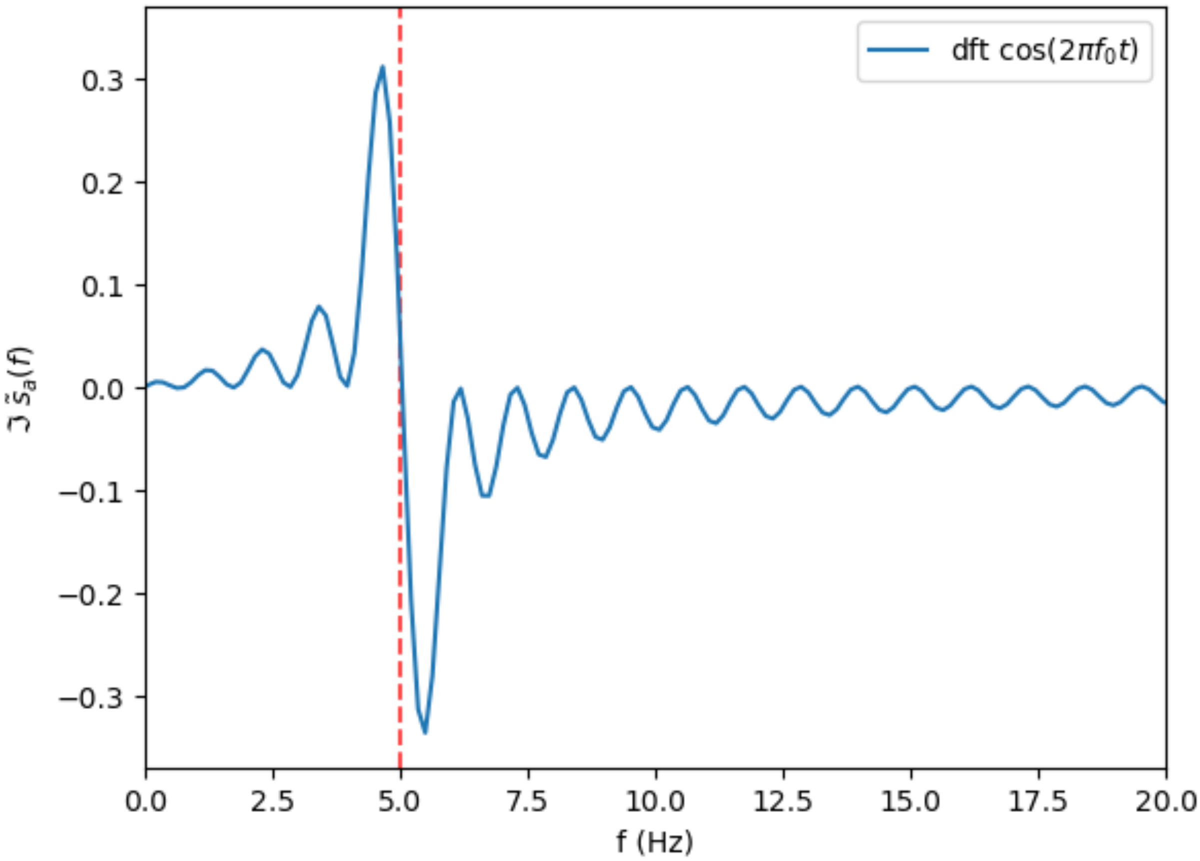
sampling frequency is low, so we aren't expecting anything fantastic if $T_a = 0.9$

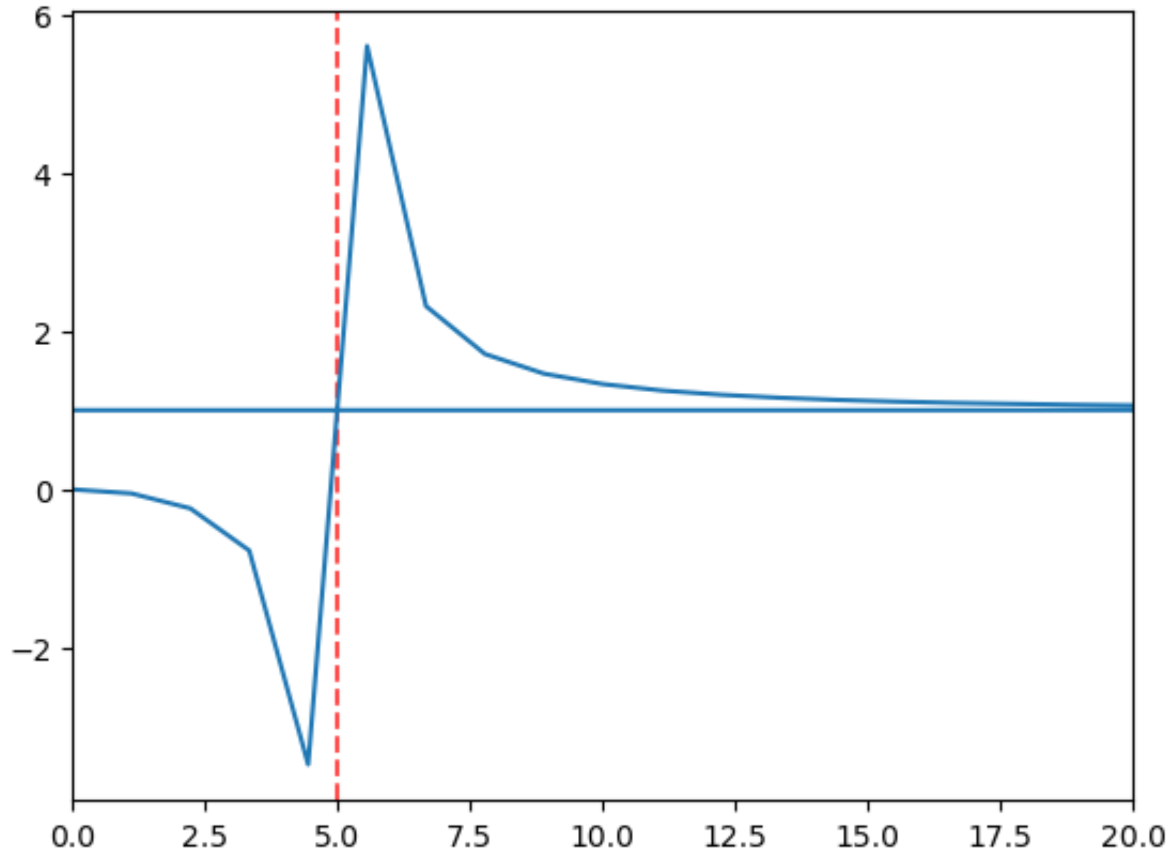
In [ ]:
```
T_a3 = 0.9
f_0 = 5
N_p3 = T_a3*f_0

print(f"{N_p3=}")

N_e3 = 128

f_e3 = N_e3/T_a3 # sampling frequency

t_tab3 = np.linspace(0, T_a3, N_e3)

signal_3 = np.cos(2*np.pi*f_0*t_tab3)
plt.plot(t_tab, signal_3)
plt.show()

f_tab3 = np.linspace(-f_e3/2, f_e3/2, 1024)

## DFT manual

sig_3_dft = DFT_mano(signal_3, f_tab3, f_e3)

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(f_tab3, sig_3_dft.real, label="dft $\cos(2\pi f_0 t)$")
plt.xlim((0, 20))
plt.legend()

plt.xlabel("f (Hz)")
plt.ylabel(r"$\Re$ $\tilde s_a(f)$")
plt.show()

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(f_tab3, sig_3_dft.imag, label="dft $\cos(2\pi f_0 t)$")
plt.xlim((0, 20))
plt.legend()

plt.xlabel("f (Hz)")
plt.ylabel(r"$\Im$ $\tilde s_a(f)$")
plt.show()

## FFT

sig_3_fft = np.fft.fft(signal_3, N_e3)
freq_domain3 = np.fft.fftfreq(N_e3, 1/f_e3)

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain3, sig_3_fft.real)
plt.xlim((0, 20))
plt.show()

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain3, sig_3_fft.imag)
plt.xlim((0, 20))
plt.show()
```

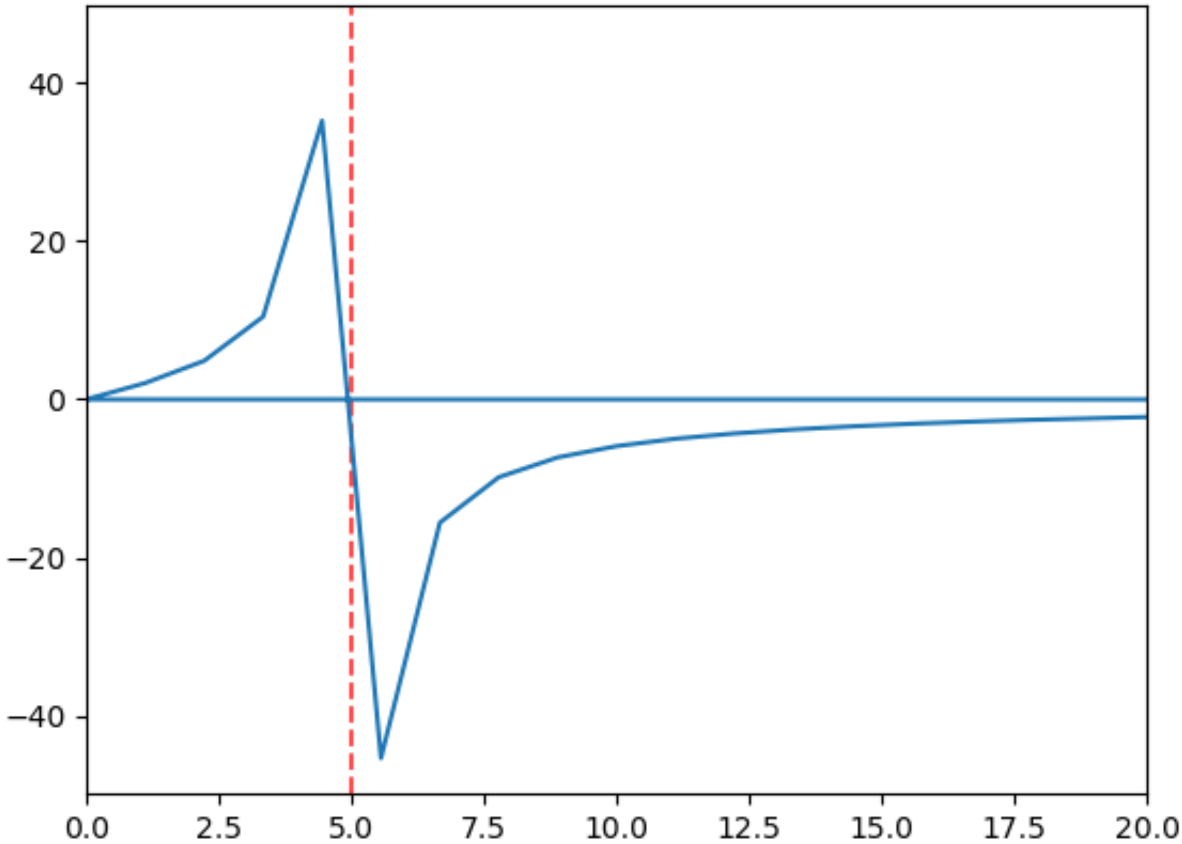
```
N_p3=4.5
```


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0


dft $\cos(2\pi f_0 t)$
$\Re\,\tilde{s}_a(f)$
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
f (Hz)


dft cos($2\pi f_0 t$)
$\Im\, \tilde{s}_a(f)$
f (Hz)
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

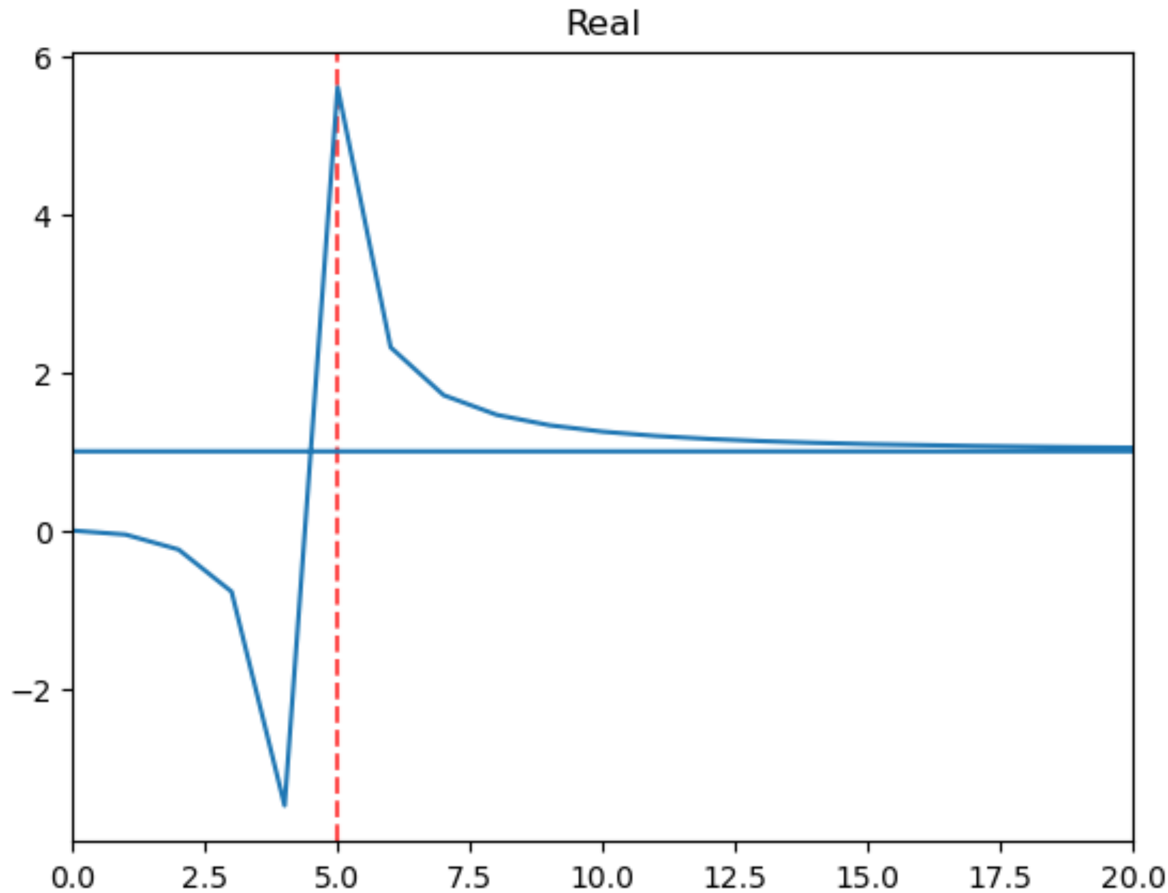

6
4
2
0
−2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

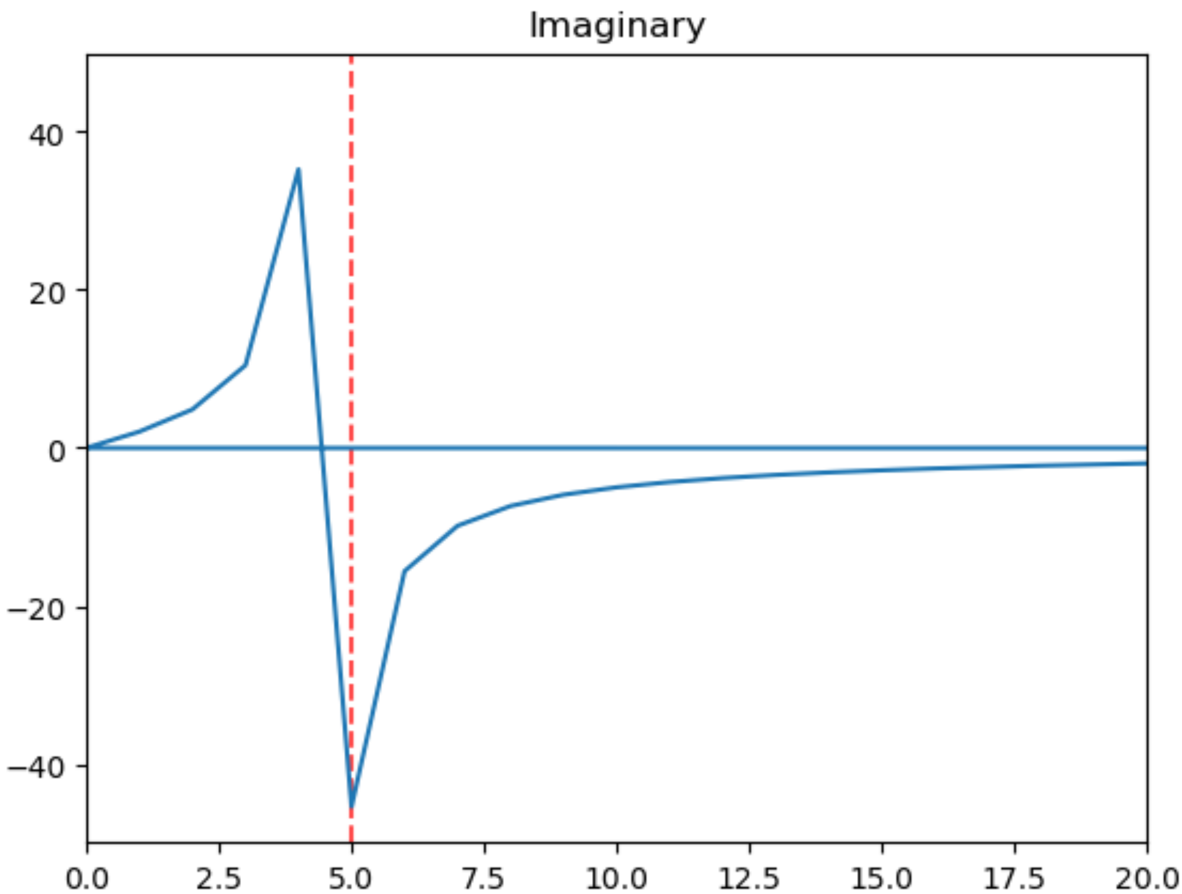


the FFT result is really off, so we pad the input with zeros. np.fft can actually do this for us.

In [ ]:
```
# resize array
signal_3_az = np.zeros(len(t_tab))#signal_3.resize()
for i in range(len(signal_3)):
    signal_3_az[i] = signal_3[i]

# resizing process is not actually necessary, numpy will pad the array
    # with extra 0s if we give n > len(signal)


print(N_e)
sig_3_az_fft = np.fft.fft(signal_3_az, N_e)


freq_domain3_az = np.fft.fftfreq(N_e, 1/f_e)

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain3_az, sig_3_az_fft.real)
plt.xlim((0, 20))
plt.title("Real")
plt.show()

plt.axvline(f_0, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain3_az, sig_3_az_fft.imag)
plt.xlim((0, 20))
plt.title("Imaginary")
plt.show()
```

128


Real
6
4
2
0
−2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Imaginary
40
20
0
−20
−40
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

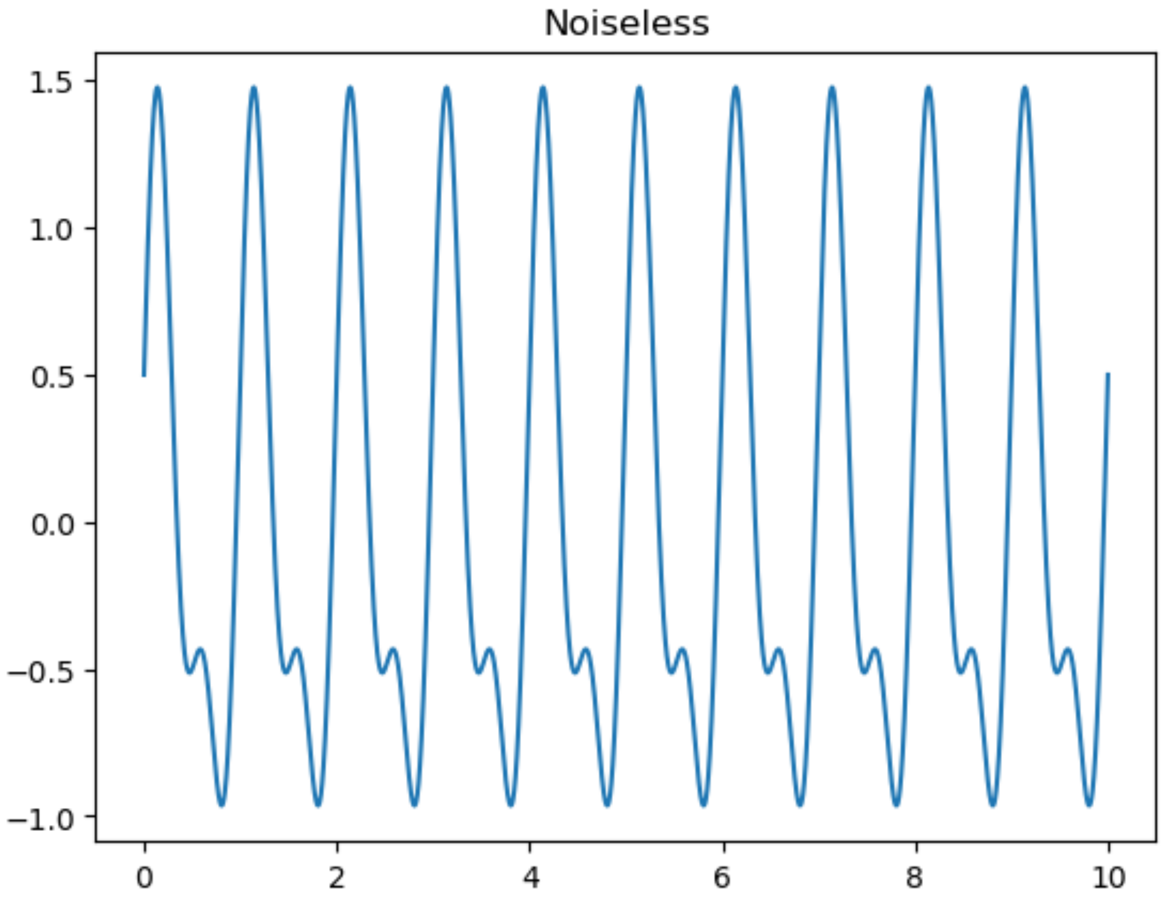
Its better, the peaks line up. Still not as good as when $T_a = 1$

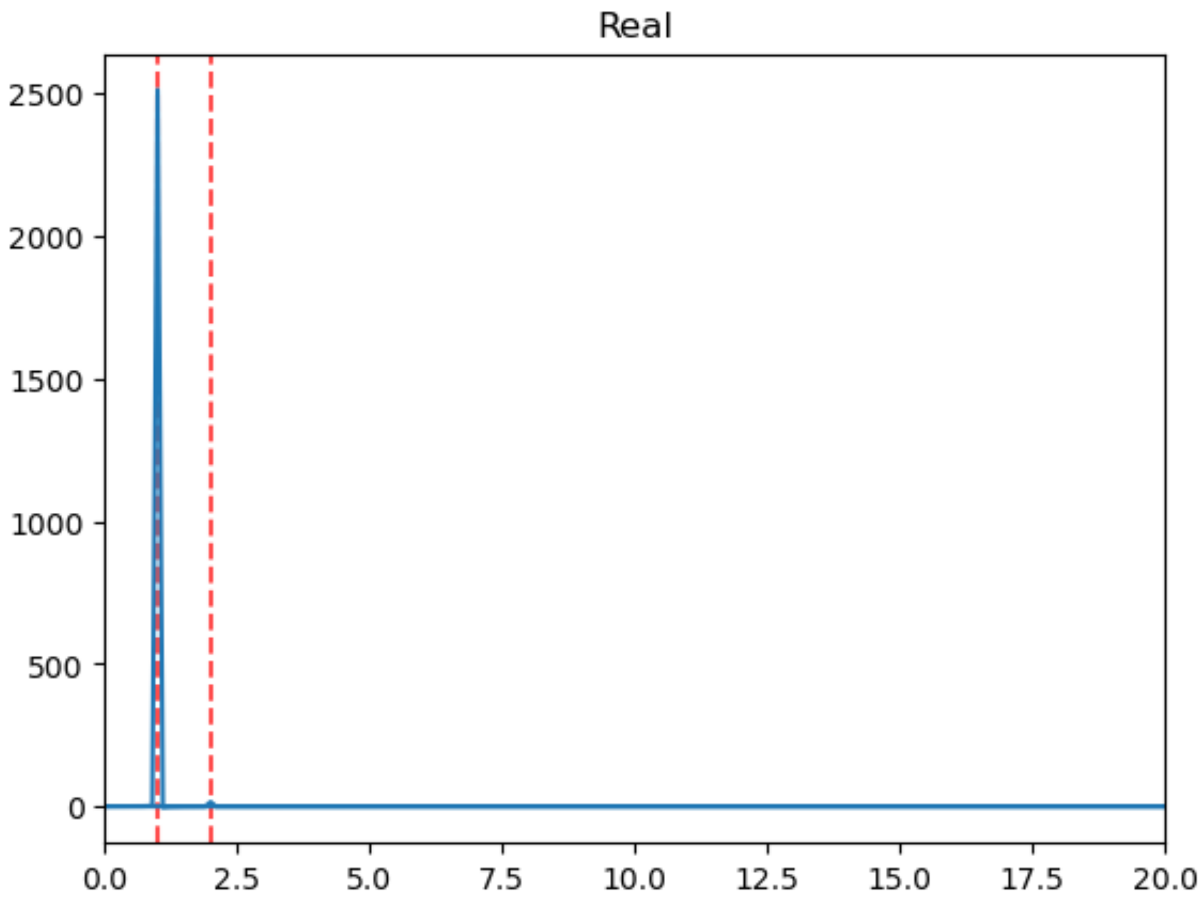
In [ ]:
```
# time DFT mano for T_a = 0.9

t0_dft = time.time()
DFT_mano(signal_3, f_tab3, f_e3)
t1_dft = time.time()

print(f"DFT mano run time {t1_dft - t0_dft:.3} s")


# time

t0_fft = time.time()
np.fft.fft(signal_3_az, N_e)
t1_fft = time.time()

print(f"numpy FFT run time {t1_fft - t0_fft:.3} s")

print(f"FFT is {(t1_dft - t0_dft)/(t1_fft - t0_fft):.4} times faster")
```

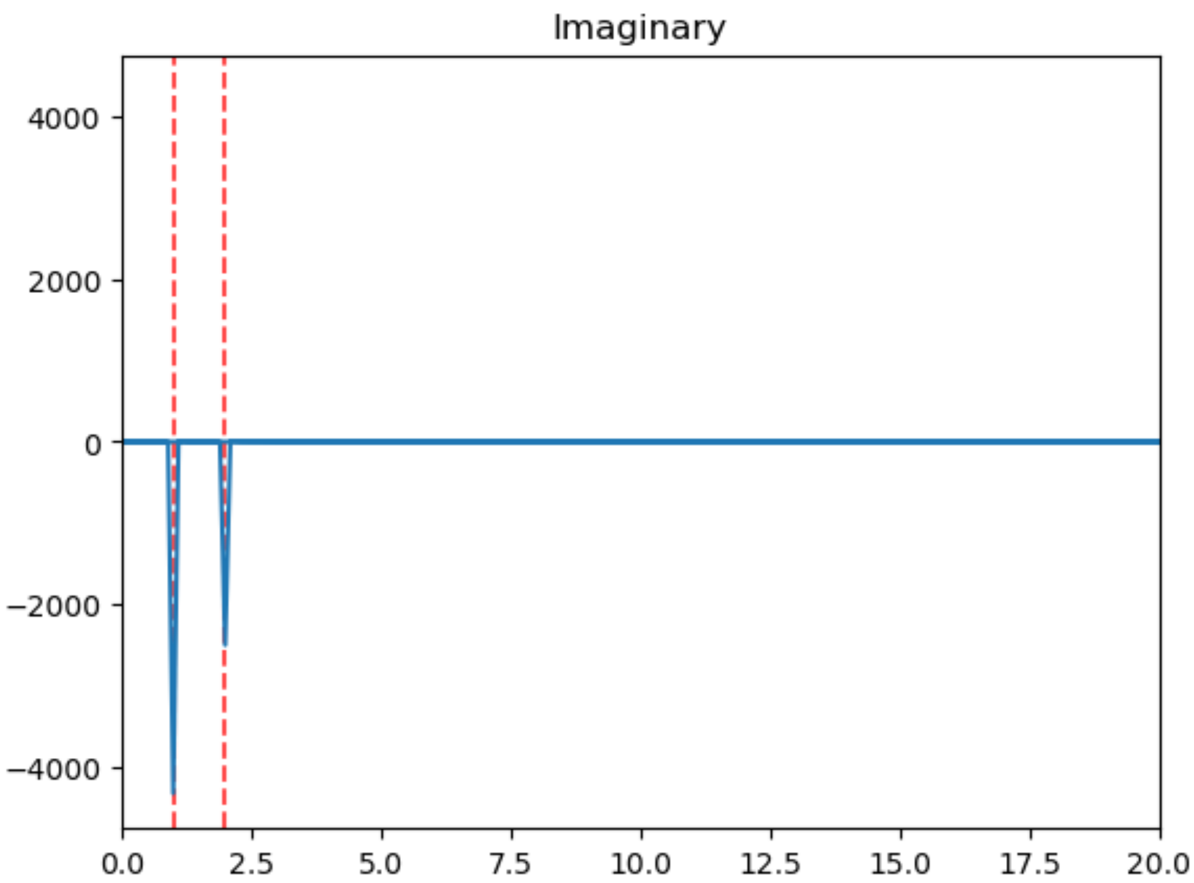
```
DFT mano run time 0.00856 s
numpy FFT run time 0.000548 s
FFT is 15.61 times faster
```

Signal filtering

In [ ]:
```
A_1 = 1
A_2 = 0.5
phi_1 = np.pi/6
phi_2 = 0
nu_1 = 1
nu_2 = 2

f_sample = 1000
t_max = 10
N_sample = f_sample*t_max


t_f = np.linspace(0, 10, N_sample)

f = A_1*np.sin(2*np.pi*nu_1*t_f + phi_1) + A_2*np.sin(2*np.pi*nu_2*t_f +
plt.plot(t_f, f)
plt.title("Noiseless")
plt.show()


f_fft = np.fft.fft(f, N_sample)


freq_domain = np.fft.fftfreq(N_sample, 1/f_sample)

plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_fft.real)
plt.xlim((0, 20))
plt.title("Real")
plt.show()
plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_fft.imag)
plt.title("Imaginary")
plt.xlim((0, 20))
plt.show()
```

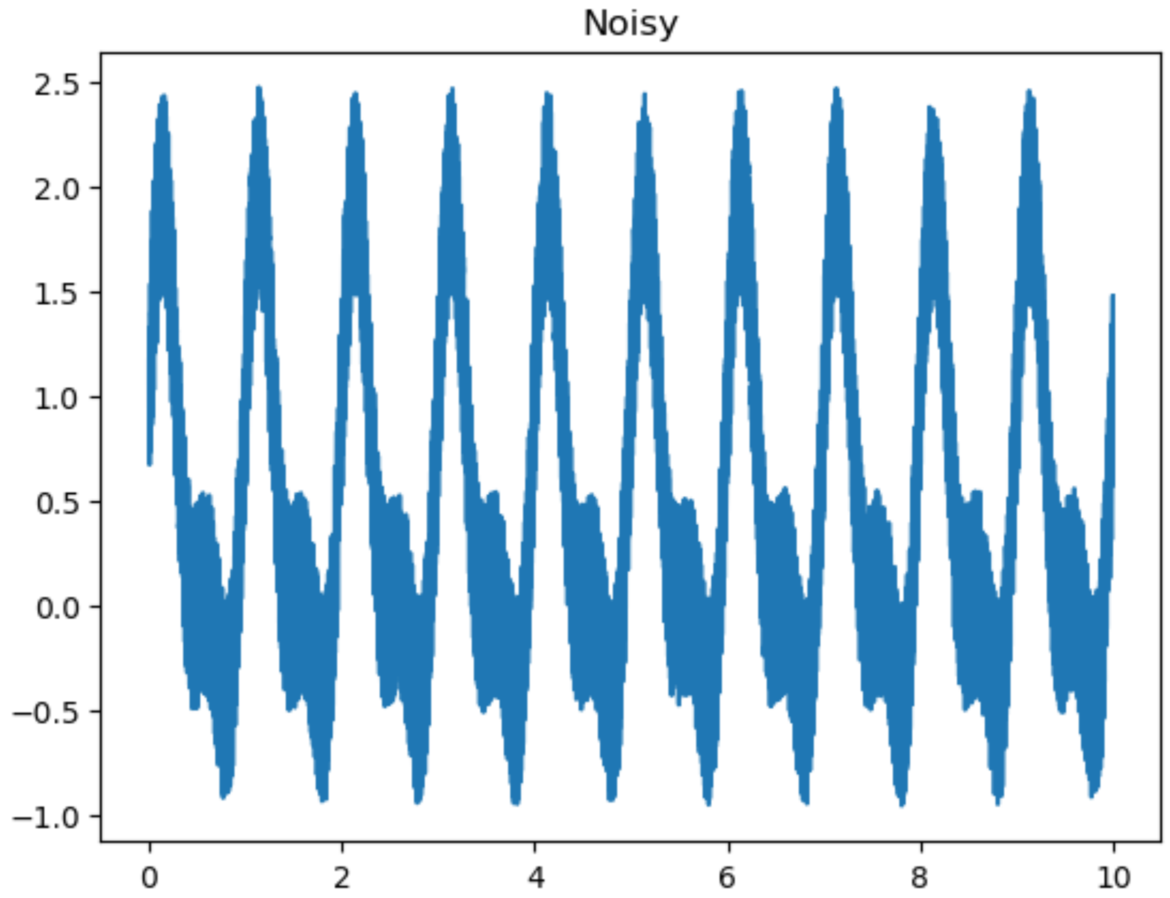

Noiseless
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10

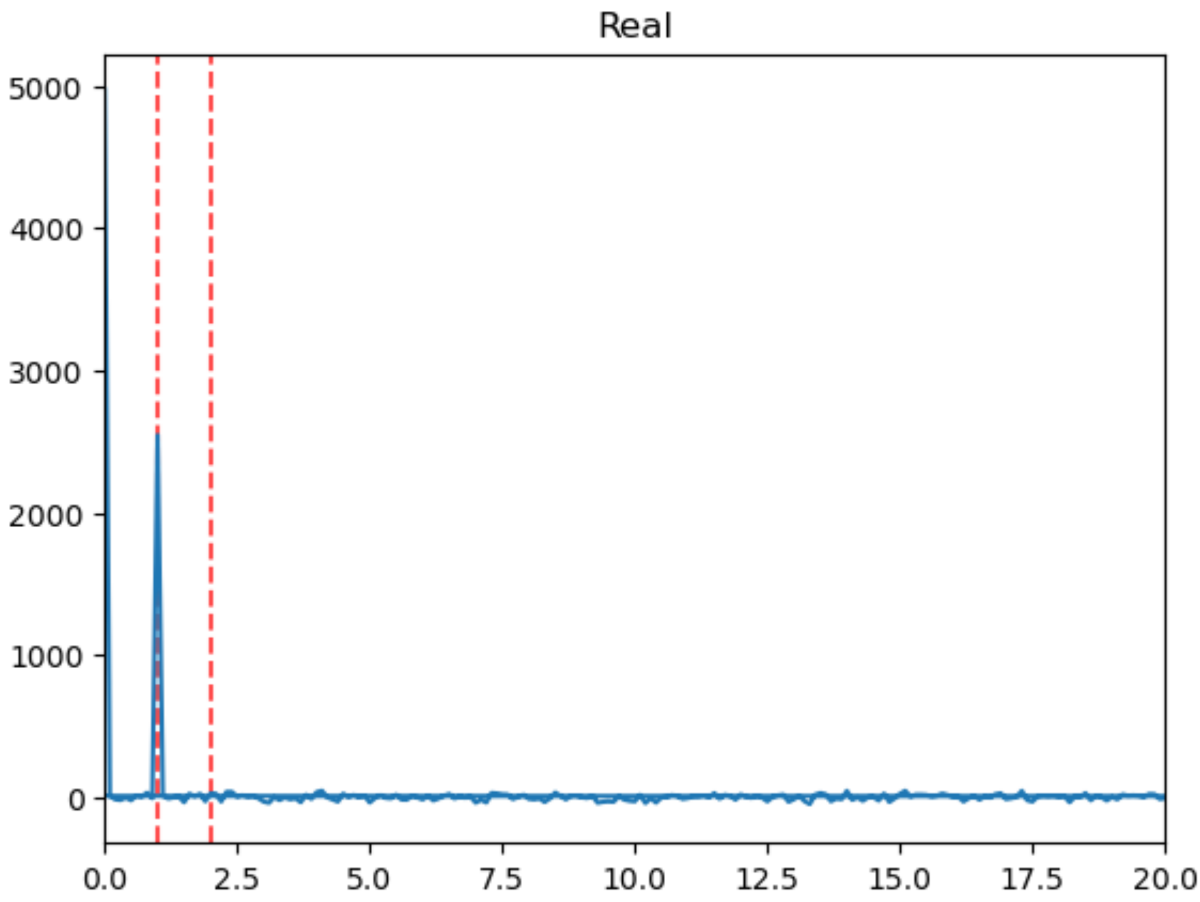

Real
2500
2000
1500
1000
500
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

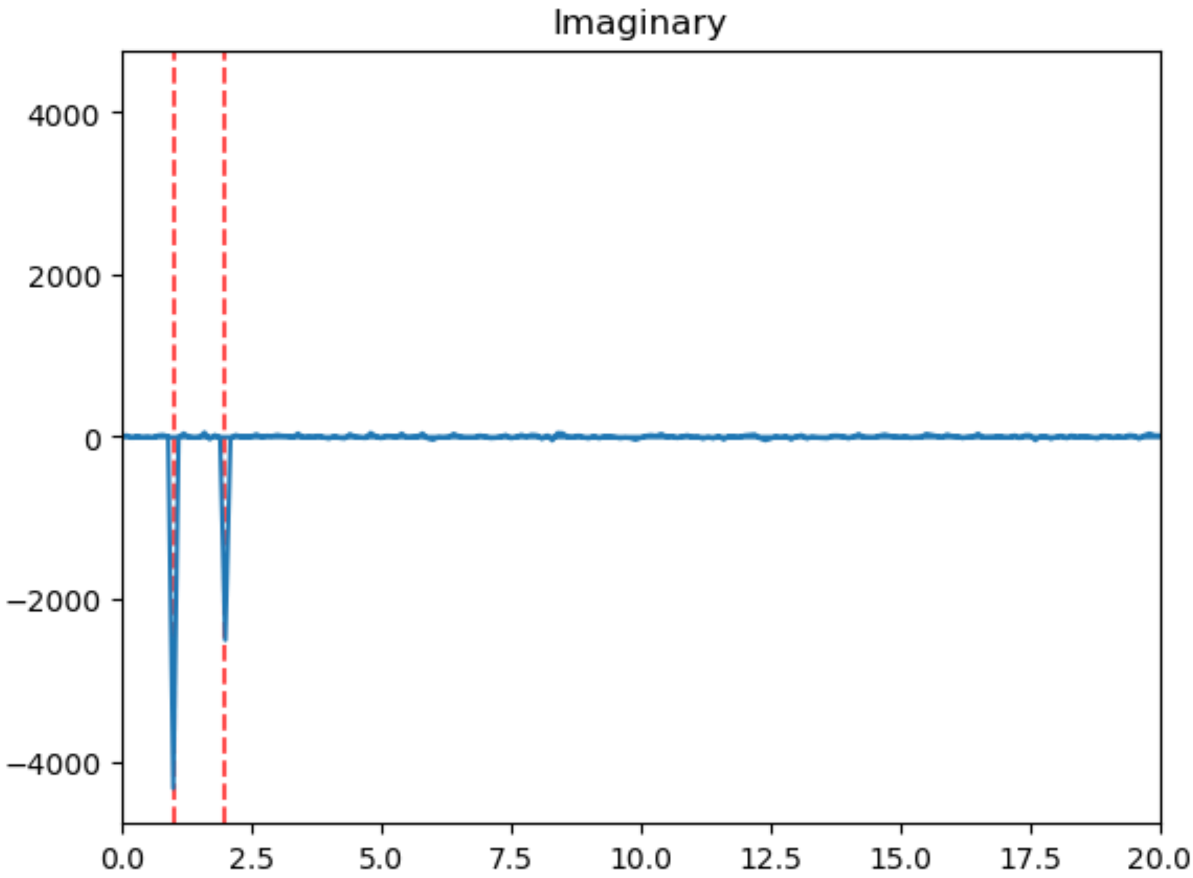


Now we add noise

In [ ]:
```
noise = np.random.random_sample(N_sample)
f_noisy = f + noise
plt.plot(t_f, f_noisy)
plt.title("Noisy")
plt.show()


f_noisy_fft = np.fft.fft(f_noisy, N_sample)


plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_noisy_fft.real)
plt.title("Real")
plt.xlim((0, 20))
plt.show()

plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_noisy_fft.imag)
plt.title("Imaginary")
plt.xlim((0, 20))
plt.show()
```


Noisy
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10

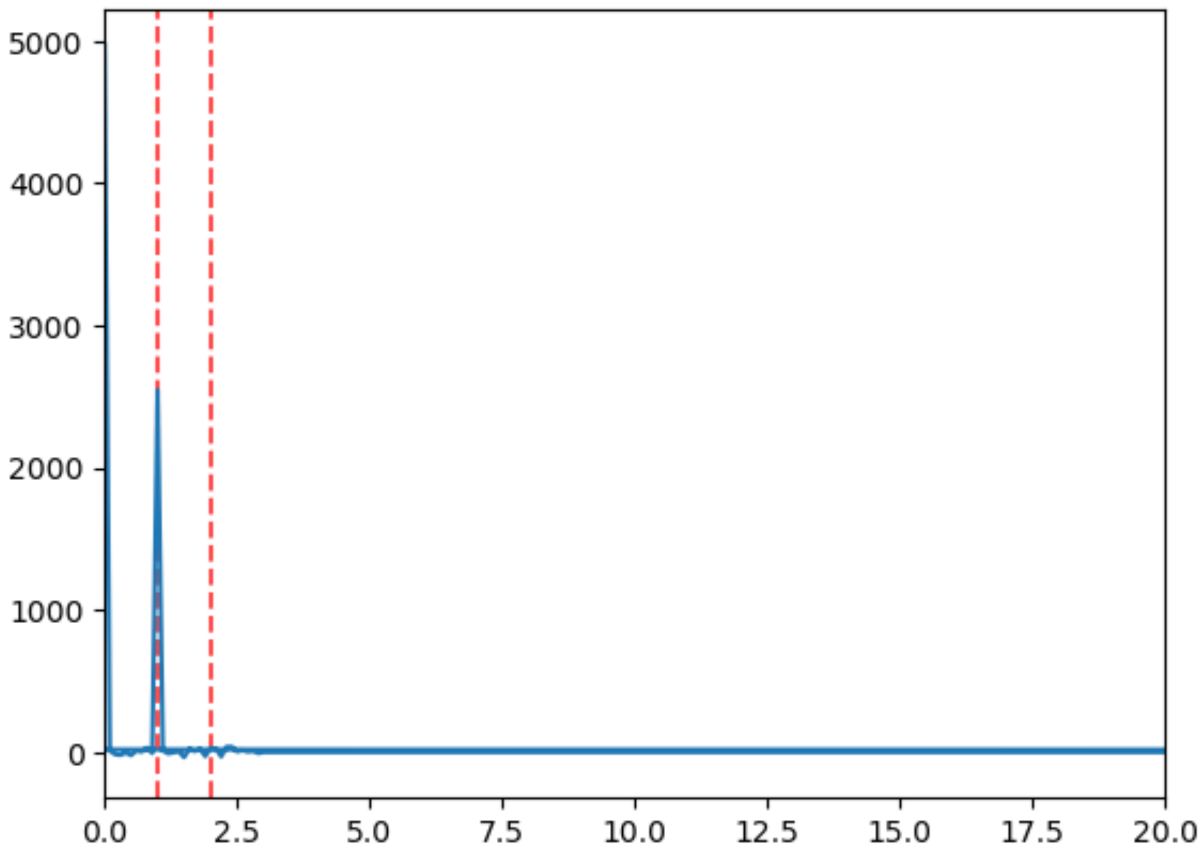

Real
5000
4000
3000
2000
1000
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

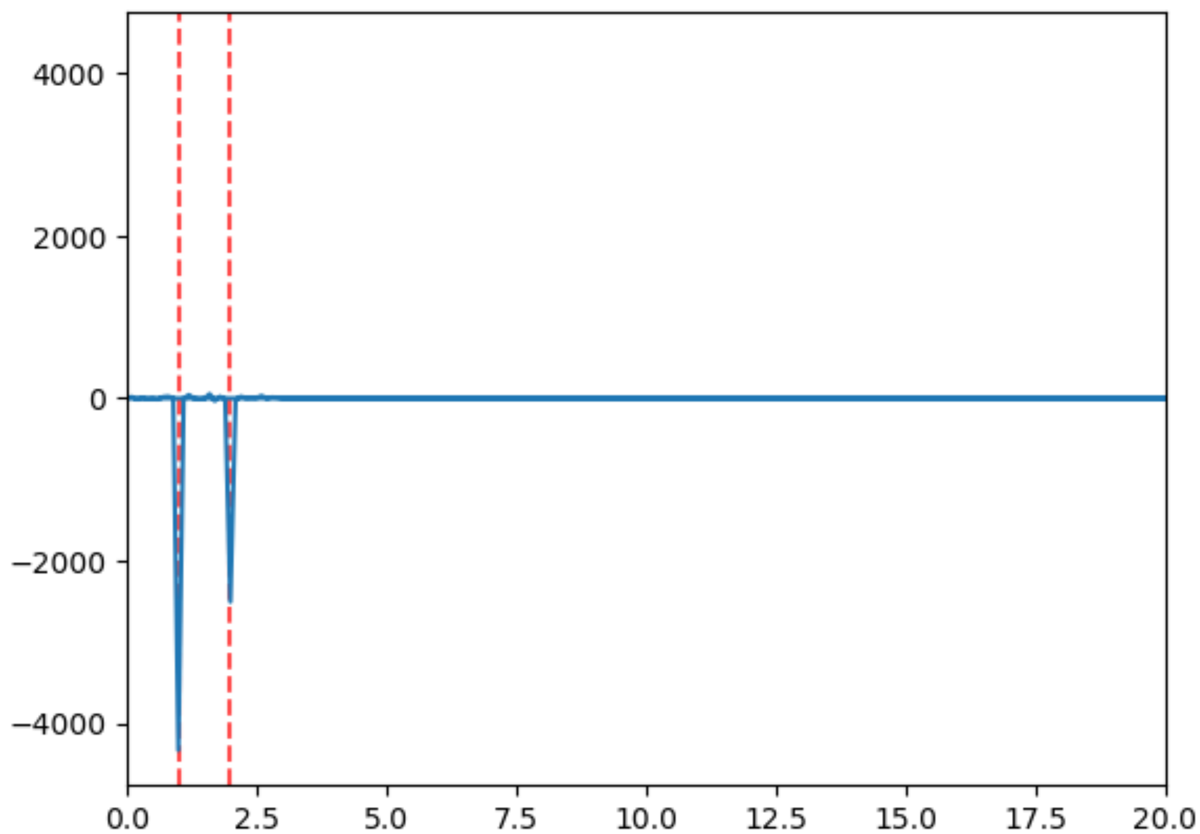


There is a peak at f=0 for the real component of the FFT - there is a constant component to the noise.

I'm passing the ft of the noisy signal through both a lowpass and a highpass filter in order to only keep the components for frequencies sufficiently close to ν_1 and ν_2

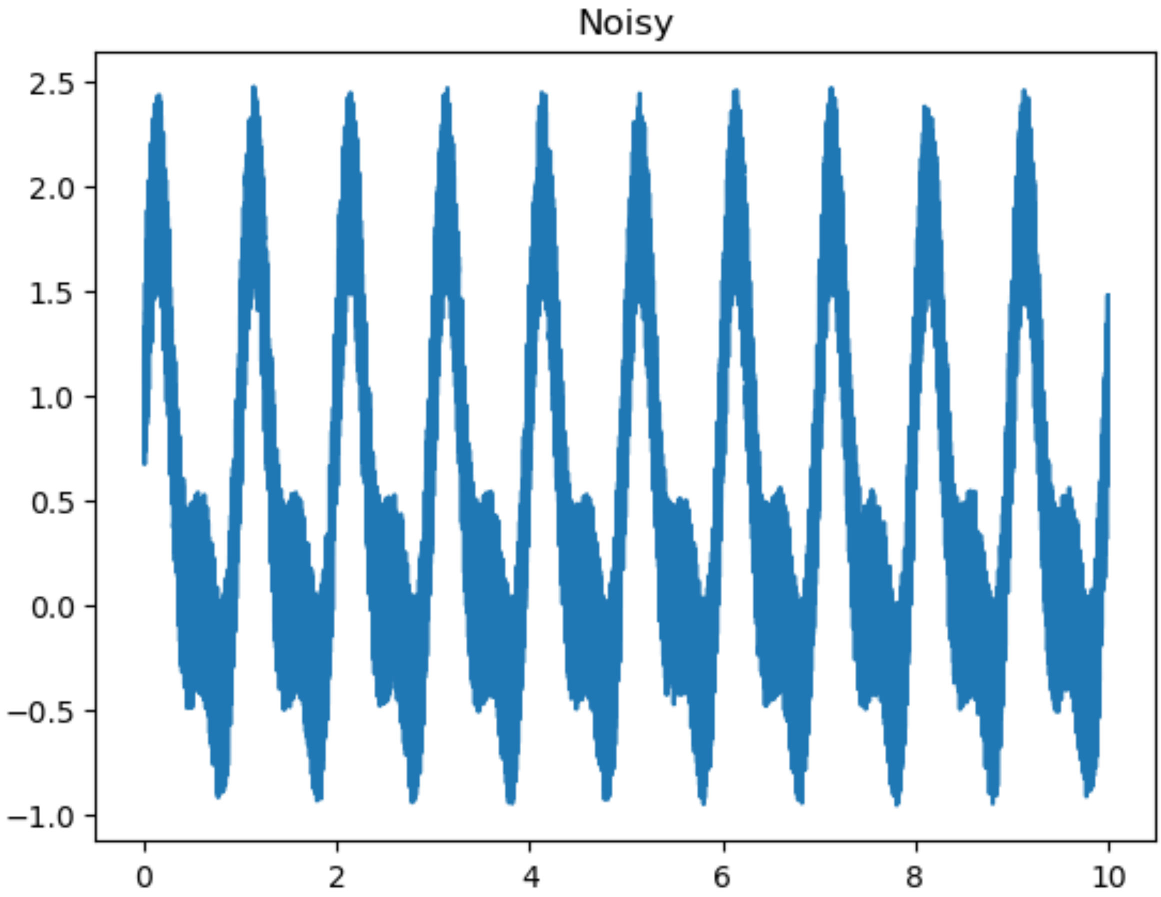
In [ ]:
```
def lowpass(fft, domain, max):
    res = np.zeros(len(fft), dtype=complex)
    for i in range(len(fft)):
        if (domain[i] < max ):
            res[i] = fft[i]
    return res

def highpass(fft, domain, min):
    res = np.zeros(len(fft), dtype=complex)
    for i in range(len(fft)):
        if (domain[i] > min):
            res[i] = fft[i]
    return res


f_noise2_fft = lowpass(f_noisy_fft, freq_domain, 1.5*nu_2)
#denoised_fft = heaviside(f_noisy_fft, freq_domain, 0.8*nu_1, 1.2*nu_1)

plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)
plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_noise2_fft.real)
plt.xlim((0, 20))
plt.show()
plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_noise2_fft.imag)
plt.xlim((0, 20))
plt.show()
```



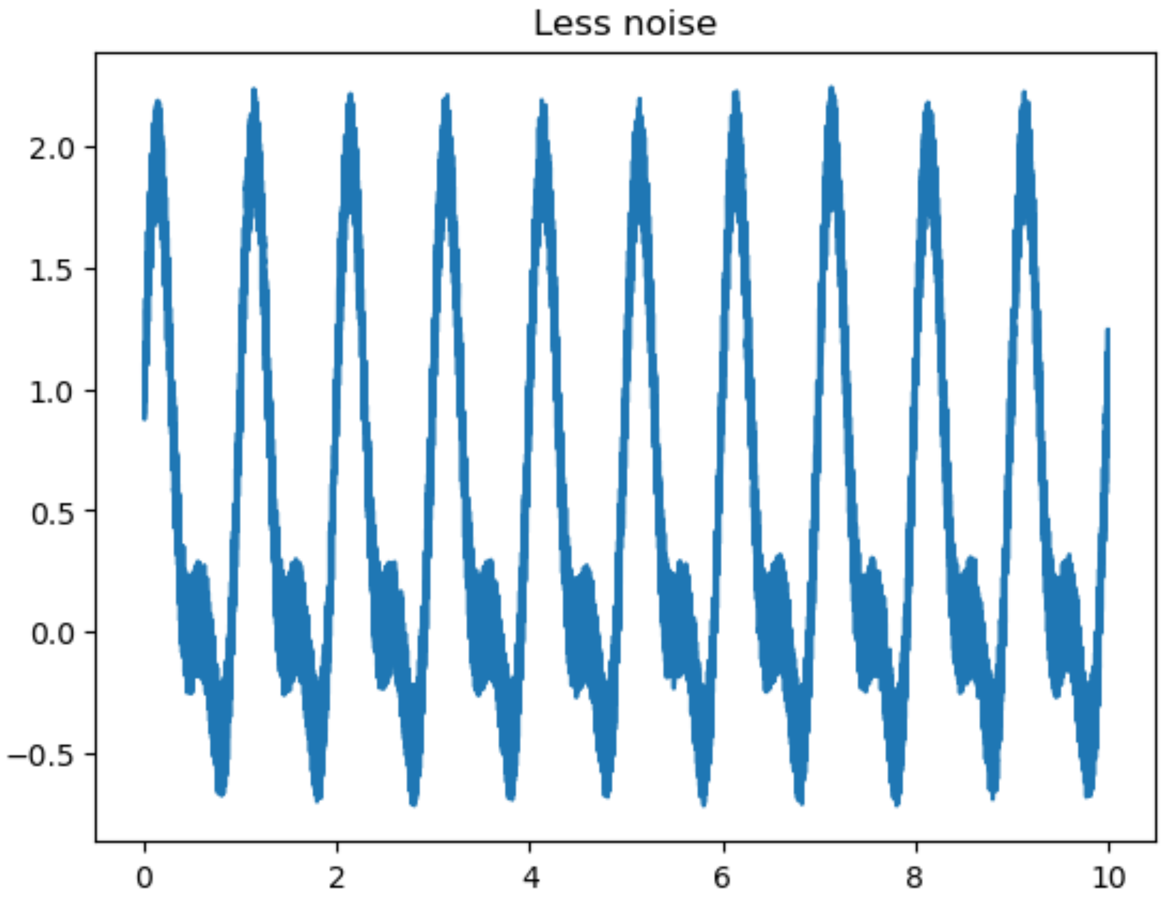


Keeping some noise to check the result

In [ ]:
```
f_noise2 = np.fft.ifft(f_noise2_fft)

plt.plot(t_f, f_noisy)
plt.title("Noisy")
plt.show()

plt.plot(t_f, f_noise2)
plt.title("Less noise")
plt.show()
```

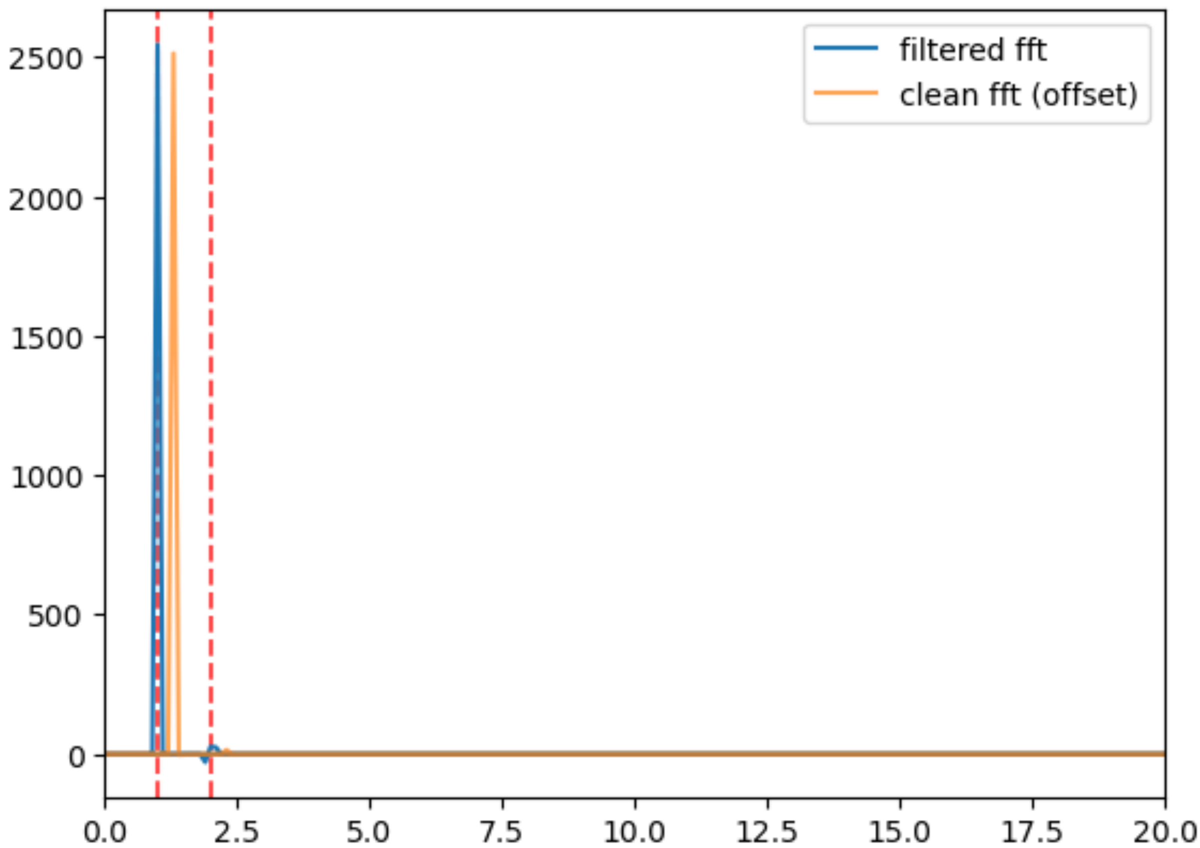



```
/Users/hervesv/opt/anaconda3/lib/python3.9/site-packages/matplotlib/cbook
/__init__.py:1335: ComplexWarning: Casting complex values to real discard
s the imaginary part
  return np.asarray(x, float)
```

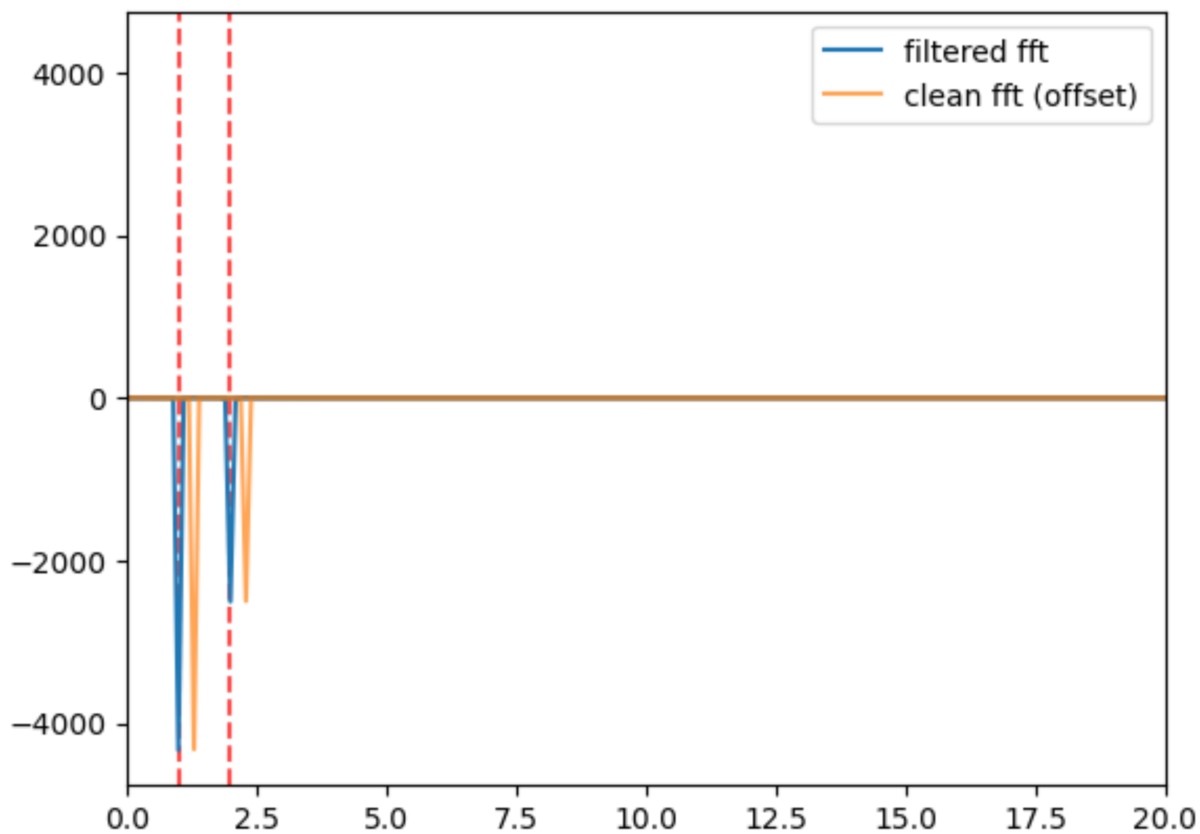


It does look better, now we filter all the noise out by combining lowpass and highpass filters

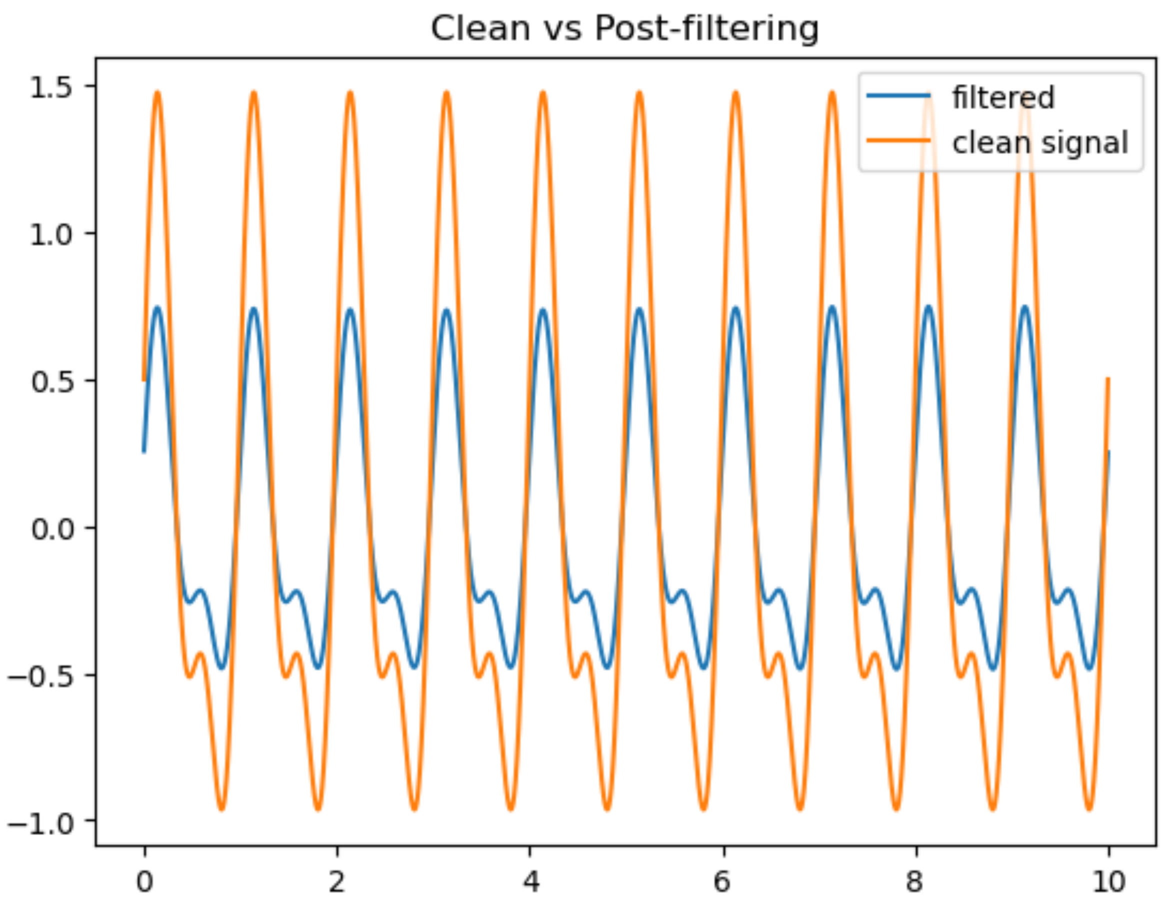
In [ ]:
```
f_nu1_fft = highpass(lowpass(f_noisy_fft, freq_domain, 1.1*nu_1), freq_do
f_nu2_fft = highpass(lowpass(f_noisy_fft, freq_domain, 1.1*nu_2), freq_do

f_denoised_fft = f_nu1_fft + f_nu2_fft

plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)
plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_denoised_fft.real, label="filtered fft")
plt.plot(freq_domain + 0.3*np.ones(len(freq_domain)), f_fft.real, alpha=0
plt.xlim((0, 20))
plt.legend()
plt.show()

plt.axvline(nu_1, c="r", linestyle='dashed', alpha=0.7)

plt.axvline(nu_2, c="r", linestyle='dashed', alpha=0.7)
plt.plot(freq_domain, f_denoised_fft.imag, label="filtered fft")
plt.plot(freq_domain + 0.3*np.ones(len(freq_domain)), f_fft.imag, alpha=0
plt.xlim((0, 20))
plt.legend()
plt.show()


f_denoised = np.fft.ifft(f_denoised_fft, N_sample)
plt.plot(t_f, f_denoised, label="filtered")
plt.plot(t_f, f, label="clean signal")
plt.title("Clean vs Post-filtering")
plt.legend()
plt.show()

plt.plot(t_f, 2*f_denoised, label="2 * filtered", zorder=2)
plt.plot(t_f, f, label="clean signal", alpha=0.7)
plt.title("Clean vs 2*Post-filtering")
plt.legend()
plt.show()
```

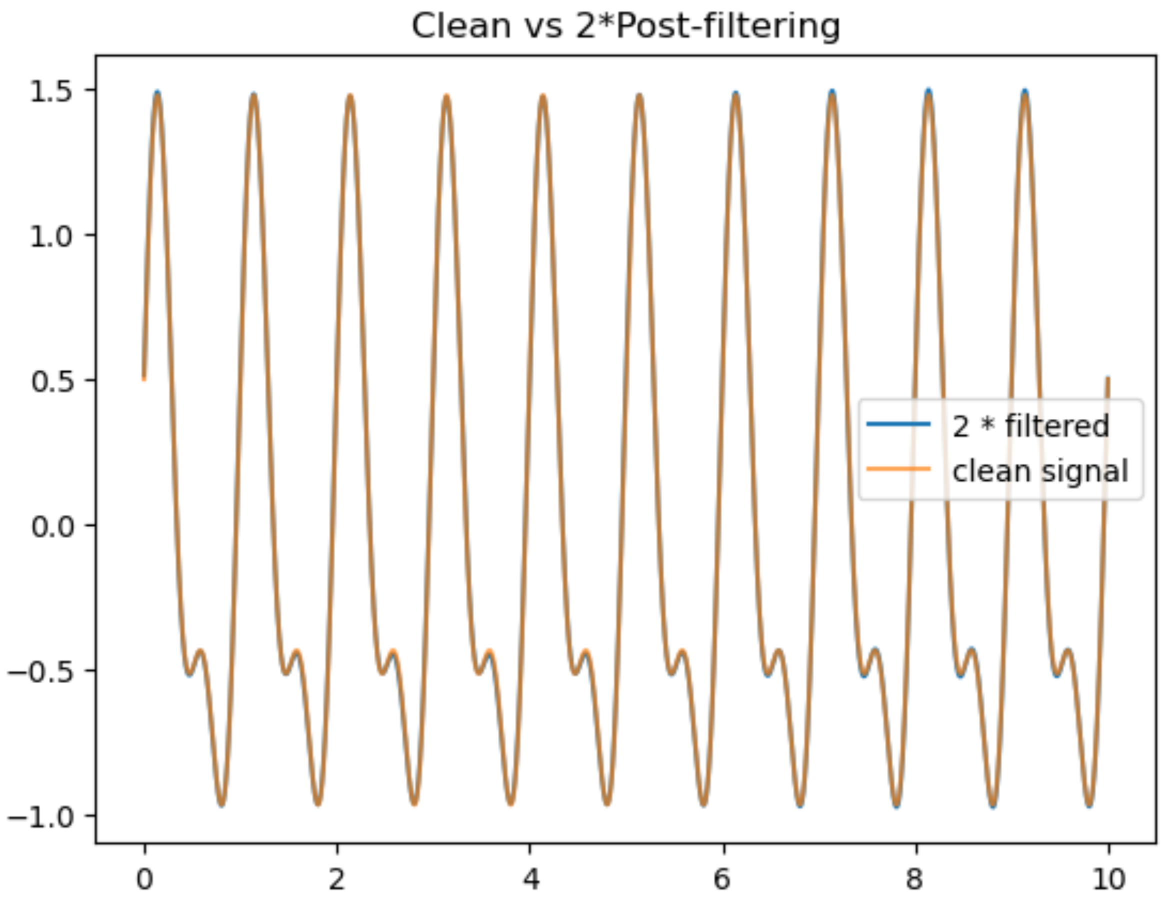


filtered fft
clean fft (offset)
2500
2000
1500
1000
500
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



filtered fft
clean fft (offset)
4000
2000
0
−2000
−4000
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Clean vs Post-filtering
filtered
clean signal
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10


Clean vs 2*Post-filtering
2 * filtered
clean signal
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10

Overall filtering is successful. A factor 2 is missing from the amplitude of the filtered signal. The Fourier transforms are the same though, so I suspect something is happening during the inverse transform

```
In [ ]:
```