In [ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
from scipy.integrate import odeint
import scipy.constants as cste
```

Scalar fields

$$T(x,y) = T_b + T_c \exp\left(-\frac{(x-x_c)^2 + (y-y_c)^2}{\sigma^2}\right) + T_f \exp\left(-\frac{(x-x_f)^2 + (y-\right.$$

In [ ]:
```python
'''
def temp(X, Xc, Xf, sigma, Tb, Tc, Tf):
    x, y = X
    xc, yc = Xc
    xf, yf = Xf
    return Tb + Tc*np.exp(-((x-xc)**2 + (y-yc)**2)/(sigma**2)) \
            + Tf*np.exp(-((x-xf)**2 + (y-yf)**2)/(sigma**2))
'''

def temp(X, Xc, Xf, sigma, Tb, Tc, Tf):
    x, y = X
    xc, yc = Xc
    xf, yf = Xf
    return Tb + Tc*np.exp(-((x-xc)**2+(y-yc)**2)/sigma**2) \
        +Tf*np.exp(-((x-xf)**2+(y-yf)**2)/sigma**2)


'''
def temp(x,y,xc,yc,xf,yf, sigma, Tb, Tc, Tf):
    #x, y = X
    #xc, yc = Xc
    #xf, yf = Xf
    return Tb + Tc*np.exp(-((x-xc)**2+(y-yc)**2)/sigma**2) \
        +Tf*np.exp(-((x-xf)**2+(y-yf)**2)/sigma**2)
'''

Tb = 20
Tc = 100
Tf = -40

Xc = (-2, 2)
Xf = (2, -2)
sigma = 2

xmin, xmax, ymin, ymax = -5,5,-5,5
```

Linspace

In [ ]:
```
Nx, Ny = 5,3
x = np.linspace(xmin, xmax, Nx)
y = np.linspace(ymin, ymax, Ny)
Tp = np.zeros((Ny, Nx)) # Ny rows, Nx cols



for i in range(Nx): # cols
    for j in range(Ny): # rows
        Tp[j, i] = temp((x[i], y[j]), Xc, Xf, sigma, Tb, Tc, Tf)

print(x)
print(y)
print(Tp)
```

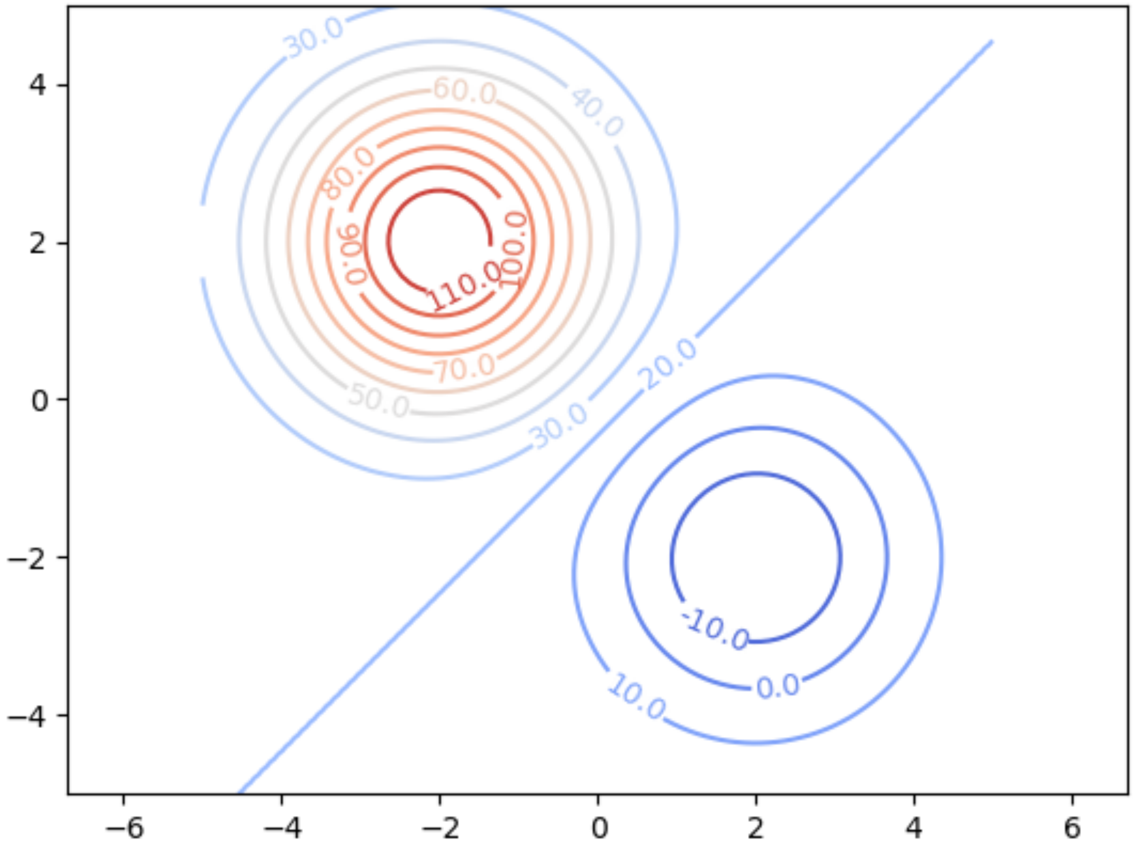
```
[-5.  -2.5  0.   2.5  5. ]
[-5.  0.  5.]
[[20.00003026 19.97376364 18.44920772 16.03946669 19.55564014]
 [23.87735037 54.46593237 28.12011699  6.40922711 18.44920772]
 [21.11089965 29.90133962 23.87735037 20.0665349  20.00003026]]
```

In [ ]:
```
## Meshgrid
```

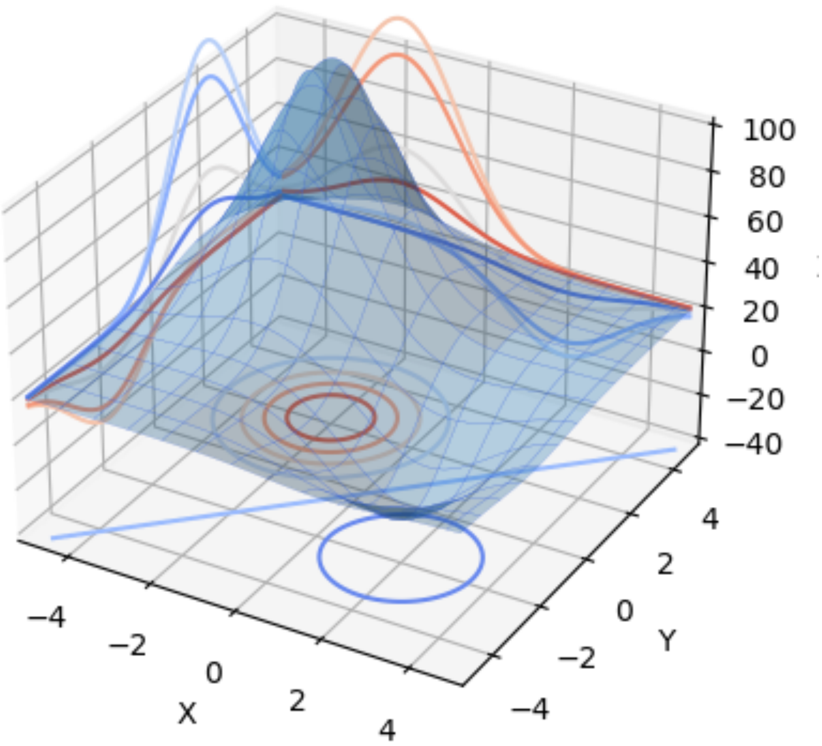
In [ ]:
```
X, Y = np.meshgrid(x, y)
print(X)
print(Y)

# X and Y are both 2*2 matrices.
# Looking up the (j,i) element of both matrices will give us the our desi
# all this to avoid an extra for loop, at the cost of memory usage

#T = temp((X,Y), Xc, Xc, sigma, Tb, Tc, Tf)
T = temp((X,Y), Xc, Xf, sigma, Tb, Tc, Tf)

print(T)
```

```
[[-5.  -2.5  0.   2.5  5. ]
 [-5.  -2.5  0.   2.5  5. ]
 [-5.  -2.5  0.   2.5  5. ]]
[[-5. -5. -5. -5. -5.]
 [ 0.  0.  0.  0.  0.]
 [ 5.  5.  5.  5.  5.]]
[[20.00003026 19.97376364 18.44920772 16.03946669 19.55564014]
 [23.87735037 54.46593237 28.12011699  6.40922711 18.44920772]
 [21.11089965 29.90133962 23.87735037 20.0665349  20.00003026]]
```

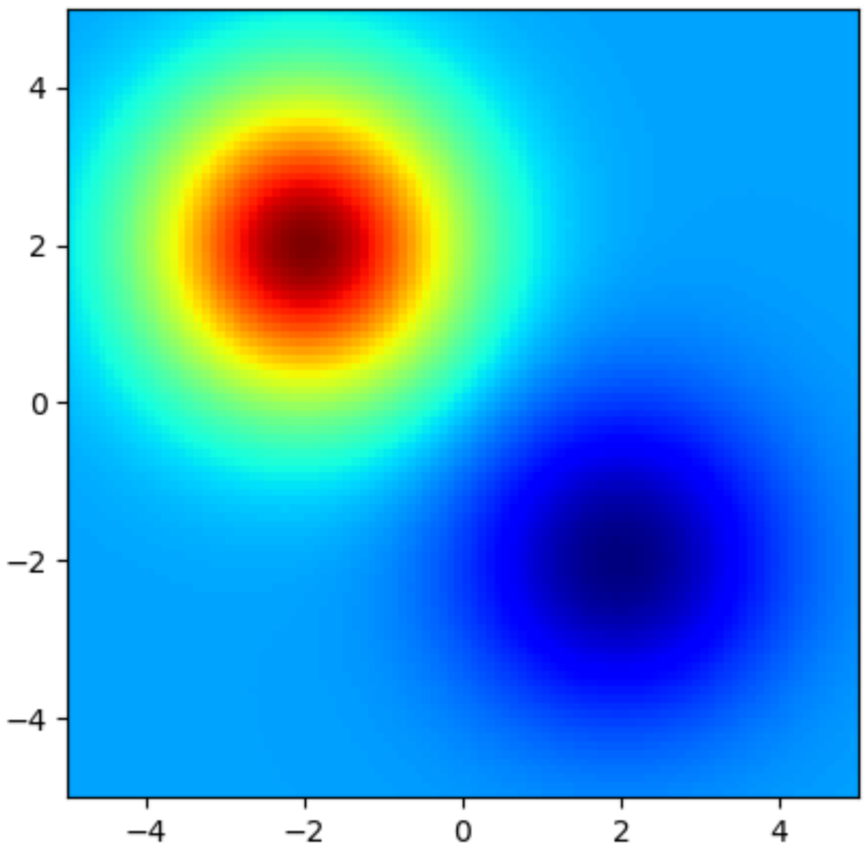
In [ ]:
```
x_t = np.linspace(xmin, xmax, 100)
y_t = np.linspace(ymin, ymax, 100)
X_t, Y_t = np.meshgrid(x_t, y_t)

T_res = temp((X_t,Y_t), Xc, Xf, sigma, Tb, Tc, Tf)

CS = plt.contour(X_t,Y_t, T_res, 16, cmap="coolwarm")
plt.clabel(CS, fontsize=10, fmt='%1.1f')
plt.axis('equal')
plt.show()
```



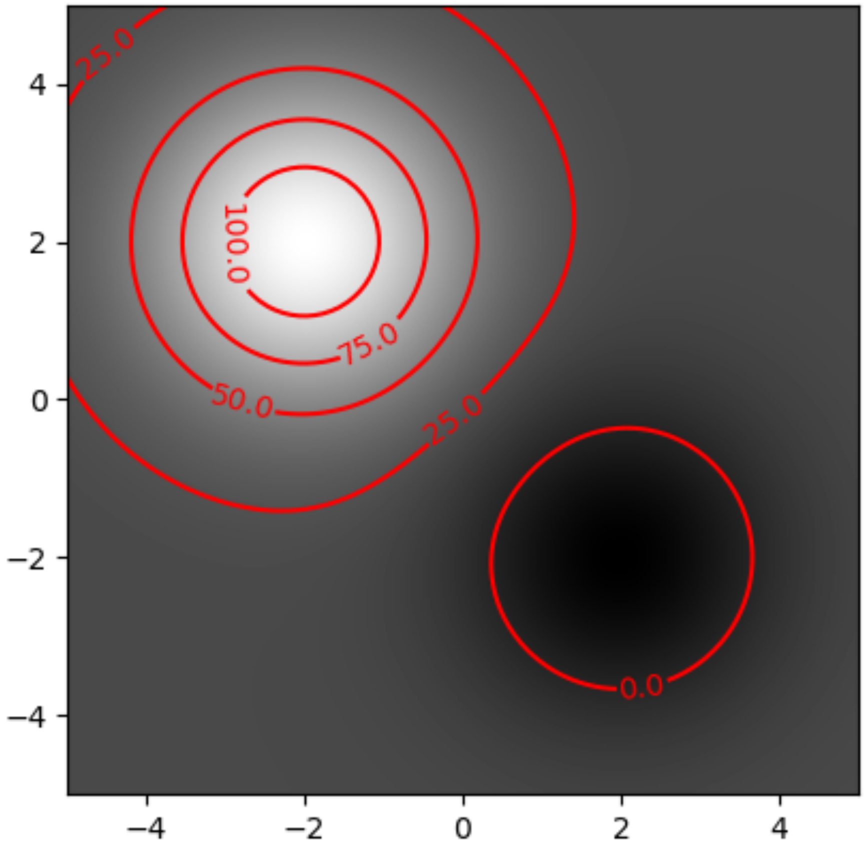
In [ ]:
```
ax = plt.figure().add_subplot(projection='3d')
# Plot the 3D surface
ax.plot_surface(X_t, Y_t, T_res, edgecolor='royalblue', lw=0.1, rstride=8
                alpha=0.3)
# Plot projections of the contours for each dimension.  By choosing offse
# that match the appropriate axes limits, the projected contours will sit
# the 'walls' of the graph.

ax.contour(X_t, Y_t, T_res, zdir='z', offset=Tf, cmap='coolwarm')
ax.contour(X_t, Y_t, T_res, zdir='x', offset=xmin, cmap='coolwarm')
ax.contour(X_t, Y_t, T_res, zdir='y', offset=ymax, cmap='coolwarm')

ax.set(xlim=(xmin, xmax), ylim=(ymin, ymax), zlim=(Tf, Tc),
       xlabel='X', ylabel='Y', zlabel='Z')

plt.show()
```



In [ ]:
```
im = plt.imshow(T_res, origin='lower', extent=(xmin, xmax, ymin, ymax), c
#plt.axis('equal')
plt.show()
```



In [ ]:
```
im = plt.imshow(T_res, interpolation="bilinear", origin="lower", cmap="gr
CS = plt.contour(X_t, Y_t, T_res, 5, colors='r') # 5 is supposedly the ve
plt.clabel(CS, fontsize=10, fmt='%1.1f')
#plt.axis('equal')
plt.show()
```



Numerical integration

$$\int_0^1 f(x)dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} = 0.75$$

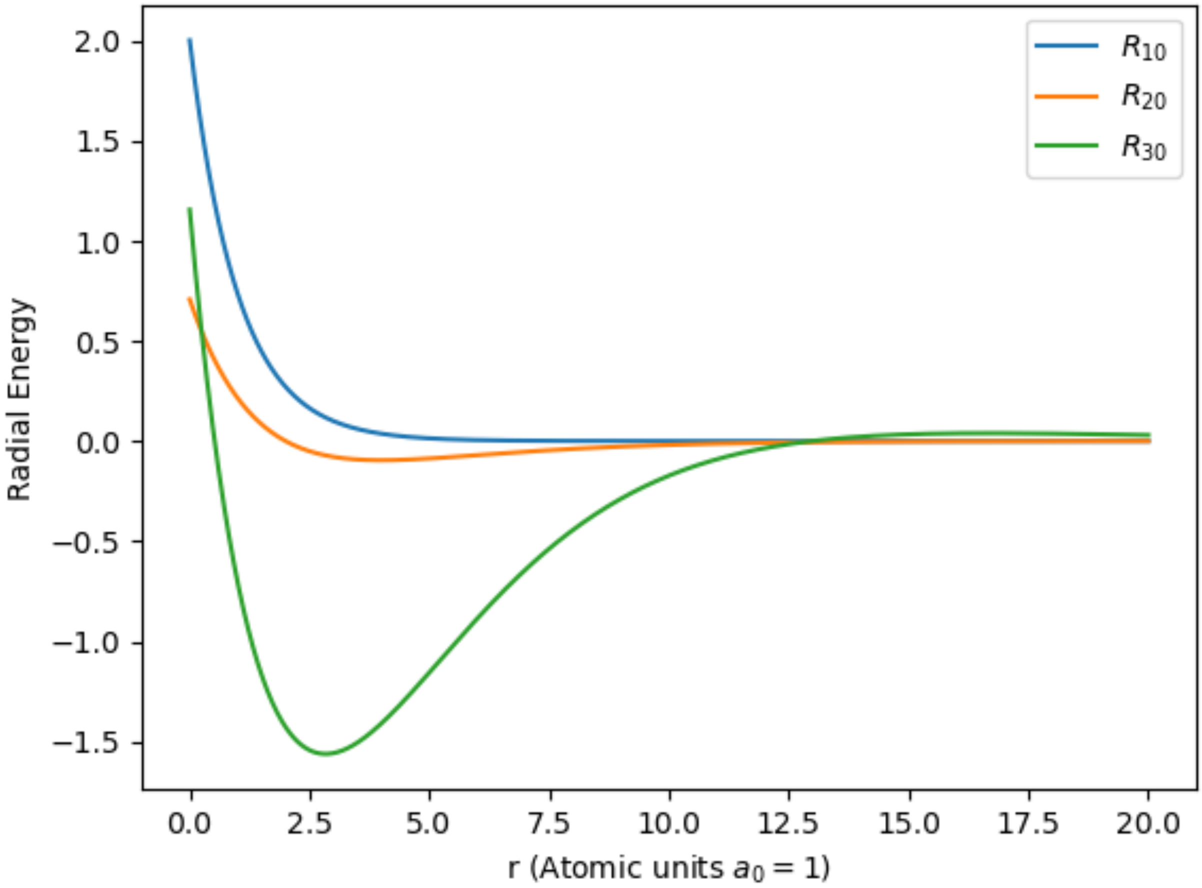
In [ ]:
```
def f(x):
    return x**3 + x

# we can integrate numerically between 0 and 1

res, err = quad(f, 0, 1)
print(res, err)
```

```
0.7499999999999999 8.326672684688672e-15
```

Atomic Orbitals

	partie angulaire	partie radiale
$1s$	$\frac{1}{2\sqrt{\pi}}$	$R_{10}(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$
$2s$	$\frac{1}{2\sqrt{\pi}}$	$R_{20}(r) = \frac{1}{a_0^{3/2}} \frac{1}{\sqrt{2}} \left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0}$
$3s$	$\frac{1}{2\sqrt{\pi}}$	$R_{30}(r) = \frac{1}{a_0^{3/2}} \frac{2}{\sqrt{3}} \left(1 - \frac{2r}{a_0} + \frac{4}{27}\left(\frac{r}{a_0}\right)^2\right) e^{-r/3a_0}$
$2p_x$	$\frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \frac{x}{r}$	$R_{21}(r) = \frac{1}{a_0^{3/2}} \frac{1}{2\sqrt{6}} \frac{r}{a_0} e^{-r/2a_0}$
$2p_z$	$\frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \frac{z}{r}$	$R_{21}(r)$
$3d_{xz}$	$\frac{1}{2}\left(\frac{15}{\pi}\right)^{1/2} \frac{xz}{r^2}$	$R_{32}(r) = \frac{1}{a_0^{3/2}} \frac{4}{81\sqrt{30}} \left(\frac{r}{a_0}\right)^2 e^{-r/3a_0}$
$3d_{z^2}$	$\frac{1}{4}\left(\frac{5}{\pi}\right)^{1/2} \frac{2z^2-x^2-y^2}{r^2}$	$R_{32}(r)$

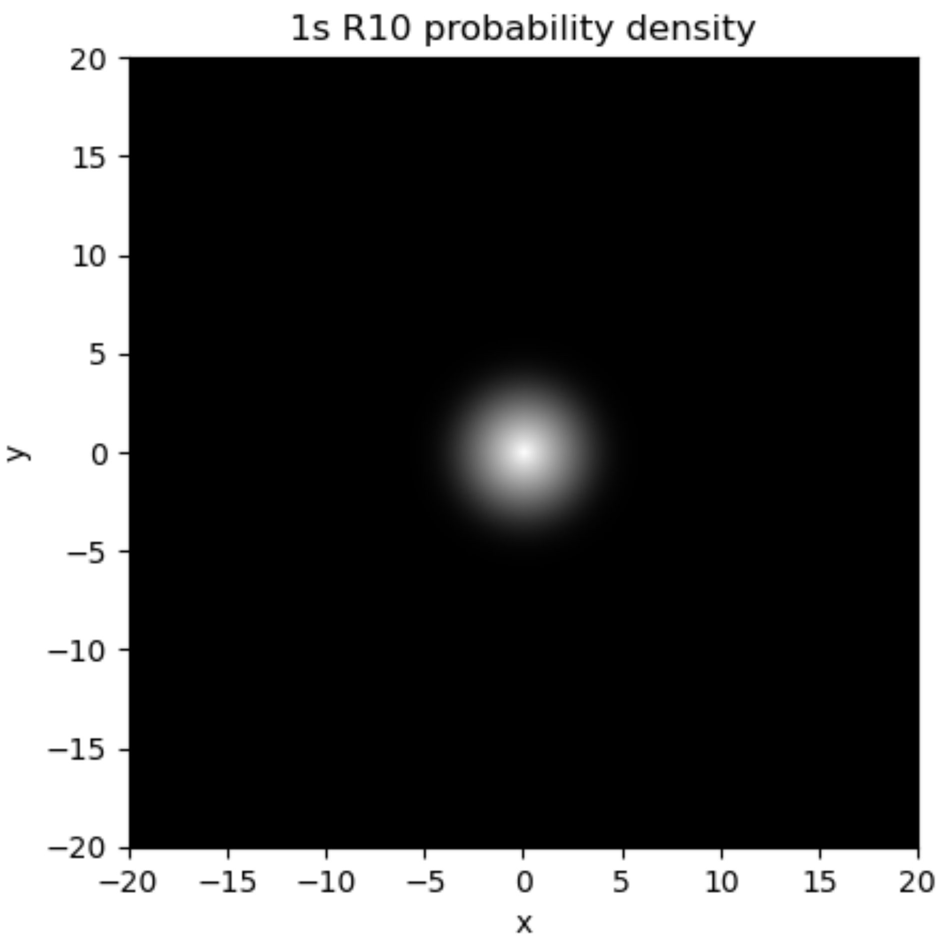
In [ ]:
```python
# working in atomic length units, a0 = 1 (bohr's radius)
r = np.linspace(0, 20, 1000)

R10 = 2*np.exp(-r)
R20 = (1/np.sqrt(2))*(1-r/2)*np.exp(-r/2)
R30 = (2/np.sqrt(3))*(1-2*r + (4/27)*(r**2))*np.exp(-r/3)

plt.plot(r, R10, label=r"$R_{10}$")
plt.plot(r, R20, label=r"$R_{20}$")
plt.plot(r, R30, label=r"$R_{30}$")
plt.legend()

plt.ylabel("Radial Energy")
plt.xlabel("r (Atomic units $a_0 = 1$)")
plt.show()
```



In [ ]:
```
X_r, Z_r = np.meshgrid(np.linspace(-20, 20, 5000), np.linspace(-20, 20, 5
R = np.sqrt(X_r**2 + Z_r**2)

R10_im = 2*np.exp(-R)
R20_im = (1/np.sqrt(2))*(1-R/2)*np.exp(-R/2)
R30_im = (2/np.sqrt(3))*(1-2*R + (4/27)*(R**2))*np.exp(-R/3)
```

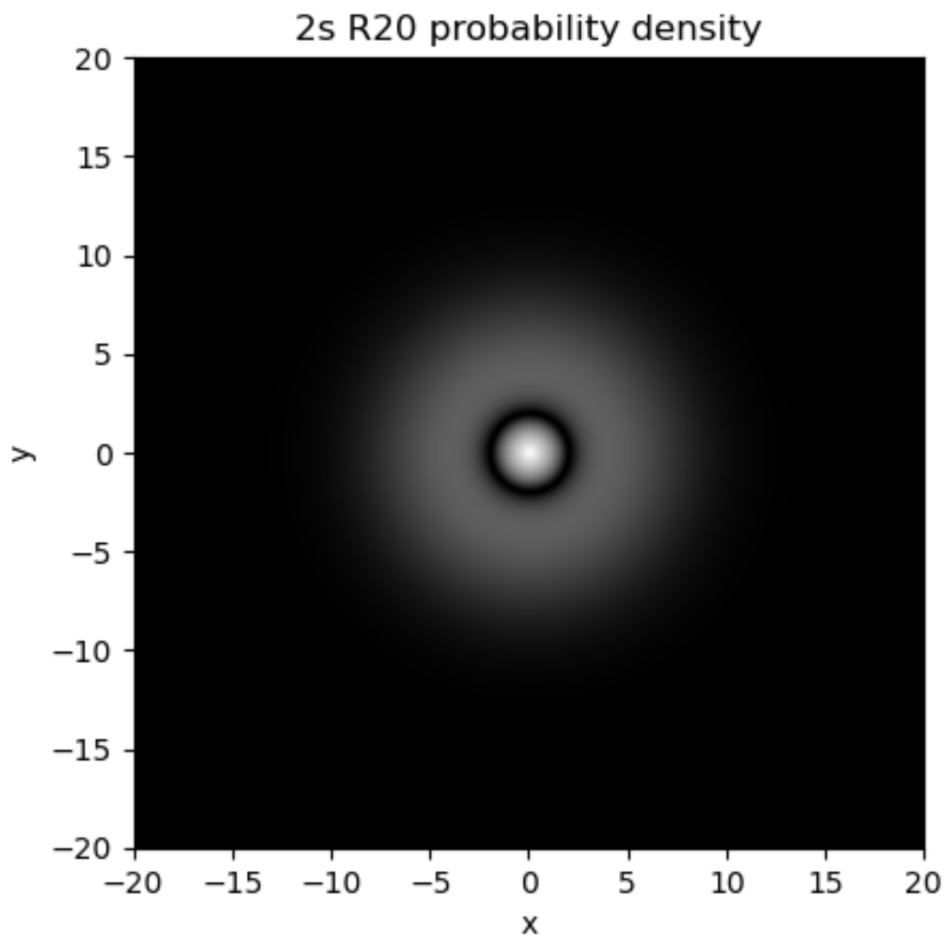
In [ ]:
```
im = plt.imshow(np.log(R10_im**2 + 1e-3), origin="lower", cmap="gray", ex
# probability is proportional to energy squared (Born's rule)
# 1e-3 offset is to adjust the value at which it is black

plt.title("1s R10 probability density")
plt.xlabel("x")
plt.ylabel("y")
```

Out[ ]:
```
Text(0, 0.5, 'y')
```



In [ ]:
```
im = plt.imshow(np.log(R20_im**2 + 1e-3), interpolation="bilinear", origi
plt.title("2s R20 probability density")
plt.xlabel("x")
plt.ylabel("y")
```

Out[ ]:
```
Text(0, 0.5, 'y')
```

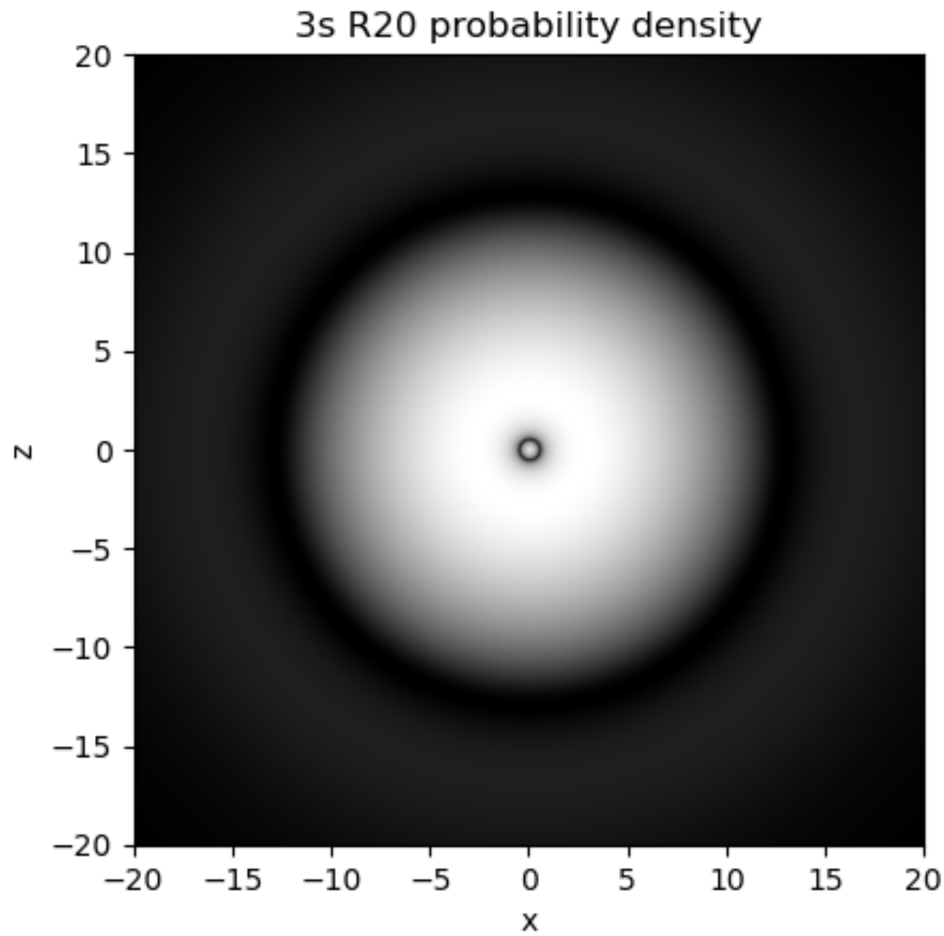


In [ ]:

```
im = plt.imshow(np.log(R30_im**2 + 1e-3), interpolation="bilinear", origi
plt.title("3s R20 probability density")
plt.xlabel("x")
plt.ylabel("z")
```

Out[ ]:

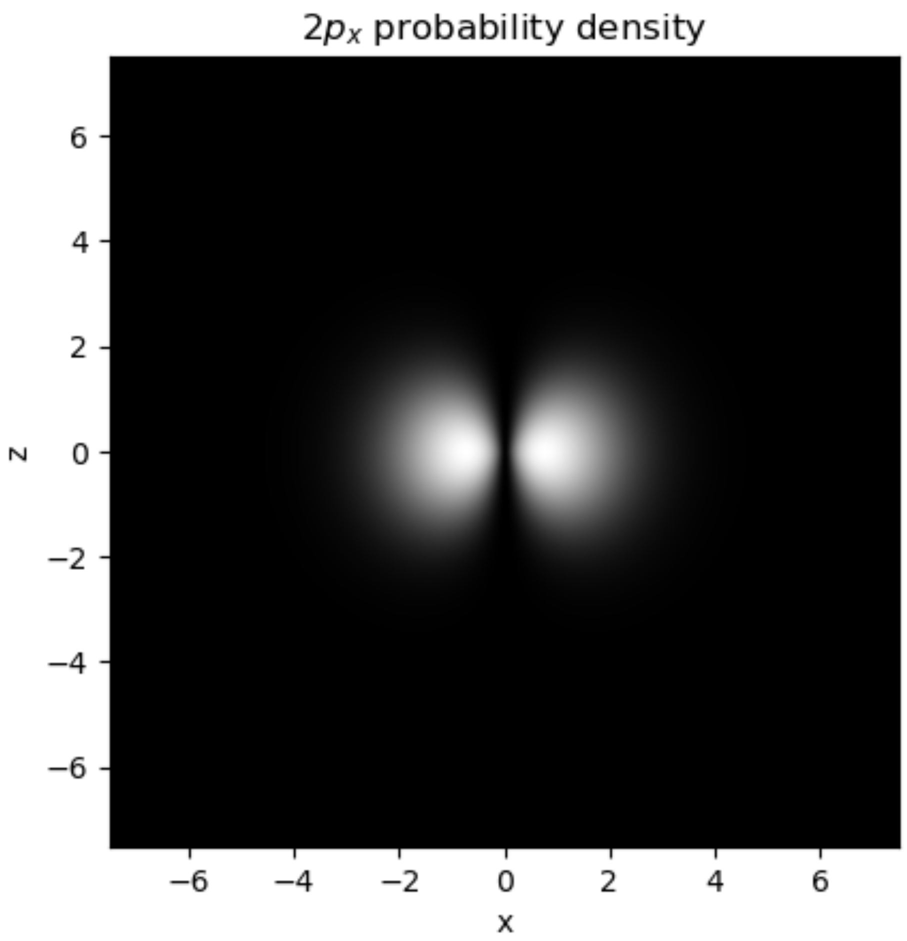
```
Text(0, 0.5, 'z')
```



In [ ]:
```
# 2px

A_px = 1/2*np.sqrt(3/np.pi)*X_r/R
R_px = 1/(2*np.sqrt(6)) * R * np.exp(-R/2)
```

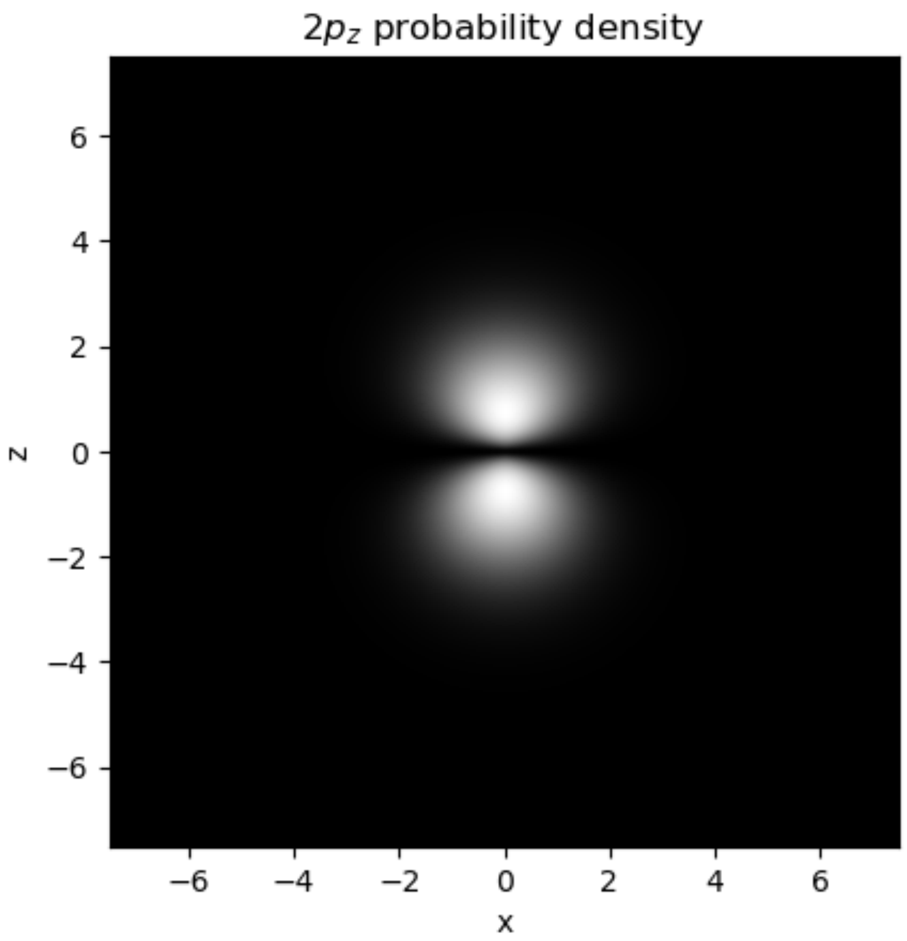
In [ ]:
```
im = plt.imshow(np.log((A_px*R_px)**2 + 1e-3), interpolation="bilinear",
plt.title("$2p_x$ probability density")
plt.xlabel("x")
plt.ylabel("z")
```

Out[ ]:
```
Text(0, 0.5, 'z')
```



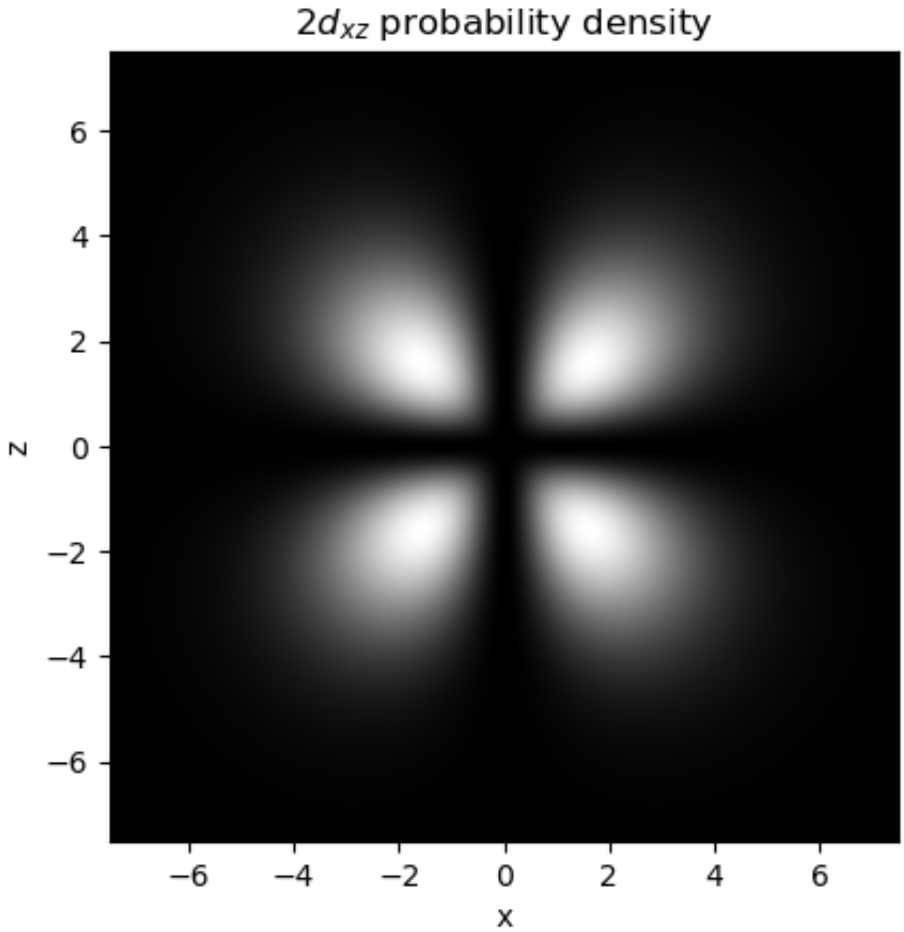
In [ ]:
```
# 2pz

A_pz = 1/2*np.sqrt(3/np.pi)*Z_r/R
R_pz = 1/(2*np.sqrt(6)) * R * np.exp(-R/2)

im = plt.imshow(np.log((A_pz*R_pz)**2 + 1e-3), interpolation="bilinear",
plt.title("$2p_z$ probability density")
plt.xlabel("x")
plt.ylabel("z")
```

Out[ ]:
```
Text(0, 0.5, 'z')
```



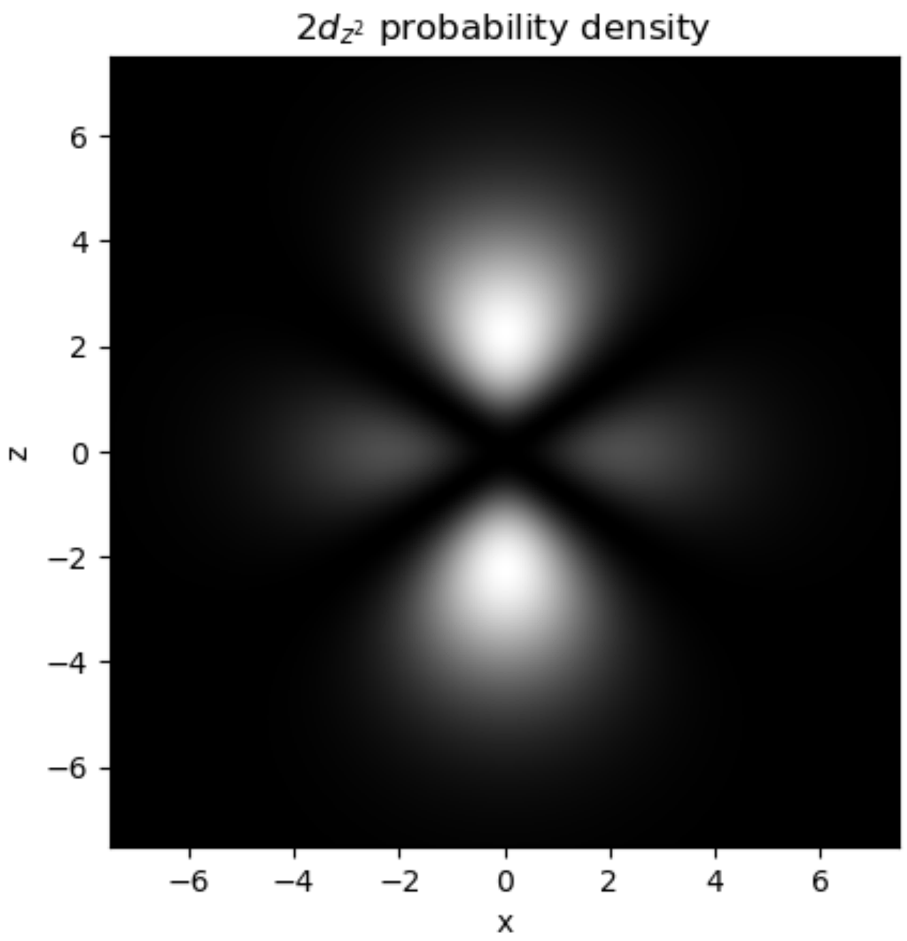
In [ ]:
```
# 2dxz

y = 0
A_dxz = 1/2*np.sqrt(15/np.pi)*X_r*Z_r/R**2
R_dxz = 4/(81*np.sqrt(30)) * R**2 * np.exp(-R/3)

im = plt.imshow(np.log((A_dxz*R_dxz)**2 + 1e-3), interpolation="bilinear"
plt.title("$2d_{xz}$ probability density")
plt.xlabel("x")
plt.ylabel("z")
```

Out[ ]:
```
Text(0, 0.5, 'z')
```



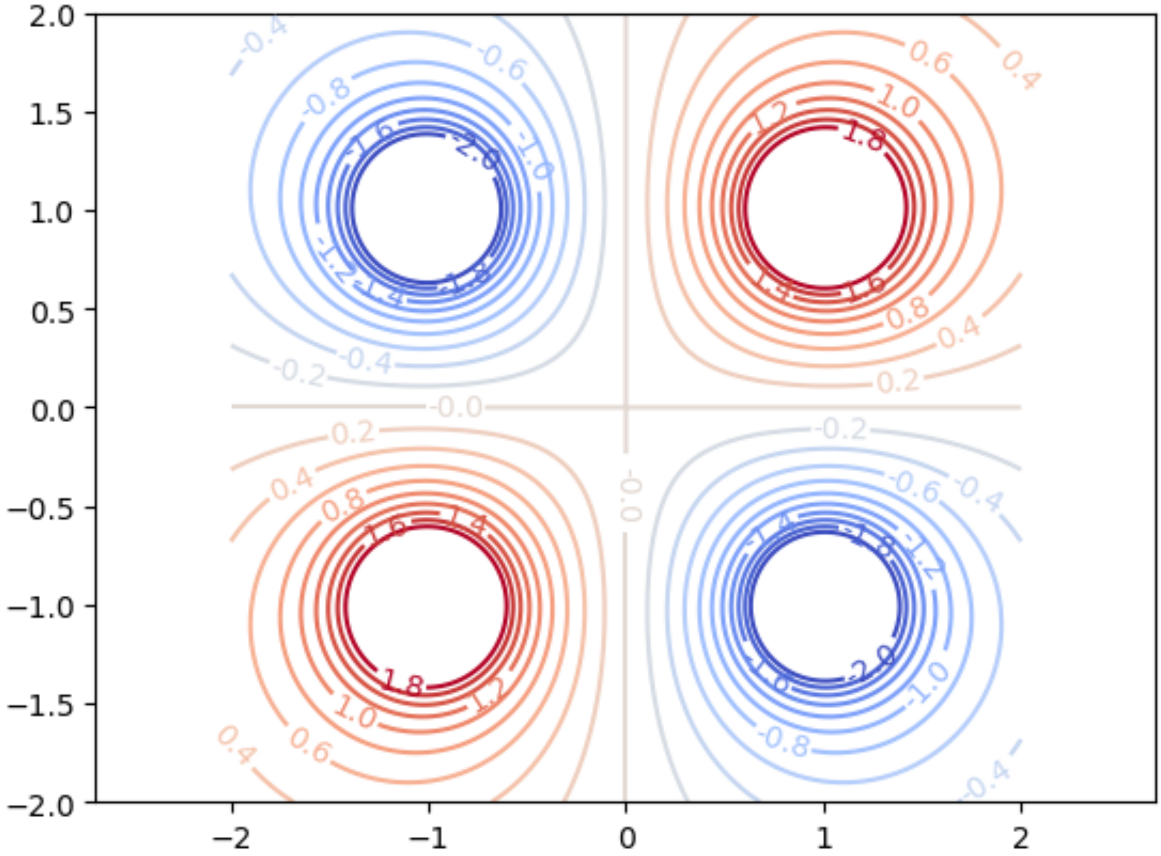
In [ ]:
```
# 2dz2

y = 0
A_dz2 = 0.25*np.sqrt(5/np.pi)*(2*Z_r**2 - X_r**2 - y**2)/R**2
R_dz2 = R_dxz

im = plt.imshow(np.log((A_dz2*R_dz2)**2 + 1e-3), interpolation="bilinear"
plt.title("$2d_{z^2}$ probability density")
plt.xlabel("x")
plt.ylabel("z")
```

Out[ ]:
```
Text(0, 0.5, 'z')
```



In [ ]:

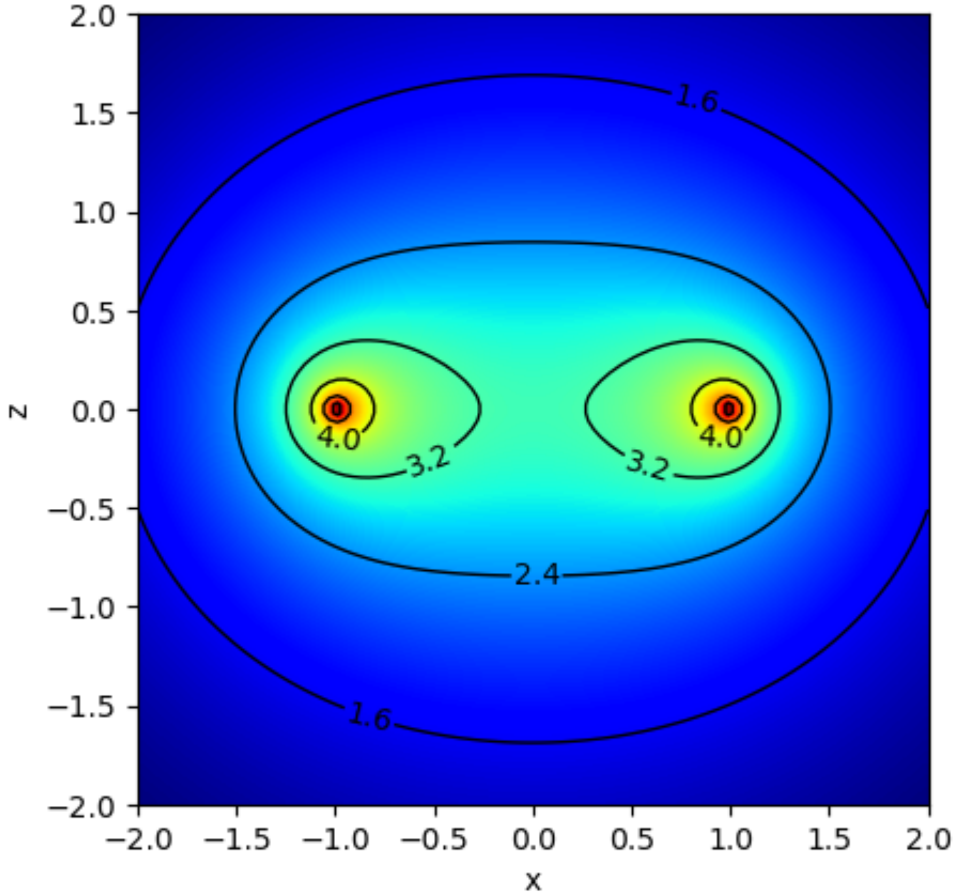
```
x_e = np.linspace(-2, 2, 1000)
y_e = np.linspace(-2, 2, 1000)

X_e, Y_e = np.meshgrid(x_e, y_e)

# (x, y, charge)
charge_pos_sign = np.array([(1, 1, 1), (1, -1, -1), (-1, -1, 1), (-1, 1,
Phi = np.zeros(shape = X_e.shape)
for i in range(len(charge_pos_sign)):
    charge_x = charge_pos_sign[i][0]
    charge_y = charge_pos_sign[i][1]
    charge = charge_pos_sign[i][2]
    Phi += charge/np.sqrt((X_e-charge_x)**2 + (Y_e-charge_y)**2)

# we can explicitely define the contour levels
contour_levels = np.arange(-2.0, 2.0, 0.2)
CS = plt.contour(X_e, Y_e, Phi, contour_levels, cmap="coolwarm")
plt.clabel(CS, fontsize=10, fmt='%1.1f')
plt.axis("equal")
plt.show()
```



Uniformely charged ring

I suppose this is a ring in the Oxy plane, we also only consider the potential in this plane

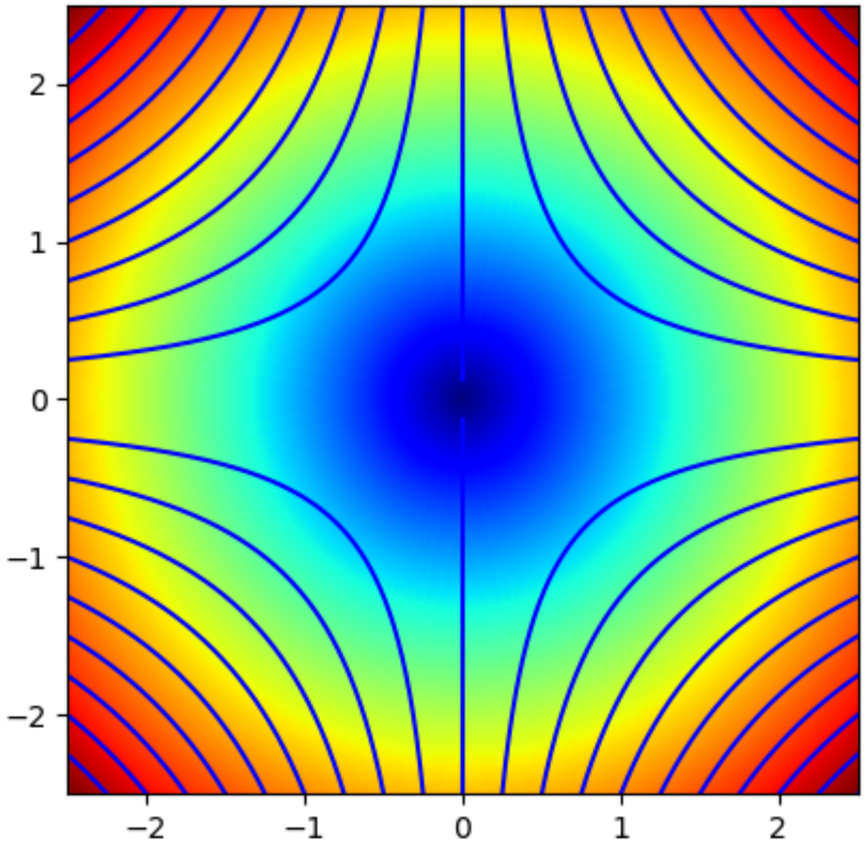
In [ ]:
```
def d_phi(theta, x, z):
    #print(np.shape(x**2))
    #print(np.shape(z**2))
    #print(np.shape(x*np.cos(theta)))
    r = np.sqrt(x**2 + z**2 + 1 - 2*x*np.cos(theta))
    return 1/r

def phi(x, z):
    J, err = quad(d_phi, 0, cste.pi, (x, z))
    return J

phi_vect = np.vectorize(phi)

x_r = np.linspace(-2.0, 2.0, 100)
z_r = np.linspace(-2.0, 2.0, 100)


# we integrate about theta to find the contributions of each charge eleme
# on the ring to the potential field on the (x,z) plane.
# This field lies in the plane and does not depend on theta

X_r, Z_r = np.meshgrid(x_r, z_r)


Phi_ring = phi_vect(X_r, Z_r)

# plotting potential on heatmap then plotting the corresponding contours

plt.rcParams["contour.linewidth"] = 1.0

im = plt.imshow(Phi_ring, interpolation="bilinear", origin="lower", cmap=
CS = plt.contour(X_r, Z_r, Phi_ring, 6, colors='k') # 5 is supposedly the
plt.clabel(CS, fontsize=10, fmt='%1.1f')
plt.xlabel("x")
plt.ylabel("z")
#plt.axis('equal')
plt.show()
```



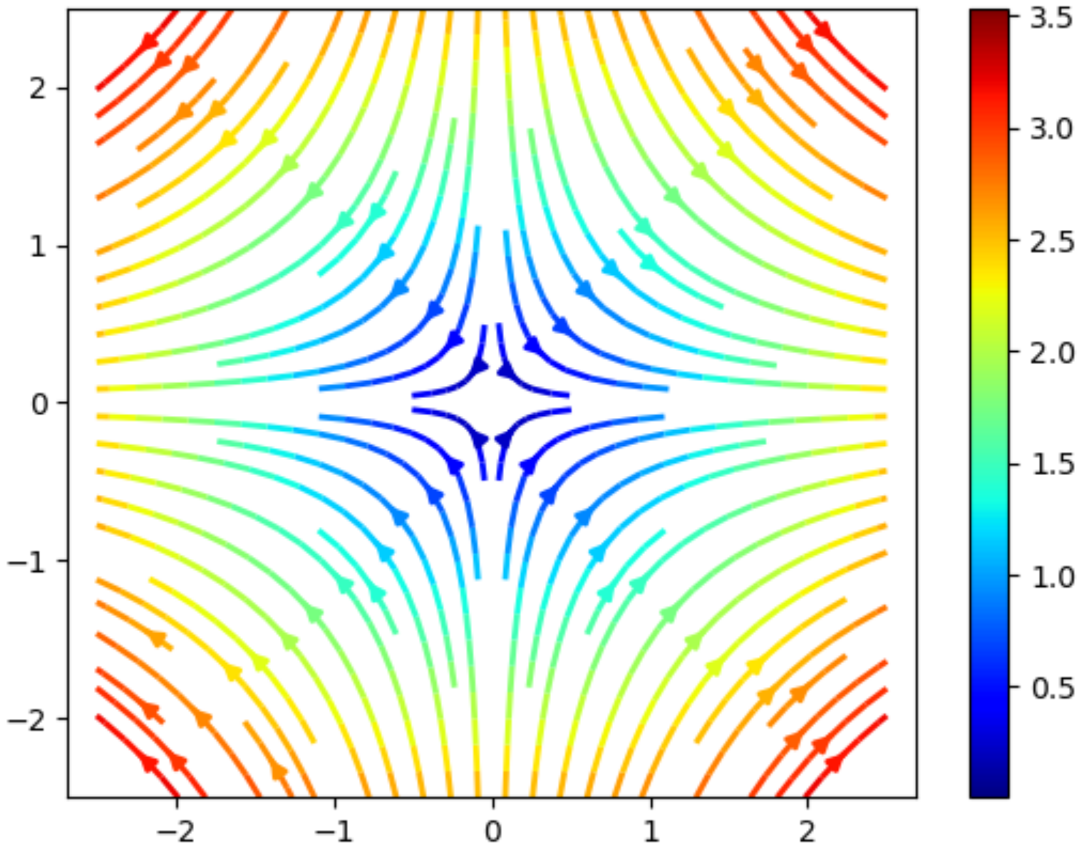
Vector Fields

B.2 Velocity field

In [ ]:
```
def velocity(X, t):
    x, y = X
    # returns [dx/dt, dy/dt]
    return [x, -y]

t = np.linspace(0, 3, 1000)

xmin, xmax, ymin, ymax = -2.5, 2.5, -2.5, 2.5
for x_ini in np.linspace(xmin, xmax, 21):
    X0_1 = [x_ini, ymin]
    X_1 = odeint(velocity, X0_1, t)
    #print(X_1.T)

    x1, y1 = X_1.T
    plt.plot(x1,y1, color="blue")

    X0_2 = [x_ini, ymax]
    X_2 = odeint(velocity, X0_2, t)
    #print(X_2.T)

    x2, y2 = X_2.T
    plt.plot(x2,y2, color="blue")

plt.xlim((xmin, xmax))
plt.ylim((ymin, ymax))


x = np.linspace(xmin, xmax, 1000)
y = np.linspace(ymin, ymax, 1000)

X, Y = np.meshgrid(x, y)
norm = np.sqrt(X**2 + Y**2)

im = plt.imshow(norm, origin='lower',cmap=plt.cm.jet, extent=(-2.5,2.5,-2
#plt.axis('equal')
plt.show()
#plt.plot(x, y)

#plt.show()
```



In [ ]:
```
Vx = X
Vy = -Y
Vnorm = np.sqrt(X**2 + Y**2)

fig0, ax0 = plt.subplots()
strm = ax0.streamplot(X, Y, Vx, Vy, color=Vnorm, linewidth=2, cmap=plt.cm
fig0.colorbar(strm.lines)
plt.axis('equal')
plt.show()
```



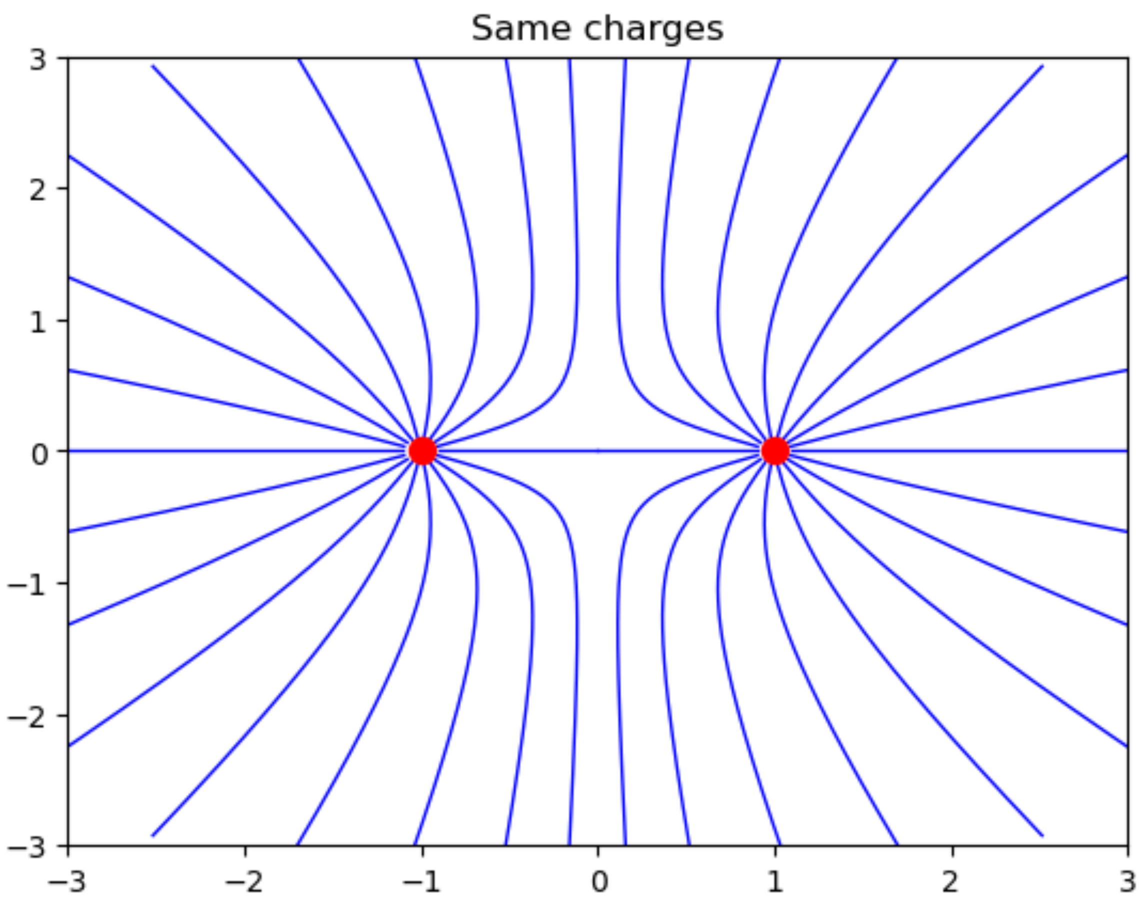
Fields in Electromagnetism

Electrostatics

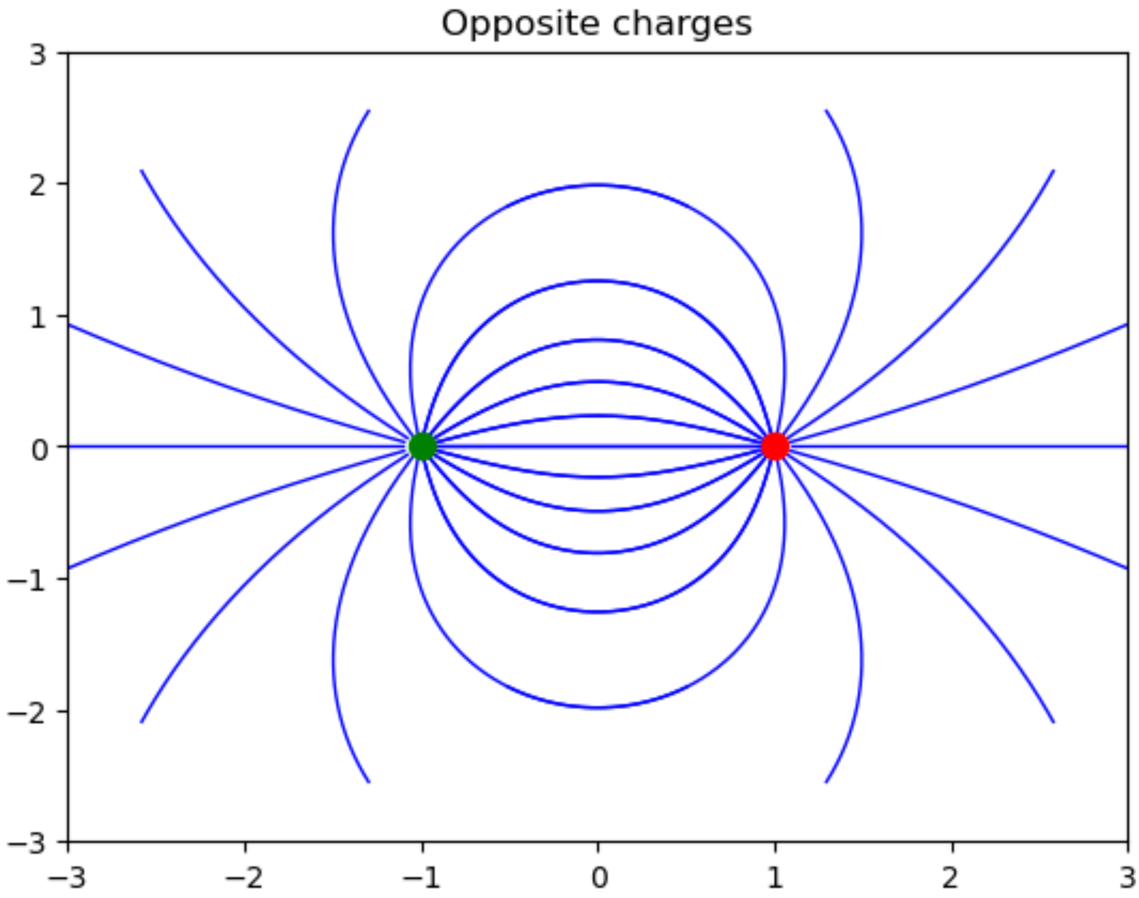
By manual integration

In [ ]:
```
def e_field_lines(X, t, X_A, X_B, q_a, q_b):
    x, y = X
    xa, ya = X_A
    xb, yb = X_B

    # prevents divergence if ra, rb too small
    r0 = 0.1

    ra = np.sqrt((x-xa)**2 + (y-ya)**2)
    rb = np.sqrt((x-xb)**2 + (y-yb)**2)

    dx = q_a*(x-xa)/(ra+r0)**3 + q_b*(x-xb)/(rb+r0)**3
    dy = q_a*(y-ya)/(ra+r0)**3 + q_b*(y-yb)/(rb+r0)**3

    return [dx, dy]

u = np.linspace(0, 10, 1000) # curvilinear abssisa (since our equations a


x_A, y_A = 1., 0.
x_B, y_B = -1., 0.
```

```
x_bounds = (-3, 3)
y_bounds = (-3, 3)

n_lines = 18 # fields lines per charge
r0 = 0.1

CI_A = np.zeros((n_lines, 2))
CI_B = np.zeros((n_lines, 2))


theta_step = 2*np.pi / n_lines
for i in range(0, n_lines):
    CI_A[i][0] = r0*np.cos(theta_step*i) + x_A
    CI_A[i][1] = r0*np.sin(theta_step*i) + y_A

for i in range(0, n_lines):
    CI_B[i][0] = r0*np.cos(theta_step*i) + x_B
    CI_B[i][1] = r0*np.sin(theta_step*i) + y_B

plt.scatter(x_A, y_A, c="r", zorder=5, s=75)
for X_ini in CI_A:
    X = odeint(e_field_lines, X_ini, u, args=((x_A, y_A), (x_B, y_B), 1,

    x, y = X.T
    plt.plot(x,y, color="blue", linewidth=1)

plt.scatter(x_B, y_B, c="r", zorder=5, s=75)
for X_ini in CI_B:
    X = odeint(e_field_lines, X_ini, u, args=((x_A, y_A), (x_B, y_B), 1,

    x, y = X.T
    plt.plot(x,y, color="blue", linewidth=1)

plt.xlim(x_bounds)
plt.ylim(y_bounds)

plt.title("Same charges")
plt.show()
```

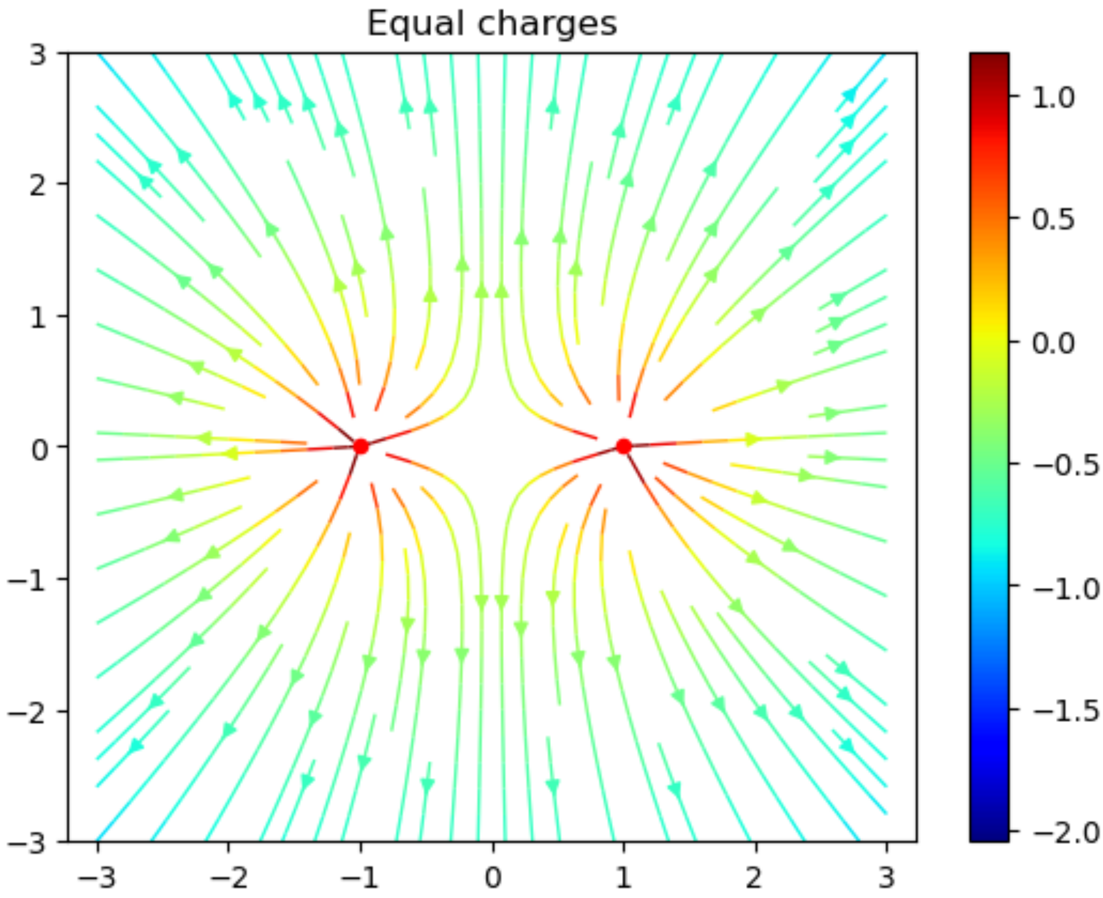


In [ ]:
```
plt.scatter(x_A, y_A, c="r", zorder=5, s=75)
for X_ini in CI_A:
    X = odeint(e_field_lines, X_ini, u, args=((x_A, y_A), (x_B, y_B), 1,

    x, y = X.T
    plt.plot(x,y, color="blue", linewidth=1)

plt.scatter(x_B, y_B, c="green", zorder=5, s=75)
for X_ini in CI_B:
    # need to invert abssisa because field goes inwards for negative char
    # we want to travel the streamline in the opposite direction
    X = odeint(e_field_lines, X_ini, -u, args=((x_A, y_A), (x_B, y_B), 1,

    x, y = X.T
    plt.plot(x,y, color="blue", linewidth=1)


plt.xlim(x_bounds)
plt.ylim(y_bounds)

plt.title("Opposite charges")
plt.show()
```

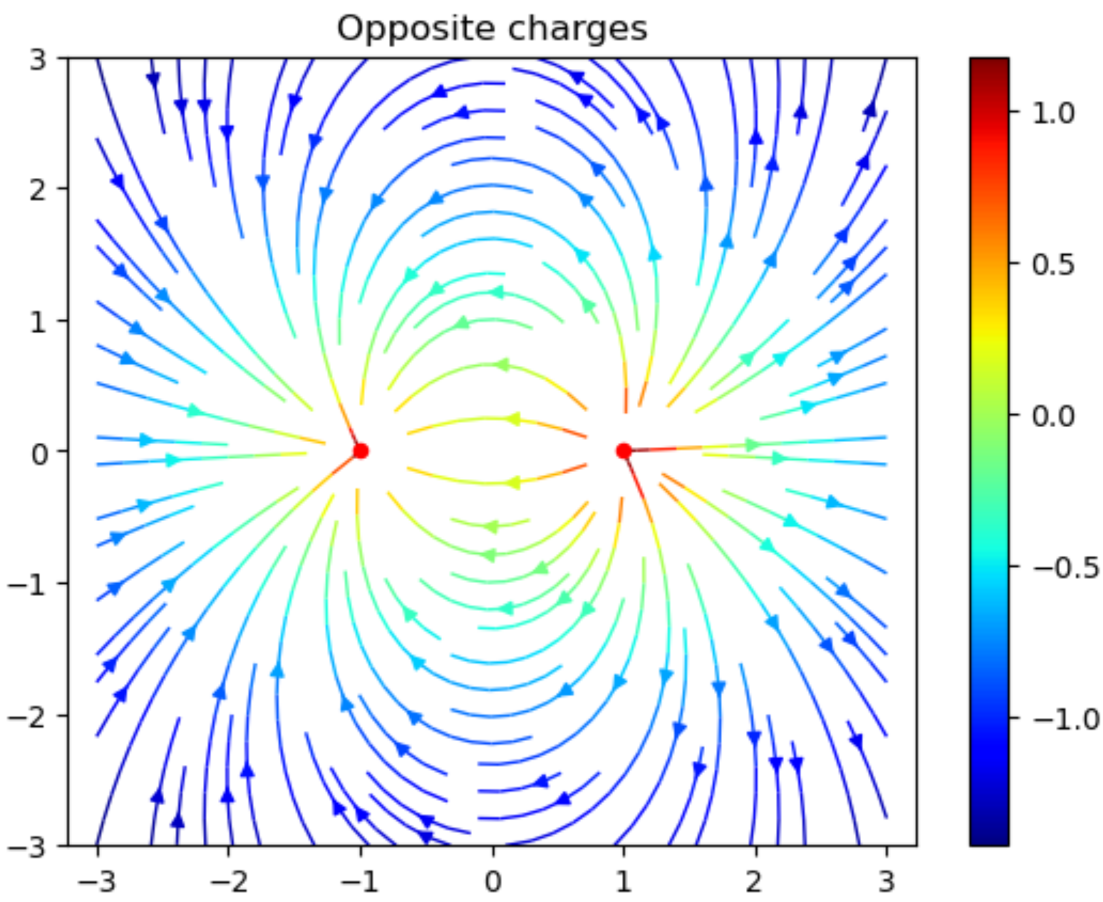


Using streamlines

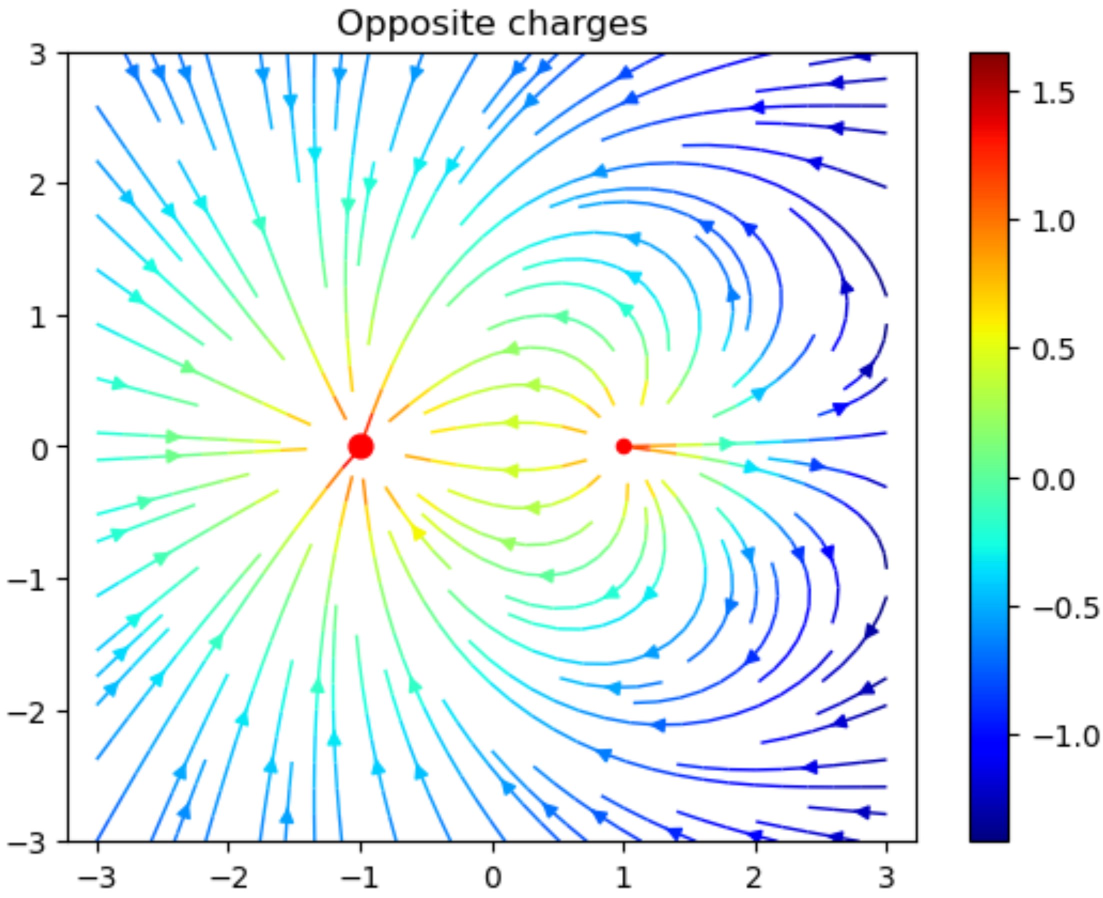
In [ ]:
```
e_field_vect = np.vectorize(e_field_lines)

x = np.linspace(x_bounds[0], x_bounds[1], 1000)
y = np.linspace(y_bounds[0], y_bounds[1], 1000)

X, Y = np.meshgrid(x,y)


q_A, q_B = 1, 1






R_A = np.sqrt((X-x_A)**2 + (Y-y_A)**2)
R_B = np.sqrt((X-x_B)**2 + (Y-y_B)**2)
E_y = q_A*(Y-y_A)/(R_A+r0)**3 + q_B*(Y-y_B)/(R_B+r0)**3
E_x= q_A*(X-x_A)/(R_A+r0)**3 + q_B*(X-x_B)/(R_B+r0)**3
#Ex, Ey = e_field_vect([X, Y], t, (x_A, y_A), (x_B, y_B), 1, 1)

E_norm = np.sqrt(E_x**2 + E_y**2)#q_A/(R_A)**2 + q_B/(R_B)**2

fig0, ax0 = plt.subplots()
strm = ax0.streamplot(X, Y, E_x, E_y, color=np.log10(E_norm), linewidth=1
fig0.colorbar(strm.lines)
plt.axis('equal')

plt.title("Equal charges")

plt.scatter(x_A, y_A, c="r", zorder=5, s=20)
plt.scatter(x_B, y_B, c="r", zorder=5, s=20)
plt.show()
```



In [ ]:
```
q_A, q_B = 1, -1


R_A = np.sqrt((X-x_A)**2 + (Y-y_A)**2)
R_B = np.sqrt((X-x_B)**2 + (Y-y_B)**2)
E_y = q_A*(Y-y_A)/(R_A+r0)**3 + q_B*(Y-y_B)/(R_B+r0)**3
E_x= q_A*(X-x_A)/(R_A+r0)**3 + q_B*(X-x_B)/(R_B+r0)**3
#Ex, Ey = e_field_vect([X, Y], t, (x_A, y_A), (x_B, y_B), 1, 1)

E_norm = np.sqrt(E_x**2 + E_y**2)#q_A/(R_A)**2 + q_B/(R_B)**2

fig0, ax0 = plt.subplots()
strm = ax0.streamplot(X, Y, E_x, E_y, color=np.log10(E_norm), linewidth=1
fig0.colorbar(strm.lines)
plt.axis('equal')

plt.title("Opposite charges")

plt.scatter(x_A, y_A, c="r", zorder=5, s=20)
plt.scatter(x_B, y_B, c="r", zorder=5, s=20)
plt.show()
```



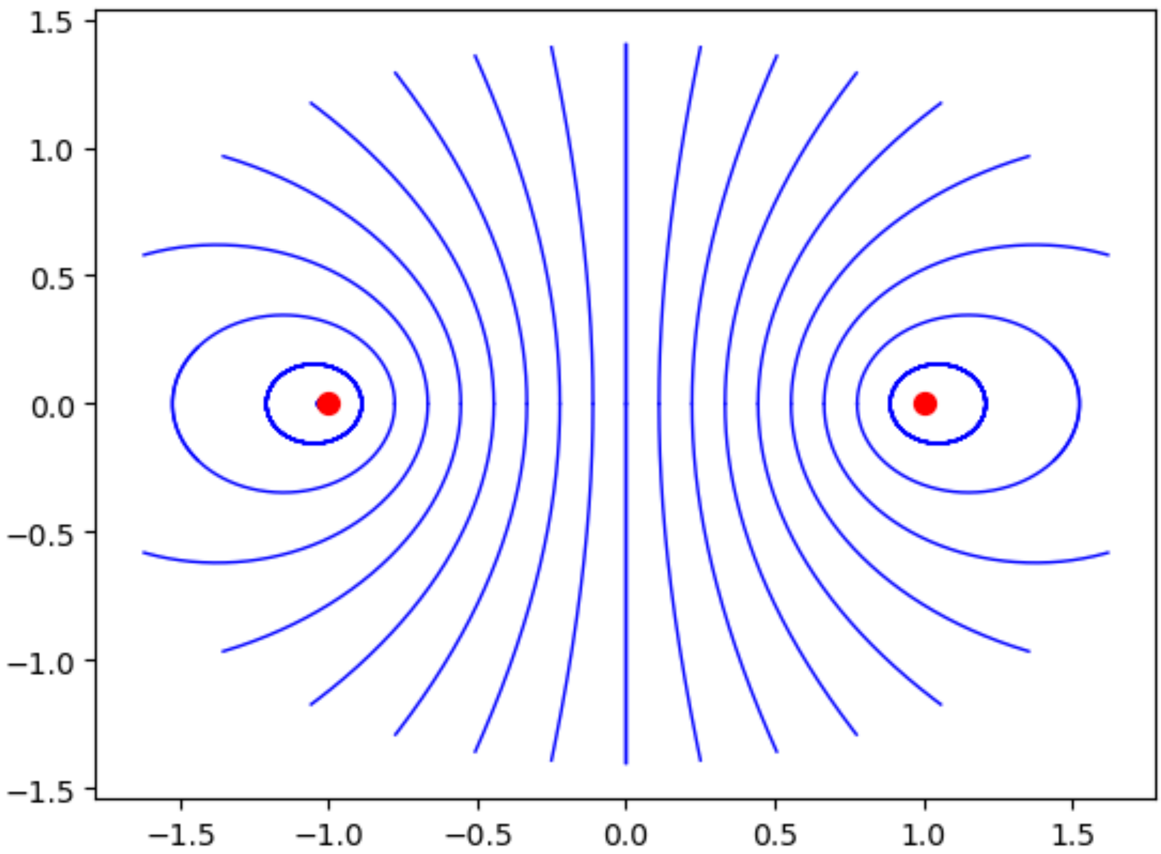
In [ ]:
```
q_A, q_B = 1, -3


R_A = np.sqrt((X-x_A)**2 + (Y-y_A)**2)
R_B = np.sqrt((X-x_B)**2 + (Y-y_B)**2)
E_y = q_A*(Y-y_A)/(R_A+r0)**3 + q_B*(Y-y_B)/(R_B+r0)**3
E_x= q_A*(X-x_A)/(R_A+r0)**3 + q_B*(X-x_B)/(R_B+r0)**3
#Ex, Ey = e_field_vect([X, Y], t, (x_A, y_A), (x_B, y_B), 1, 1)

E_norm = np.sqrt(E_x**2 + E_y**2)#q_A/(R_A)**2 + q_B/(R_B)**2

fig0, ax0 = plt.subplots()
strm = ax0.streamplot(X, Y, E_x, E_y, color=np.log10(E_norm), linewidth=1
fig0.colorbar(strm.lines)
plt.axis('equal')

plt.title("Opposite charges")

plt.scatter(x_A, y_A, c="r", zorder=5, s=20)
plt.scatter(x_B, y_B, c="r", zorder=5, s=60)
plt.show()
```



Magnetostatics

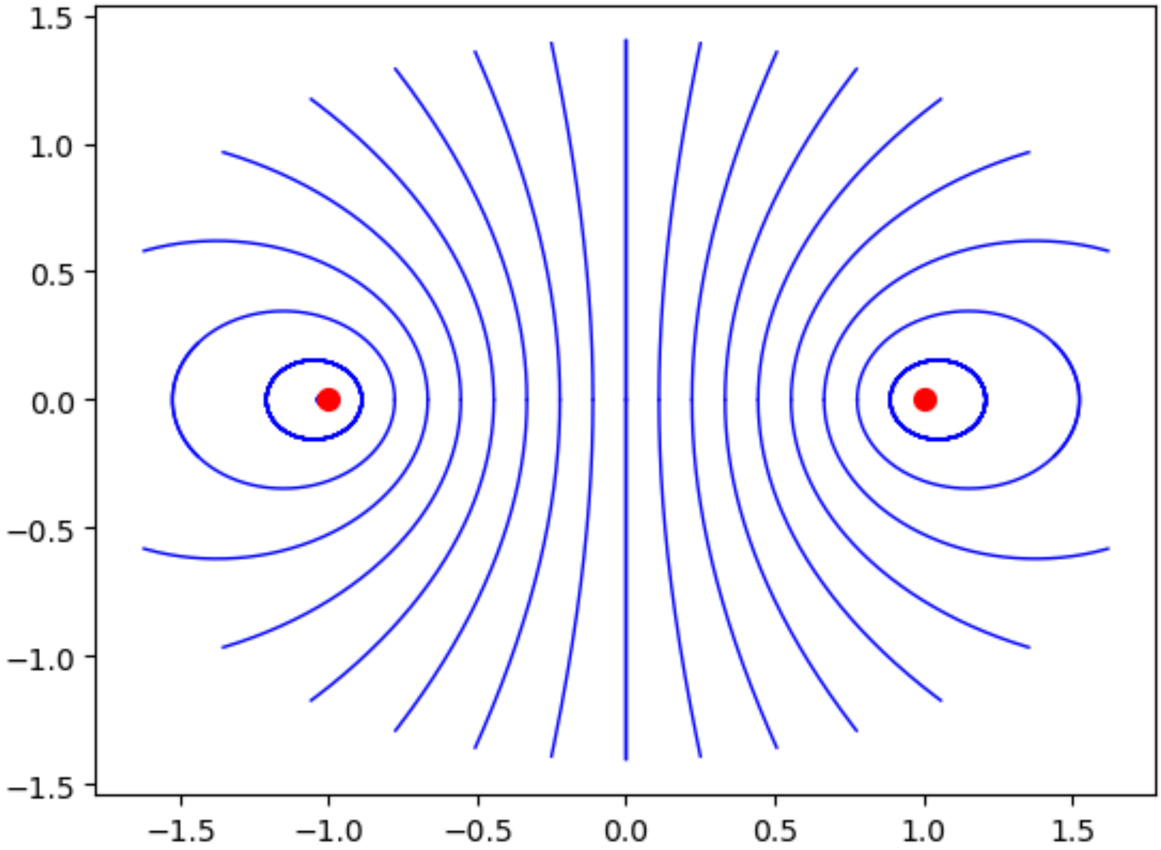
In [ ]:
```
def dBx(theta, x, z, a):
        return z*np.cos(theta)/(x**2 + z**2 + a**2 - 2*a*x*np.cos(theta)

def dBz(theta, x, z, a):
    return (a - x*np.cos(theta))/(x**2 + z**2 + a**2 - 2*a*x*np.cos(theta


def mag_field_lines(X, t, a, e):
    x, z = X

    Bx, err = quad(dBx, 0, np.pi, (x, z, a))
    Bz, err = quad(dBz, 0, np.pi, (x, z, a))

    return [Bx, Bz]

a = 1
u_mag = np.linspace(0, 1, 1000)
CI = np.array([[x, 0] for x in np.linspace(0, 1, 10)])
#print(len(CI))
sols = []
plt.scatter(a, 0, c="r", zorder=5, s=50)
plt.scatter(-a, 0, c="r", zorder=5, s=50)
print(sols)
c = 0
for X_ini in CI:
    X = odeint(mag_field_lines, X_ini, u_mag, args=(a, 0))
    x, y = X.T
    sub = (x, y)
    sols.append(sub)

    #print(len(x), len(y))
    #sols[c][0] = x
    #sols[c][1] = y
    plt.plot(x,y, color="blue", linewidth=1)

    plt.plot(x, -y, color="blue", linewidth=1)

    plt.plot(-x, y, color="blue", linewidth=1)

    plt.plot(-x, -y, color="blue", linewidth=1)

    c += 1
plt.show()
```

```
[]
```



In [ ]:

```
plt.scatter(a, 0, c="r", zorder=5, s=50)
plt.scatter(-a, 0, c="r", zorder=5, s=50)
for sol in sols:

    x, y = sol
    plt.plot(x,y, color="blue", linewidth=1)

    plt.plot(x, -y, color="blue", linewidth=1)

    plt.plot(-x, y, color="blue", linewidth=1)

    plt.plot(-x, -y, color="blue", linewidth=1)
```


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5