Additional project

Fitting a disk distribution to a galaxy rotation curve, perhaps adding DM component.

Take the rotation curve of ngc3198 (moodle), assuming ngc3198 to be at a distance of 13Mpc and assume that the mass distribution can be characterized as the sum of a Plummer potential (Eq 2.44a) for the dark matter and a Kuzmin disk (Eq 2.68a) from the Potential Theory link.

The circular velocites are as below

Plummer : $v_c^2(r) = r^2 \, G \, M_P \, / \, (r^2 + b^2)^{3/2}$

Kuzmin disk : $v_c^2(r) = r^2 \, G \, M_K \, / \, (r^2 + (a+z)^2)^{3/2}$

M_P and M_K are respectively the halo and disk masses.

b and a are size parameters for respectively the halo (Plummer) and disk (Kuzmin)

You can start by fitting the beginning (rising part) of the rotation curve with only the disk potential. Then add Dark Matter (Plummer) as necessary.

Assume that the velocities are at the center of the plane of the galaxy (z=0)

PROJECTS

project 1 (Coma)

project 1 (A496)

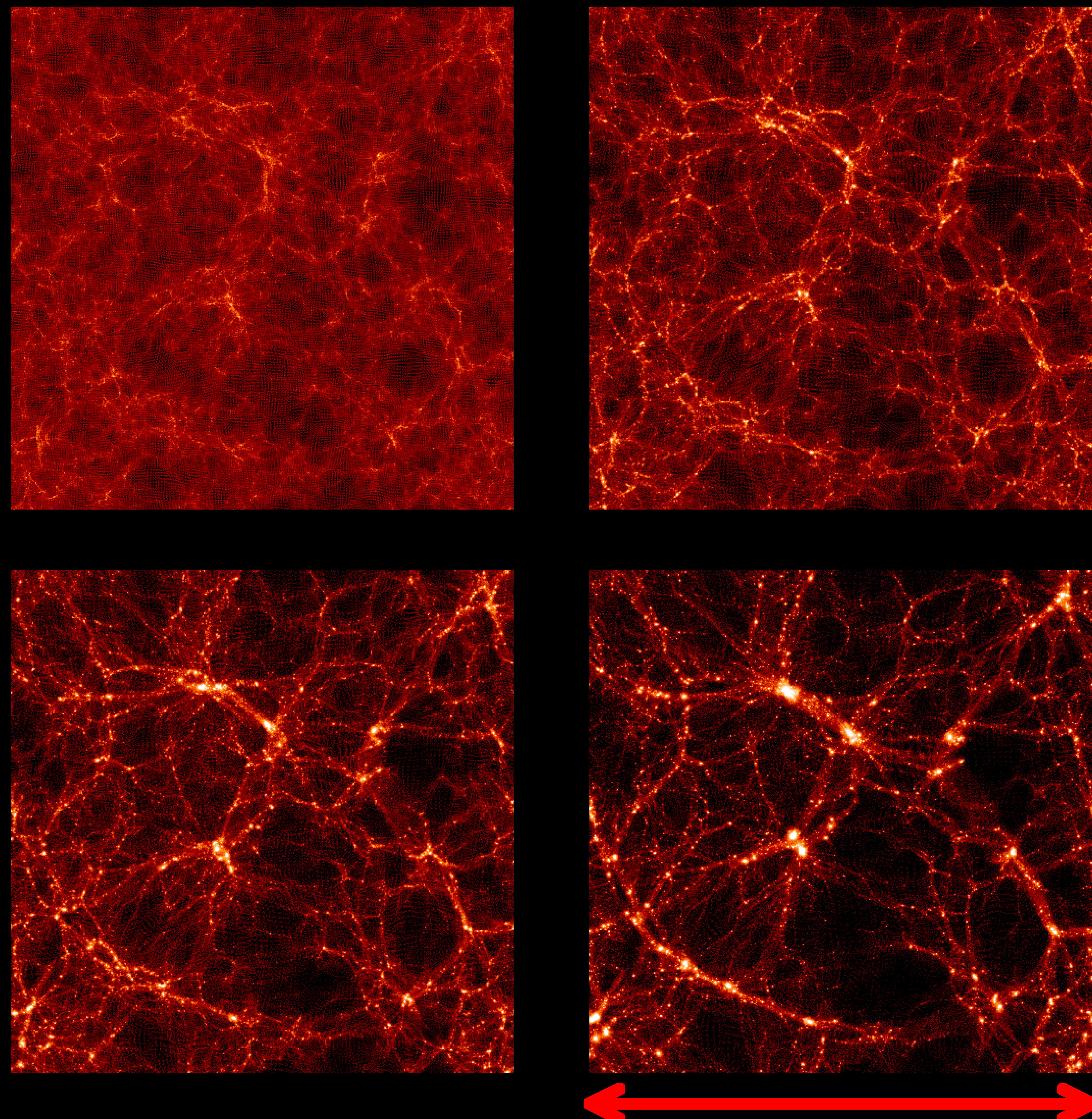
project on cluster simulation

MOND

Fitting a disk distribution + DM to a galaxy rotation curve

z = 5
Age universe =
1.2 billion years

z = 2
Age universe =
3.5 billion years



z = 1
Age universe =
6.2 billion years

z = 0
Age universe =
14.5 billion years

100 h^{-1} Mpc

Dark matter only (ΛCDM)

Simulations : R. Teyssier (CEA - SAp)

Mass of a galaxy cluster

Original and easiest method: *using galaxy velocities*. Typically, we assume what is called Virial equilibrium: $2T + U = 0$. The mass is then dependent on the velocity distribution and the size of the cluster.

An *approximate* calculation goes as follows:

$T = 1/2 \sum m_i v_i^2 \cong 1/2\, Nmv^2$ (N identical galaxies)

$U = -Gm^2/R$ for each pair of objects

$\Longrightarrow U \cong -N(N-1)\, Gm^2/2R \sim -N^2\, Gm^2/2R$

with $2T + U = 0$ and $Nm = M$,

$Nmv^2 = N^2\, Gm^2/2R \Longrightarrow M \sim 2Rv^2/G$

Generally R is taken as the outer cluster radius and v is the standard deviation of the velocities.

Other means: (1) hydrostatic equilibrium (2) gravitational lensing

Converting magnitudes to luminosities

m is the apparent magnitude

M is the "absolute" magnitude, defined as $M=m+5-5\log(d_{pc})$

such that M is equal to m for an object at 10pc

M is an *intrinsic* property, like a luminosity

fluxes are linked to magnitudes via $m_1-m_2=2.5\log(F_2/F_1)$

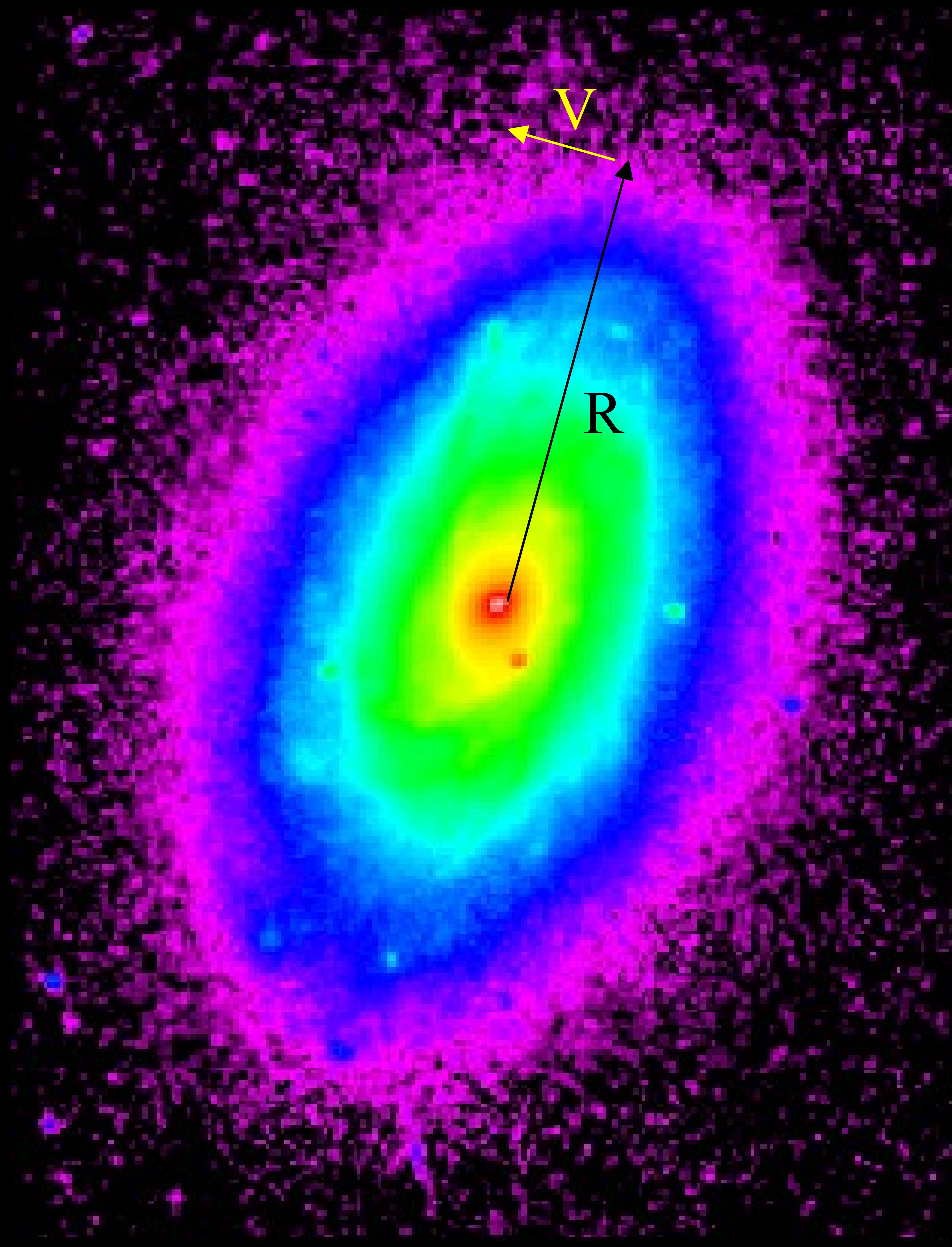
so for objects are at the same distance $m_1-m_2=2.5\log(L_2/L_1)$

where F represents flux (e.g. W/m^2) and L luminosity (e.g. W)

If we take absolute magnitudes, such that the objects are considered at a distance of 10pc, $M_1-M_2=2.5\log(L_2/L_1)$, so if M_1 is the sun and M_2 the galaxy in the cluster, $L_{gal}/L_{sun} = 10^{0.4(5.48-Mgal)}$.

This enables you to calculate the luminosity of a galaxy given m and the distance to the cluster calculated from the velocity and H_0 .

Dark Matter in galaxies



The Problem

$M(r<R) \sim R\, V^2 / G$
where R is the distance of the point where V is measured.

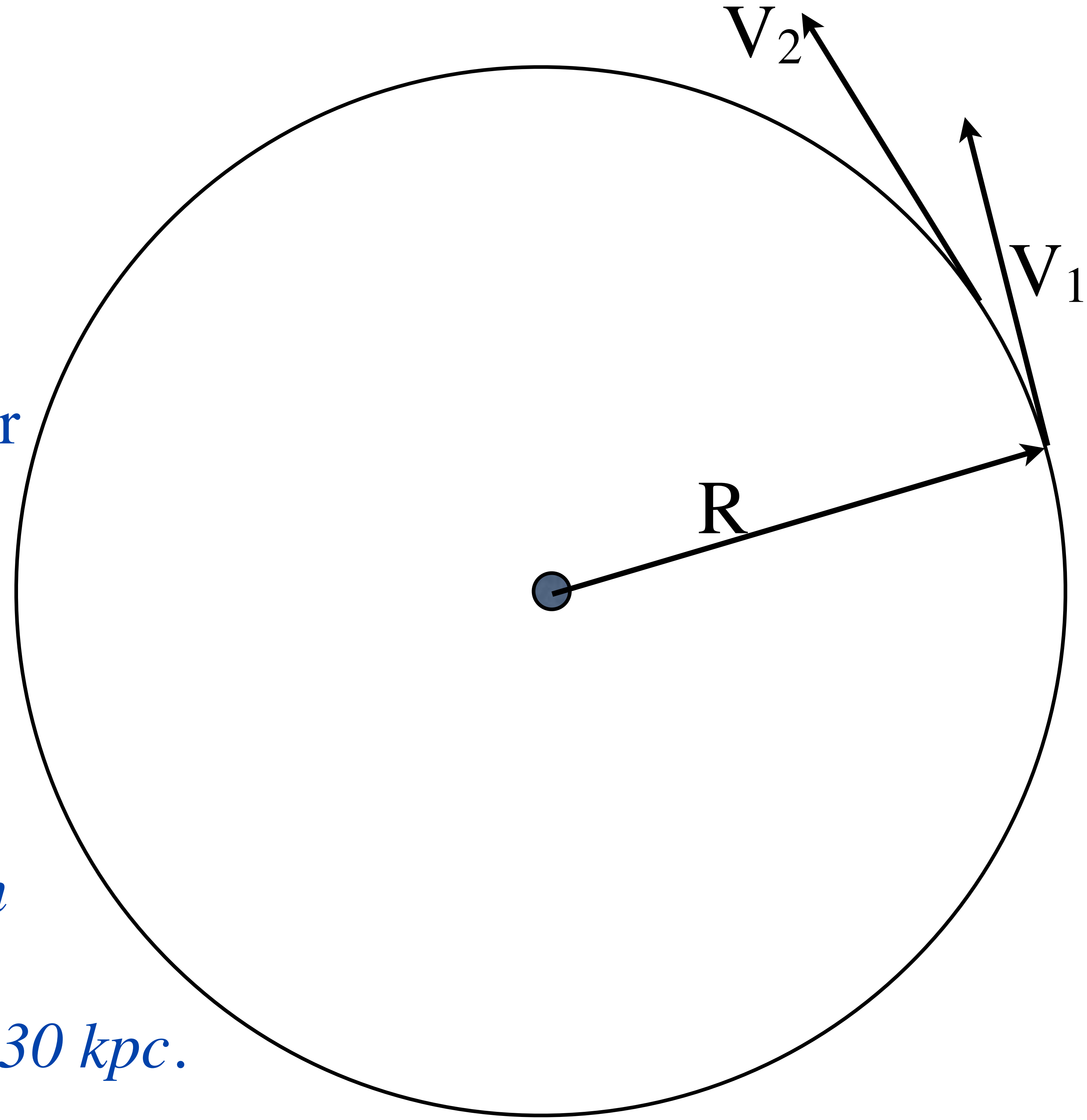
For a spherical mass distribution, $F=GM(r<R)/R^2$ and $F=mv^2/R$ such that $M(r<R) \sim R\, V^2 / G$

When R increases but V stays constant, $M \sim R$ even without obvious mass.

Now use this to calculate the mass of a a spiral galaxy, with measurements of rotation velocity of 220 km/s at 10, 20 and 30 kpc from the center.

pc = 3.0856 x 10^{16} m

$M(r<R) \sim R\, V^2 / G$



Strictly, this is only true for a spherical distribution of matter but overestimates the mass by only a small amount.

For March 9th, estimate the mass of a galaxy which has a rotation velocity of 220km/s at R=10, 20, and 30 kpc.

What is a Galaxy?

Typically 2 conditions should be met.

A galaxy is a self-gravitating mixture of stars and gas with a mass greater than that of a star cluster.

A galaxy is not part of a larger object that we would call a galaxy (e.g. a particularly massive globular cluster vs. a low-mass dwarf galaxy).

Generally, distinctions are obvious but the definition above is not absolutely definite for very small galaxies / large star clusters.

Dynamics

To maintain a particle in a circular orbit of radius r and velocity v, a force $F = m\, v^2/r$ is required.
For a symmetric mass distribution, the force of gravity is $F = -m\delta\Phi/\delta R$ where Φ is the gravitational potential.

More generally, $F=-m\, \mathrm{grad}(\Phi)$.
Hence $F = GMm/R^2$ where M=mass within $r<R$
==> for a spherical distribution
$F = GMm/R^2 = m\, v^2/r$ so, since $r=R$
$v^2 = GM/R$

$$\Phi(x,y,z) = G \int \rho(x,y,z)\, dx\, dy\, dz \,/\, (x^2 + y^2 + z^2)^{1/2}$$

The gravitational potential is a scalar field (no direction, just numbers) so if there are several masses or mass distributions, the total grav potential is equal to the sum of the individual potentials (no direction to worry about).

So $\Phi_{tot} = \Phi 1 + \Phi 2 + \Phi 3$ ... and thus
$V^2_{circ} = V_{1circ}^2 + V_{2circ}^2 + V_{3circ}^2$ ...

Note the squares are summed, not the velocities.

Going from density (i.e. mass distribution) to gravitational potential is far from straightforward.

Dynamics

$v^2 = -r\,\delta\Phi/\delta r$, so

$\delta\,(\Phi 1+\Phi 2+\Phi 3)/\delta r = \delta\Phi 1/\delta r + \delta\Phi 2/\delta r + \delta\Phi 3/\delta r,$

$v_{totcirc}{}^2 \quad = v1^2 + v2^2 + v3^2$

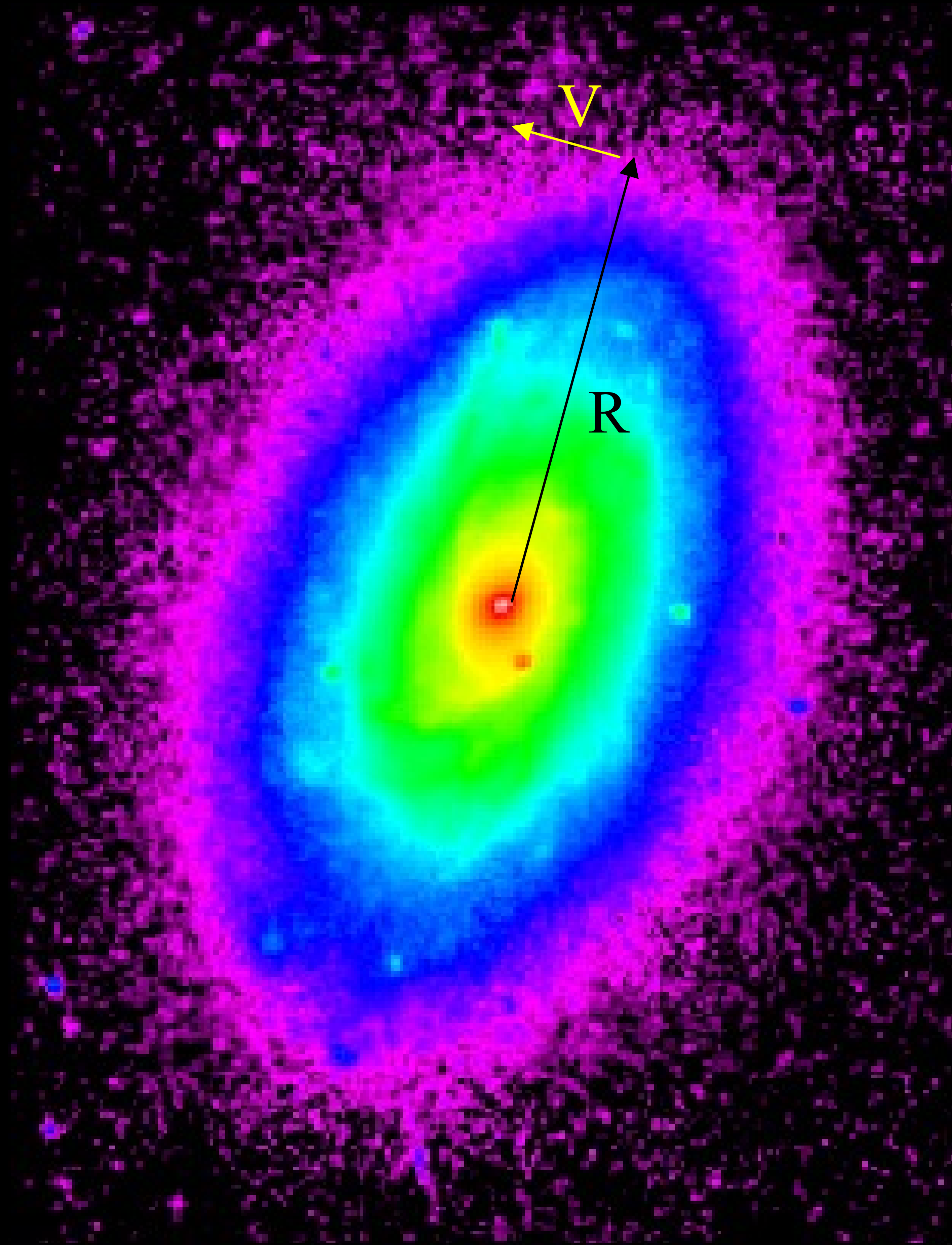
$\quad = -r\,\delta\Phi 1/\delta r + -r\,\delta\Phi 2/\delta r + -r\,\delta\Phi 3/\delta r$

Here $\Phi 1$, $\Phi 2$, and $\Phi 3$ are 3 components of the total grav potential of the galaxy (e.g. bulge, disk, halo).
==> The gravitational potential is essential to understand the dynamics of a galaxy.

HOWEVER... we measure the projected (2D) brightness distribution. We try to estimate the density from the surface brightness and then the gravitational potential.

In practice, we fit projected density profiles to the brightness.

Dark Matter in galaxies

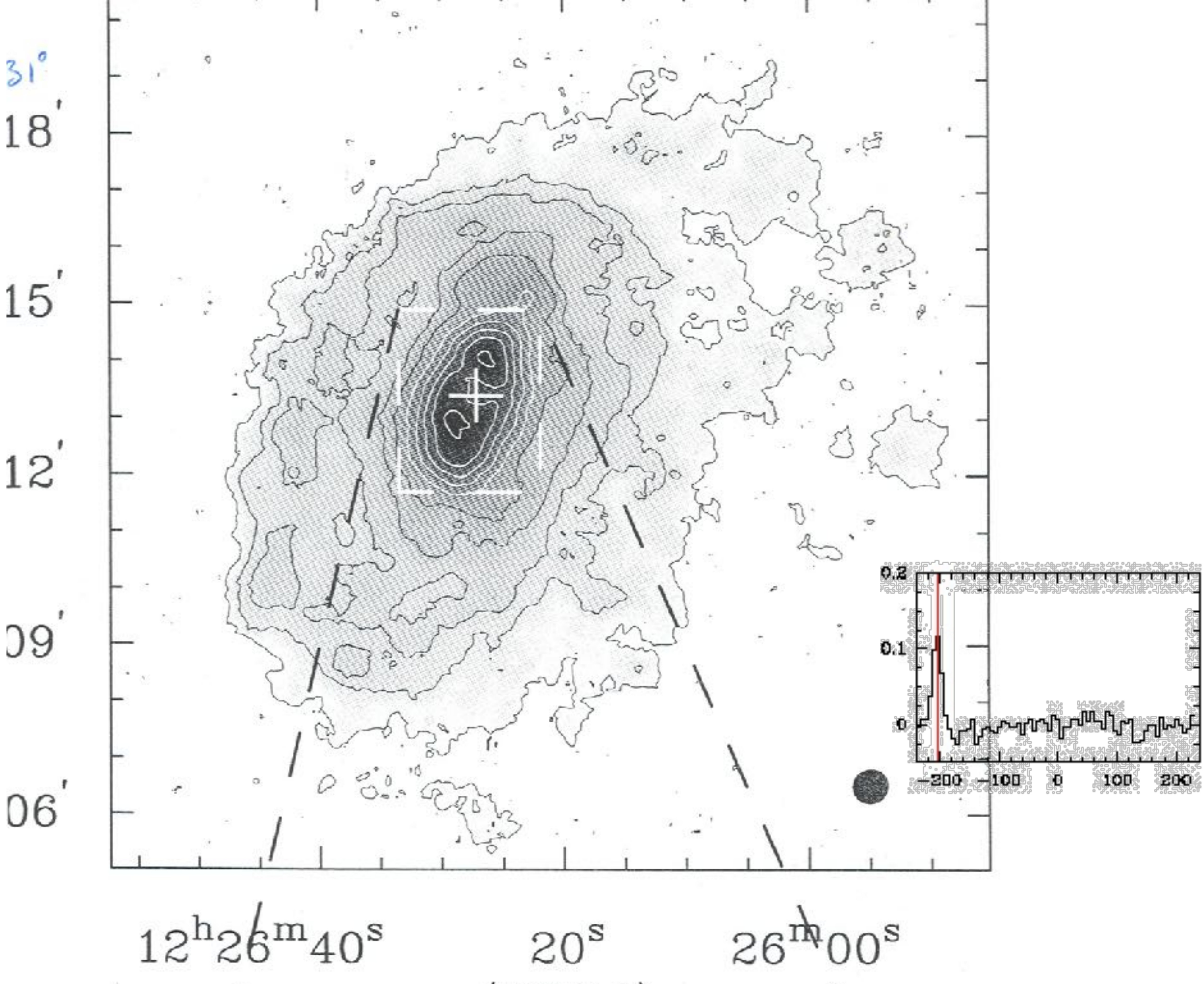


The Problem

$M(r<R) \sim R\, V^2 / G$
where R is the distance of the point where V is measured.

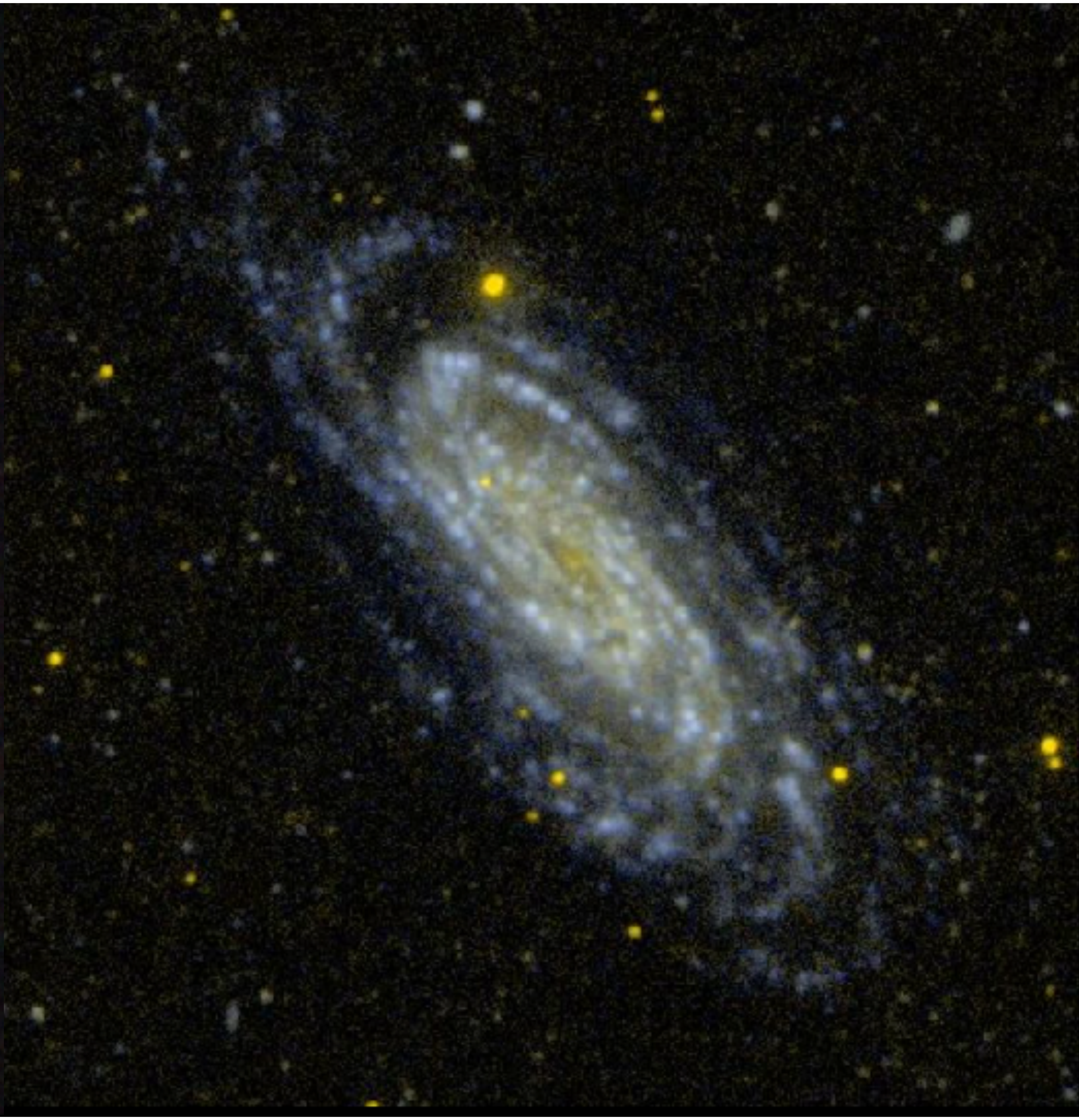
For a spherical mass distribution, $F=GM(r<R)/R^2$ and $F=mv^2/R$ such that $M(r<R) \sim R\, V^2 / G$

When R increases but V stays constant, $M \sim R$ even without obvious mass.

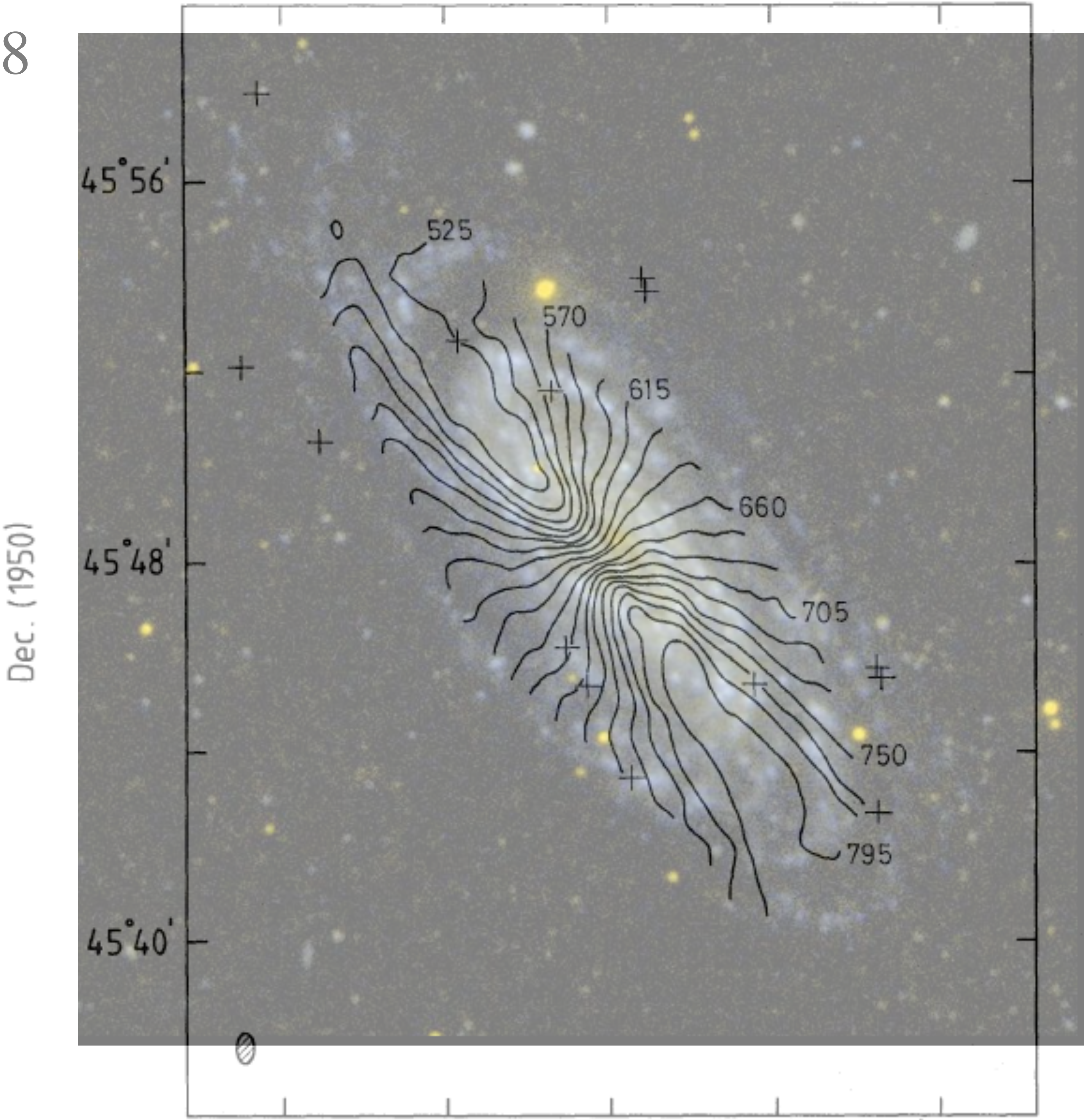


31°
18′
15′
12′
09′
06′
12h26m40s
20s
26m00s
0.2
0.1
0
−200
−100
0
100
200

NGC 3198



NGC 3198



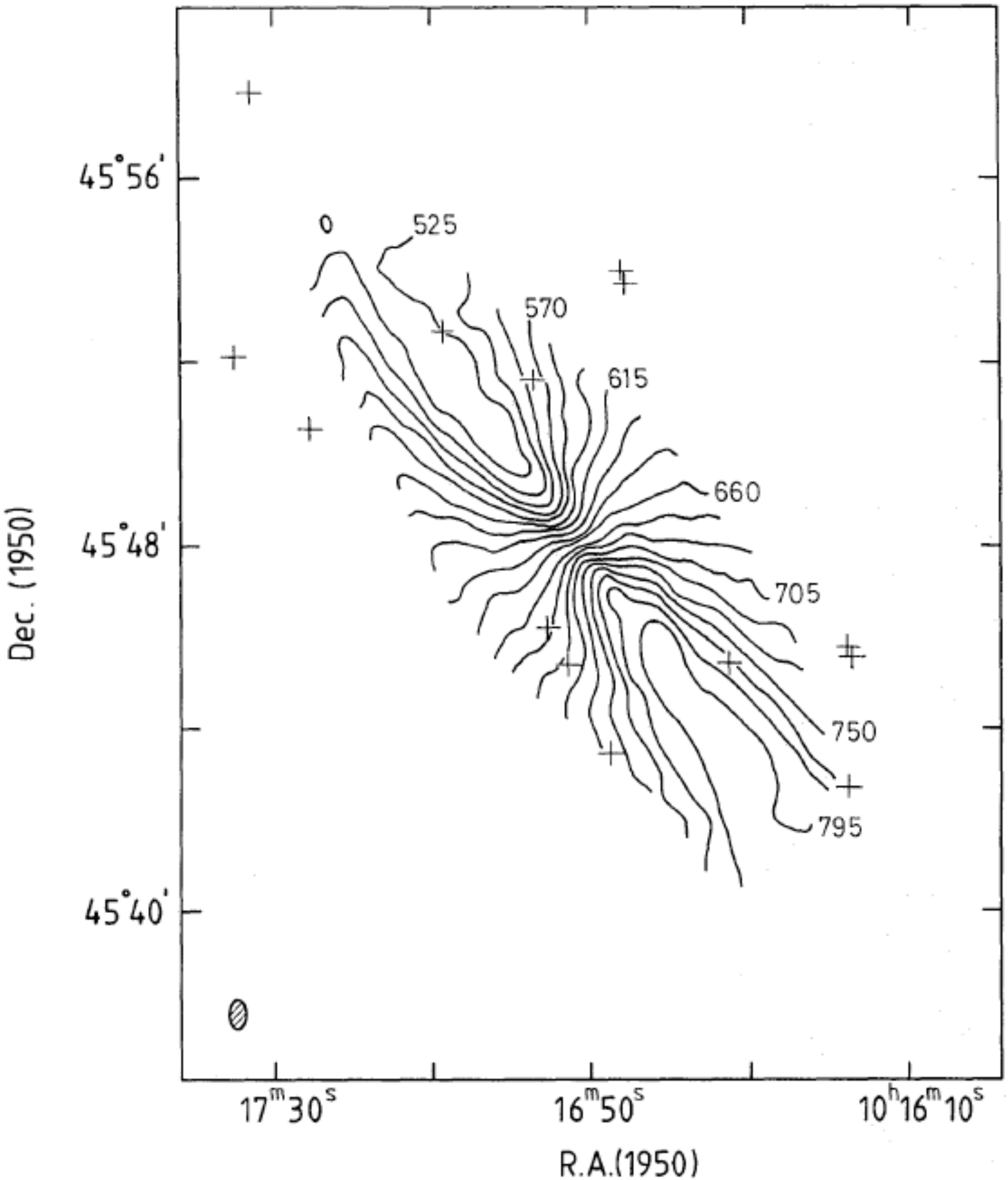
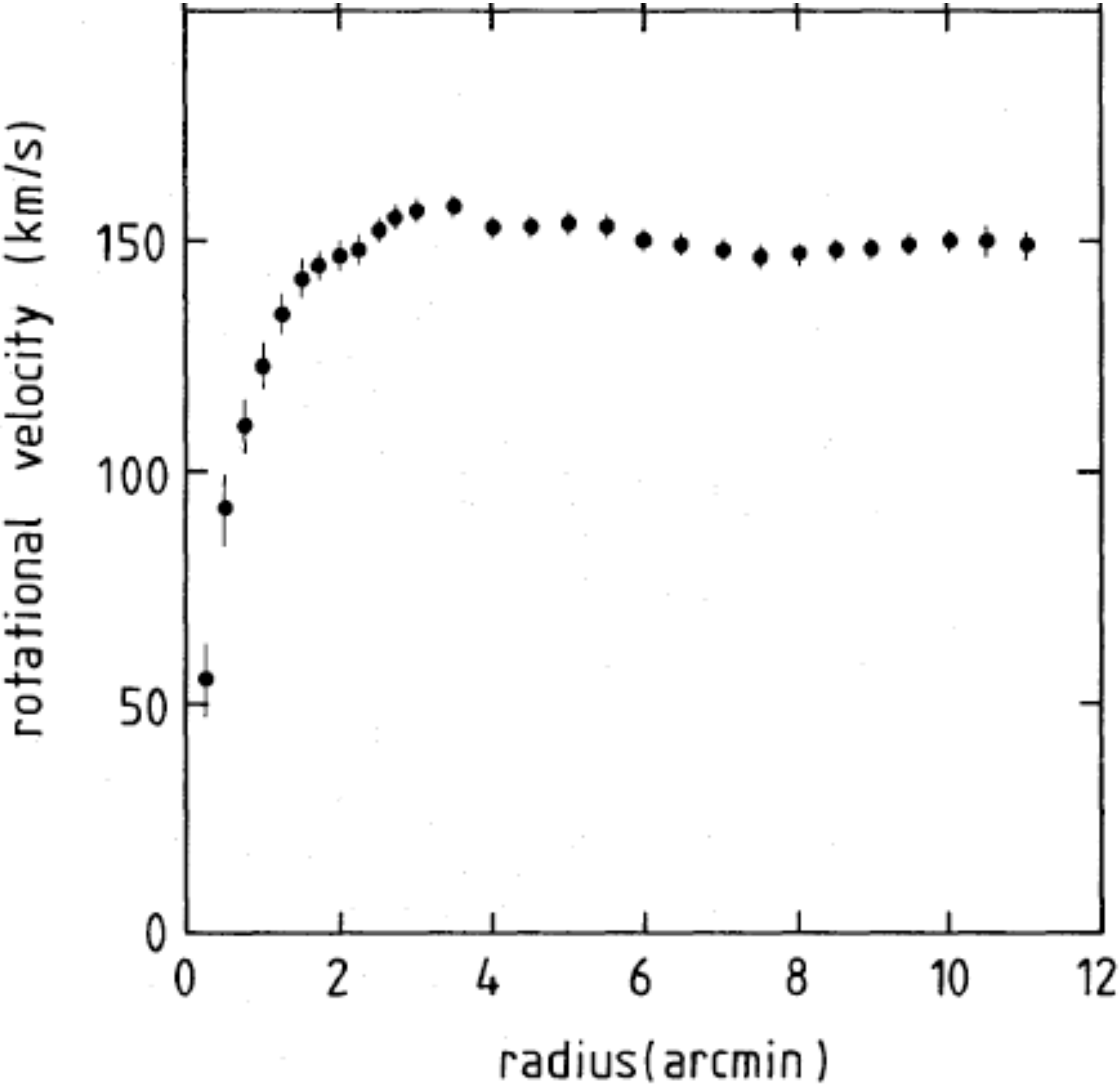
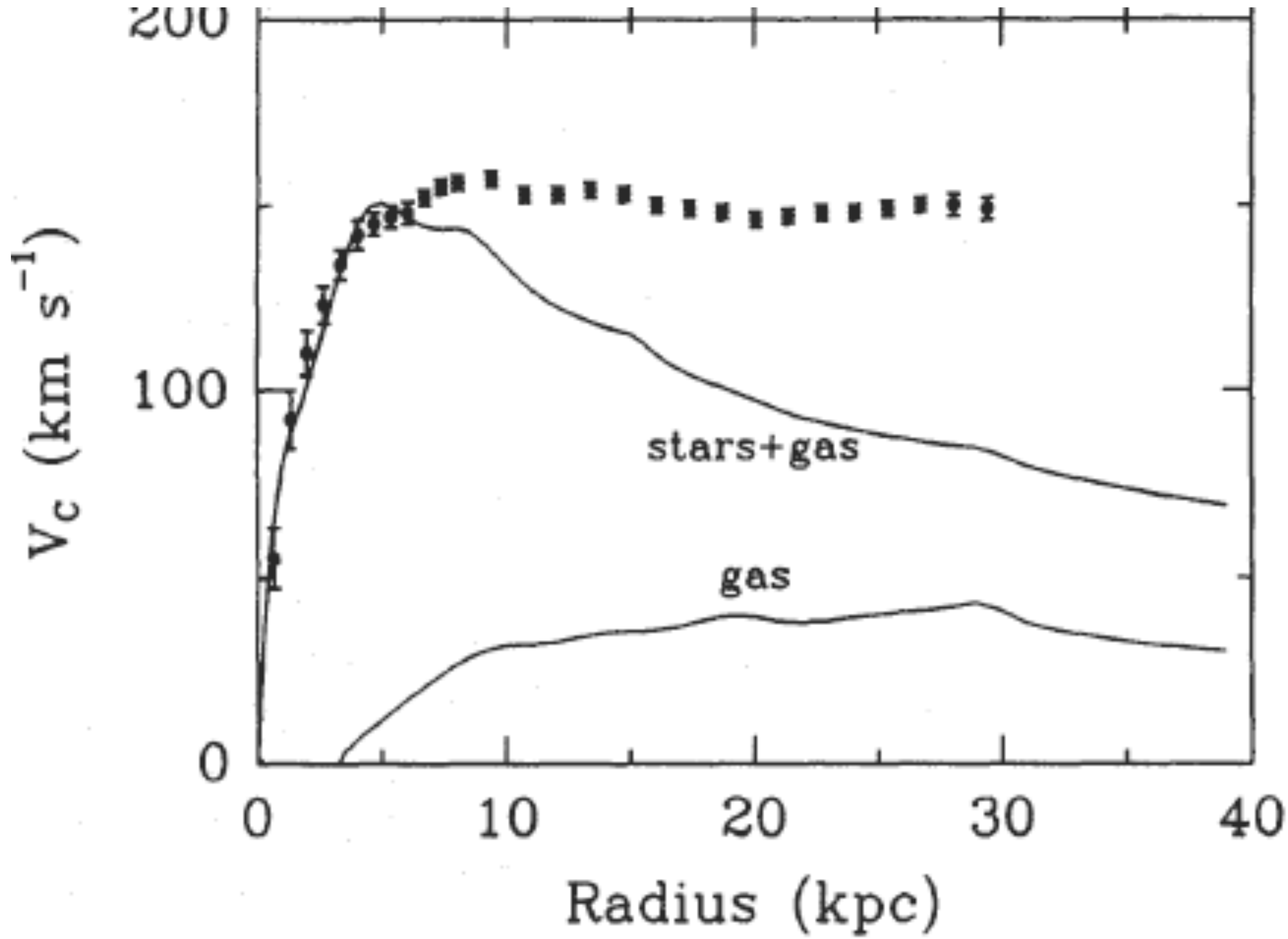


Fig. 5. Full resolution radial velocity field derived by fitting Gaussians to each line profile. The heliocentric radial velocities ($km\,s^{-1}$) are indicated


rotational velocity (km/s)
150
100
50
0
0
2
4
6
8
10
12
radius(arcmin)


200
100
0
0
10
20
30
40
Radius (kpc)
V_c (km s^{-1})
stars+gas
gas