In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy.integrate import quad
from scipy.integrate import odeint
import scipy.constants as cste
```

Ex. 1

a)

In [ ]:
```
B_0 = 400 * 1e-6 # T
R_j = 71492 * 1e3 # m

x = np.linspace(-R_j, +R_j, 100)
y = np.linspace(-R_j, +R_j, 100)

X, Y = np.meshgrid(x, y)

r = np.sqrt(X**2 + Y**2)
theta = np.arctan2(Y, X)
```

b)

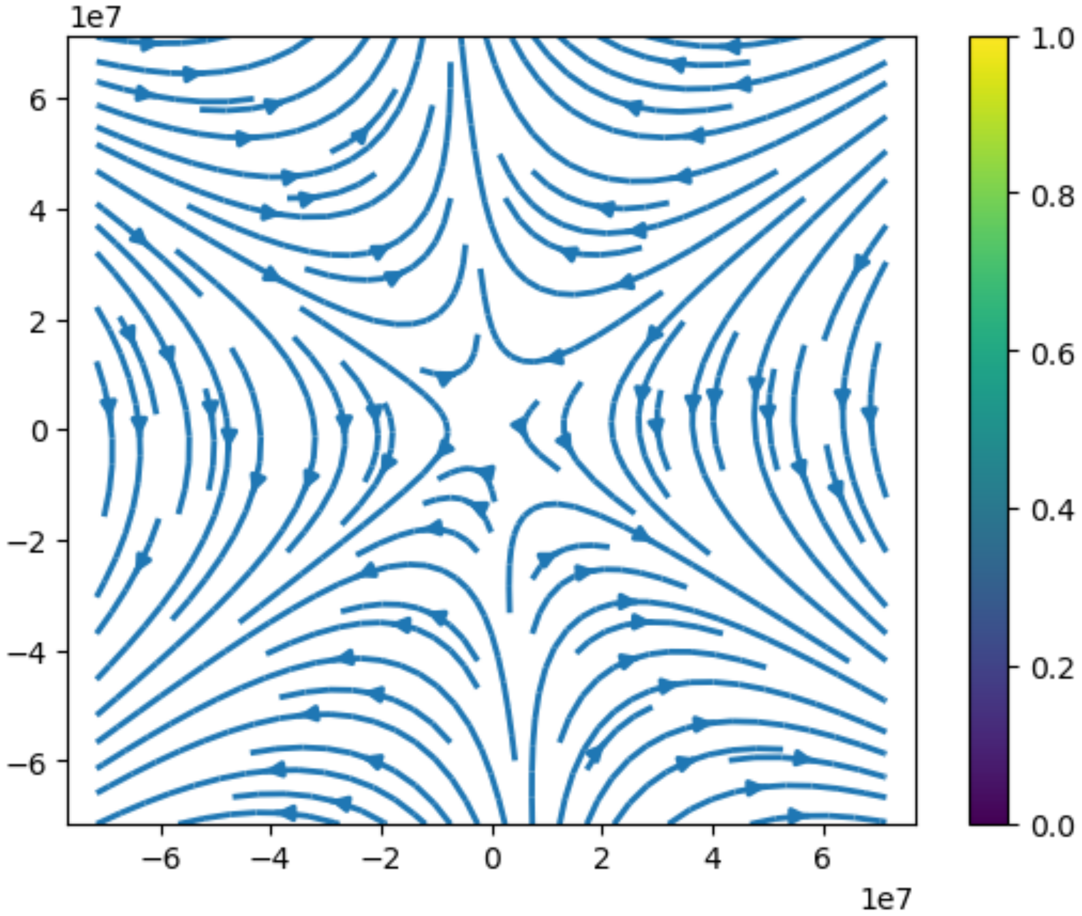
In [ ]:
```
alpha = np.deg2rad(-9.6)

B_r = -2 * B_0 * (R_j/r)**3 * np.cos(theta + alpha)
B_theta = -1 * B_0 * (R_j/r)**3 * np.sin(theta + alpha)
B_phi = 0 * r

B_norm = np.sqrt(B_r**2 + B_theta**2)
```

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{bmatrix}$$

here it seems we consider θ to be the azimuthal angle, not ϕ

$\vec{e_\theta}$ is orthogonal to the xOy plane, so I'm guessing we are plotting in the xOz plane ?

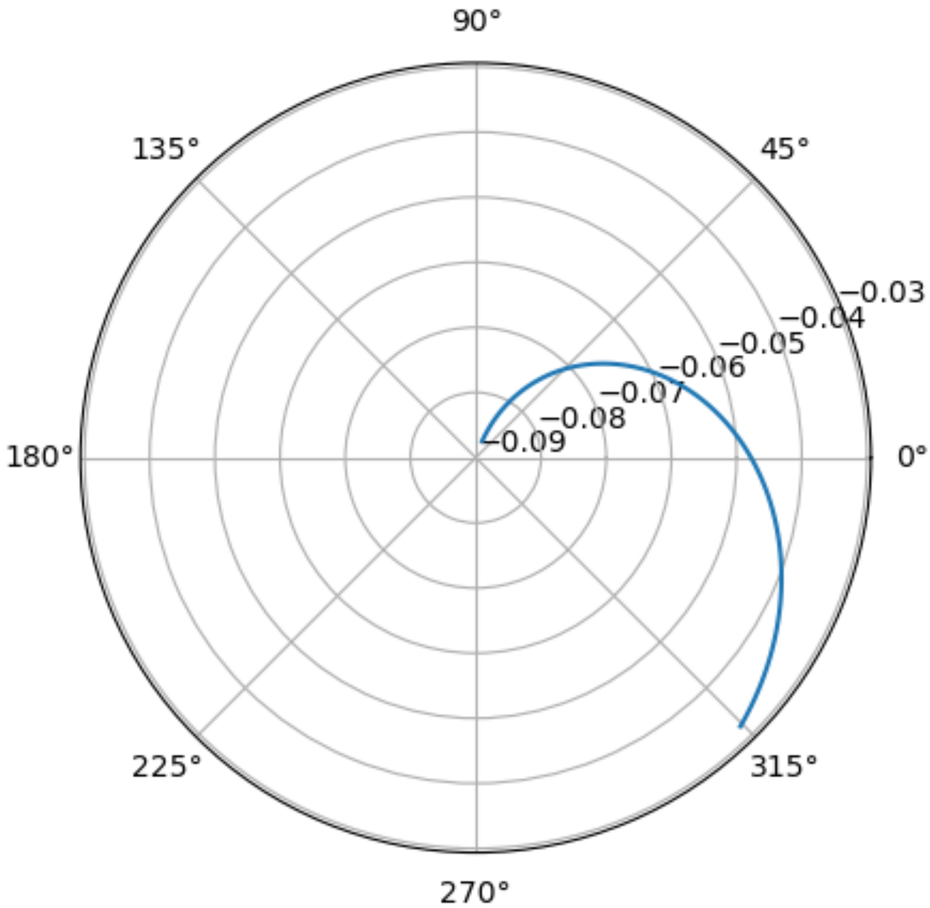
In [ ]:
```
# phi = 0
B_x = np.sin(theta)*B_r + np.cos(theta)*B_theta
B_y = np.cos(theta)*B_r - np.sin(theta)*B_theta

fig0, ax0 = plt.subplots()
strm = ax0.streamplot(X, Y, B_x, B_y, linewidth=2, cmap=plt.cm.jet)
fig0.colorbar(strm.lines)
plt.axis('equal')
plt.show()
```

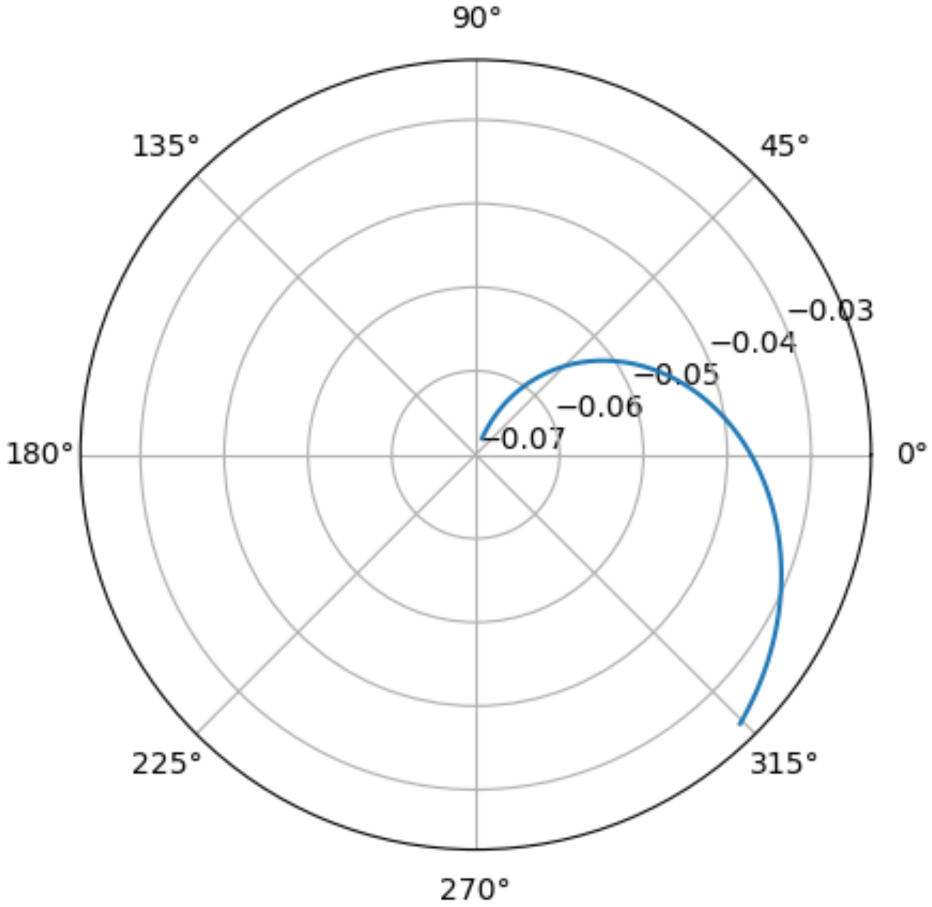


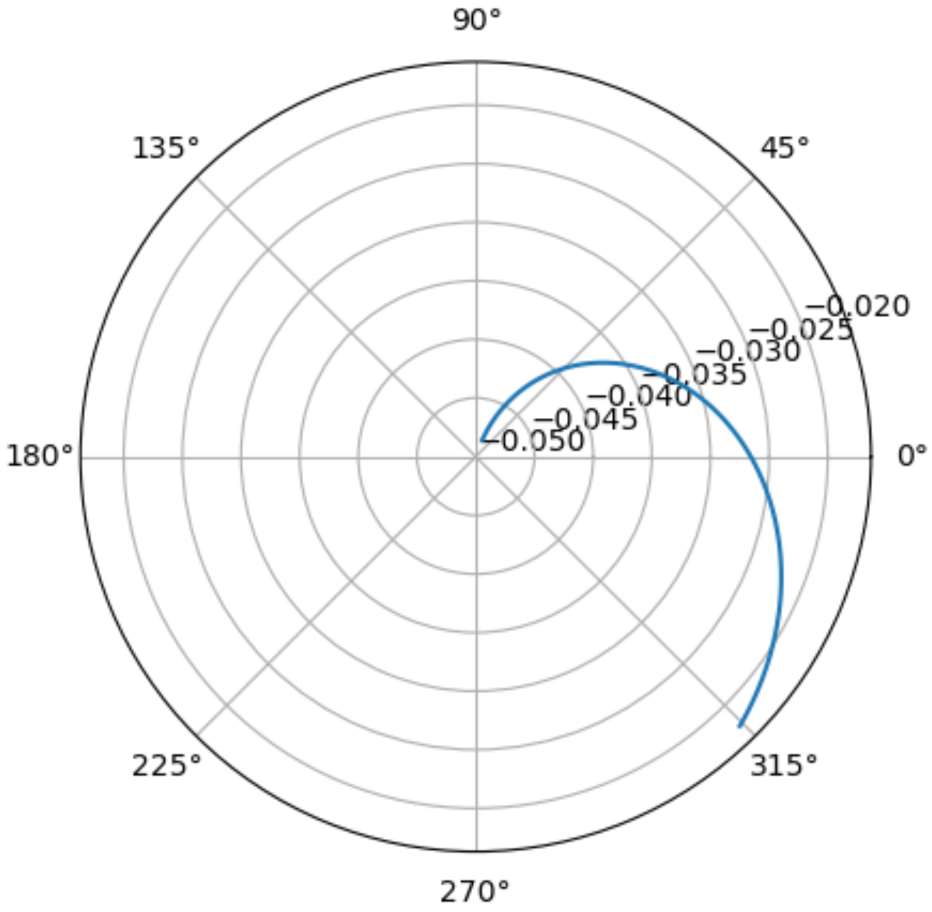
c)

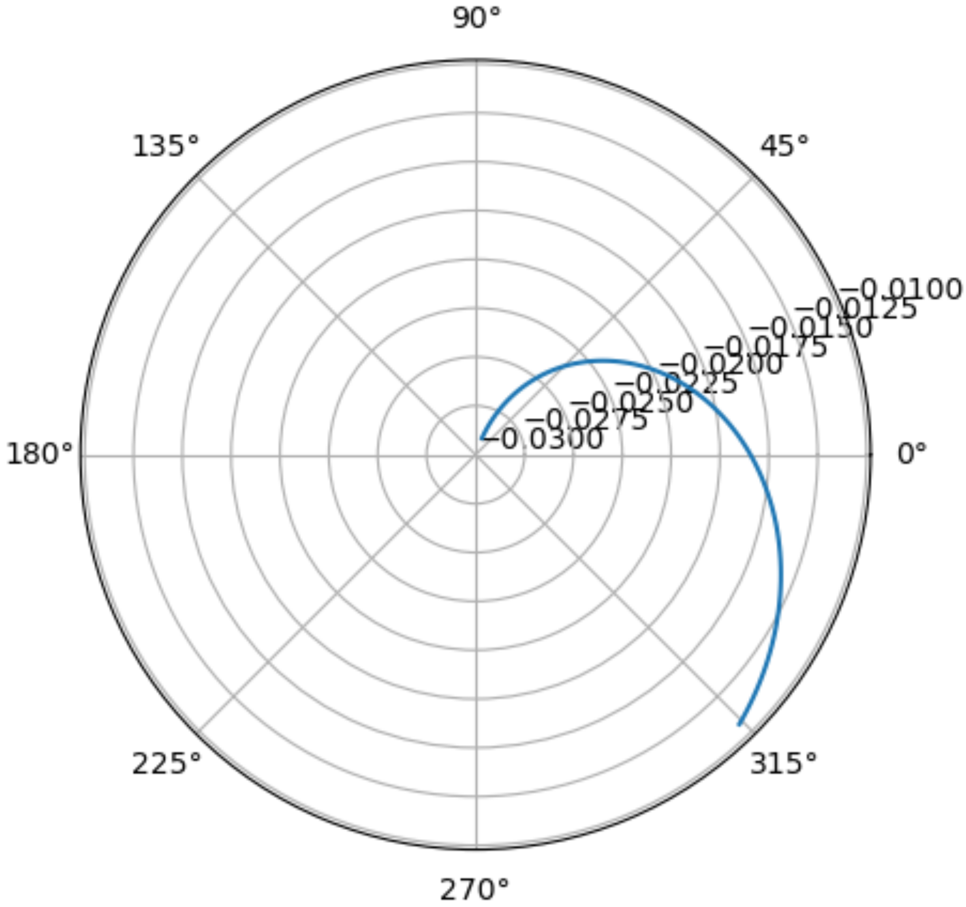
don't know what I did here

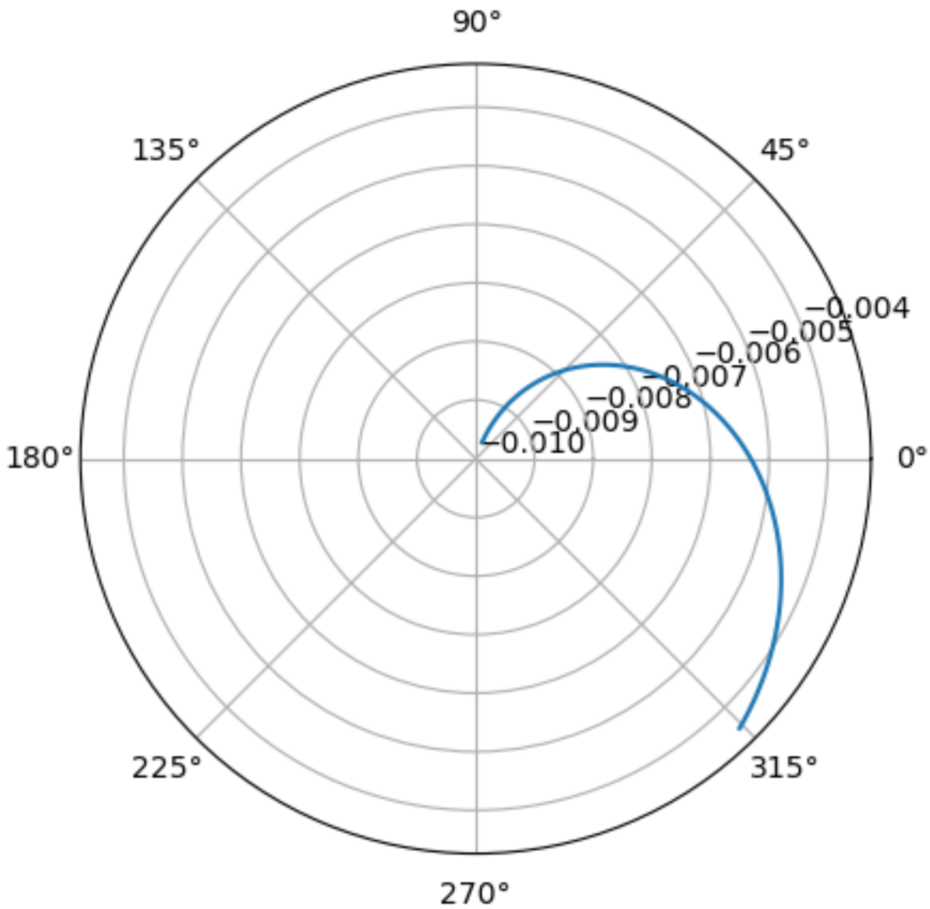
In [ ]:
```
def mag_field_lines(X, t, R_j):
    r, theta = X

    Br = -2*(R_j/r)**2 * np.cos(theta)
    Btheta = -(R_j/r)**3 * np.sin(theta)

    return [Br, Btheta]


u_mag = np.linspace(0, 1, 100)
# initial conditions
CI = np.array([[R_j, theta0] for theta0 in np.linspace(np.deg2rad(-5), np
for X_ini in CI:
    X = odeint(mag_field_lines, X_ini, u_mag, args=(R_j,))
    x, y = X.T
    fig, ax = plt.subplots(subplot_kw={'projection': 'polar'})
    ax.plot(x, y)
plt.show()
```

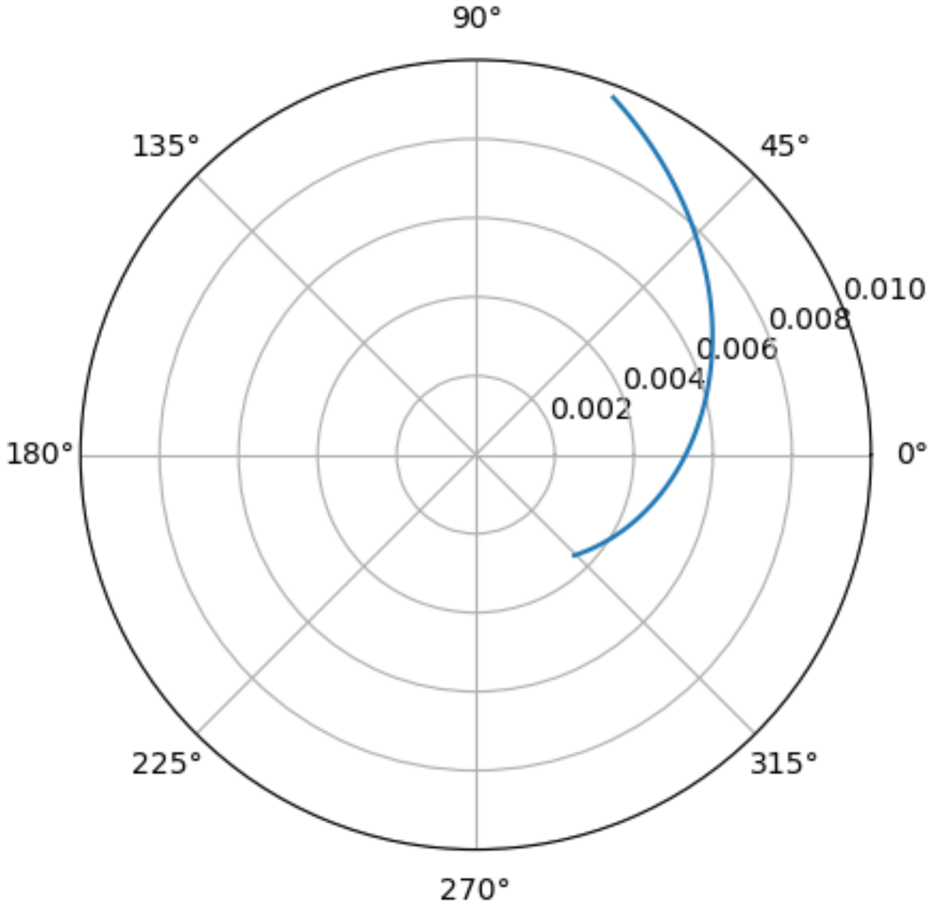


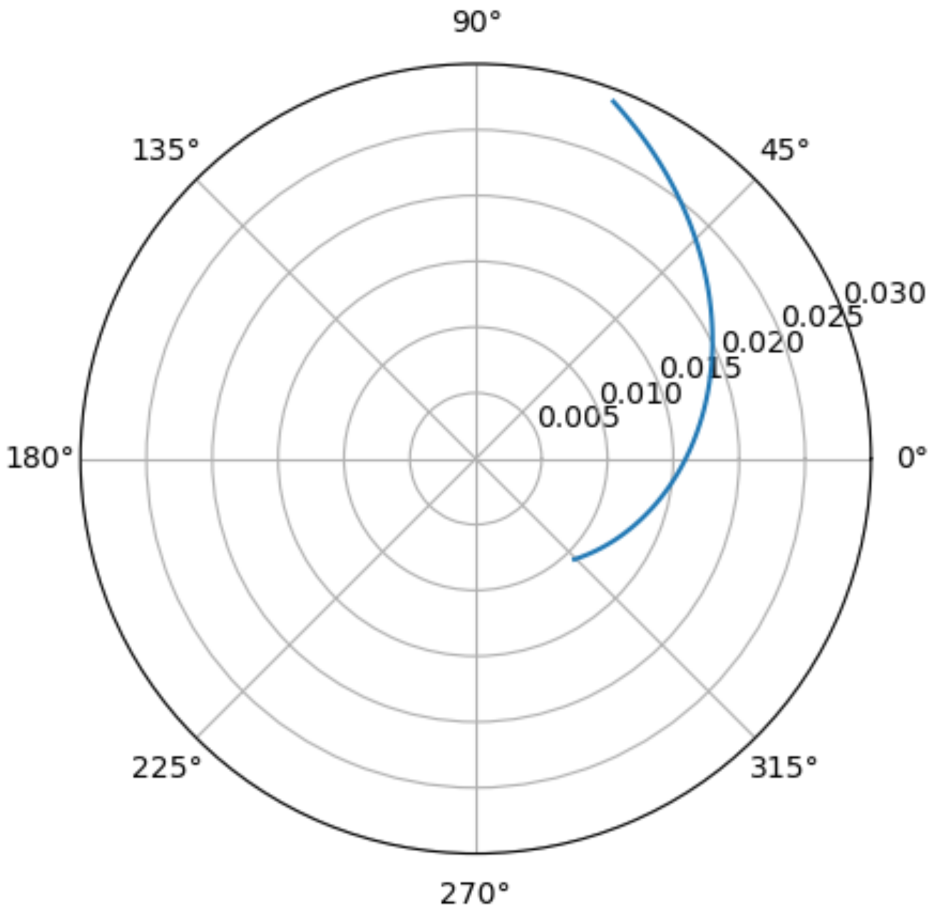

90°
45°
135°
0°
180°
225°
270°
315°
−0.03
−0.04
−0.05
−0.06
−0.07

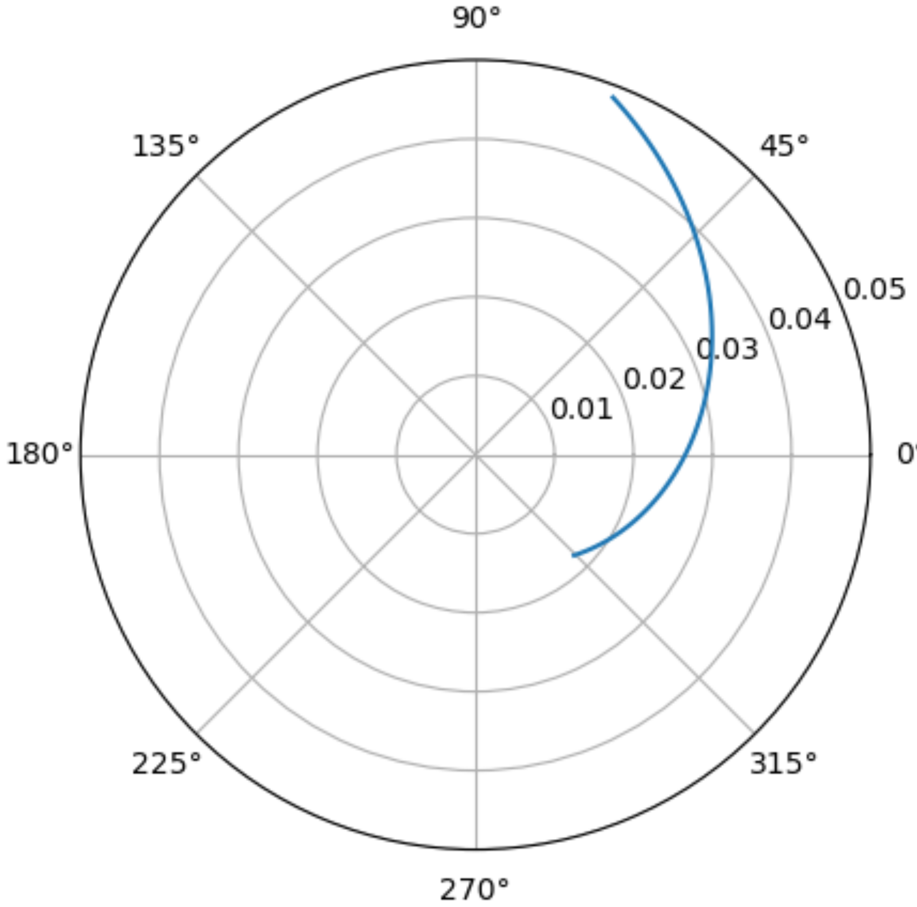

90°
45°
135°
0°
180°
225°
270°
315°
−0.020
−0.025
−0.030
−0.035
−0.040
−0.045
−0.050

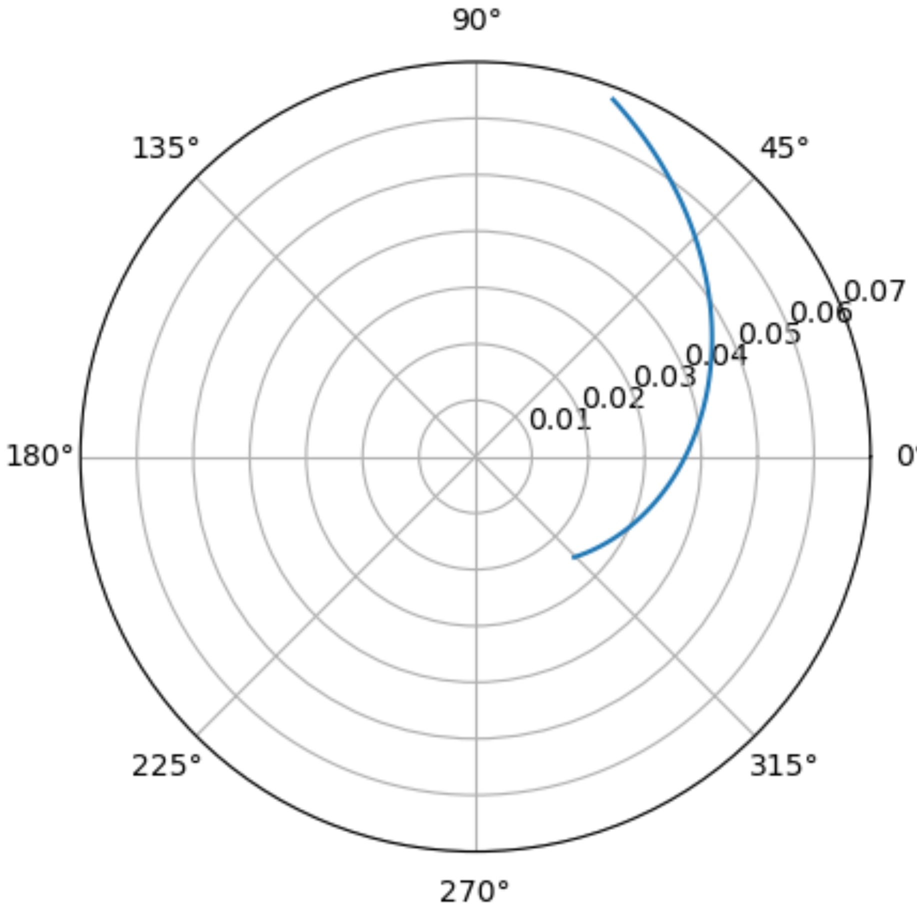

90°
45°
135°
0°
180°
225°
315°
270°
−0.0100
−0.0125
−0.0150
−0.0175
−0.0200
−0.0225
−0.0250
−0.0275
−0.0300

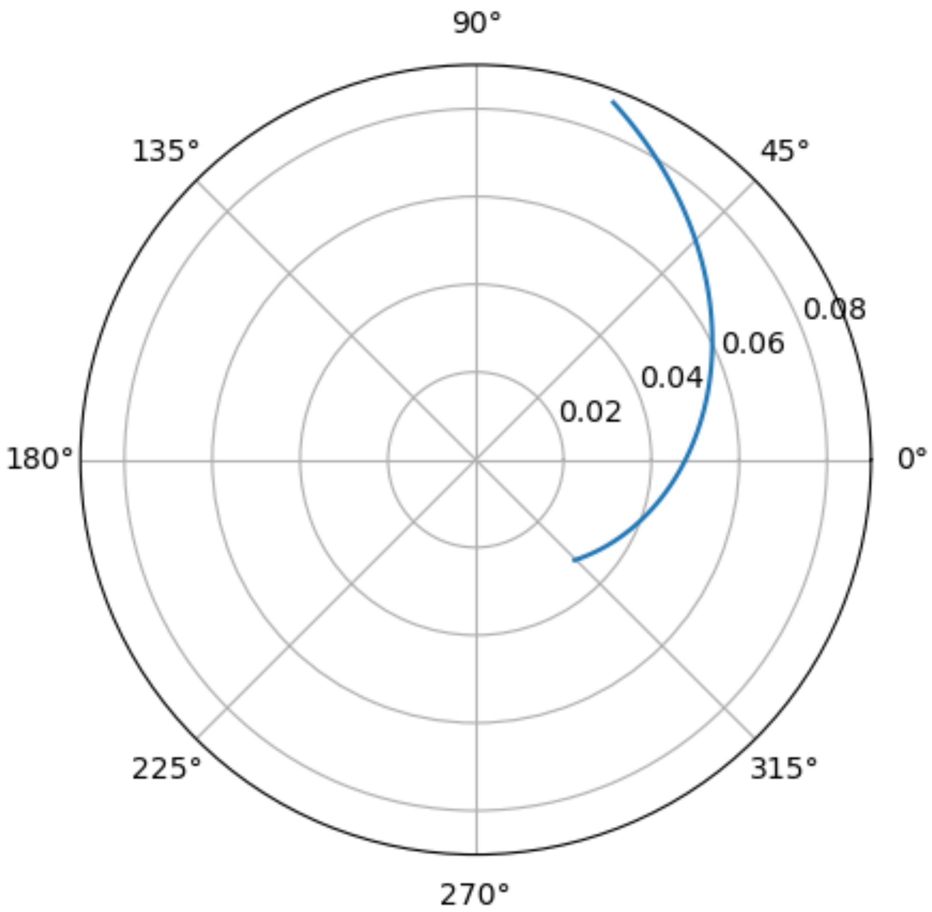

90°
45°
135°
0°
180°
225°
315°
270°
−0.004
−0.005
−0.006
−0.007
−0.008
−0.009
−0.010


90°
135°
45°
0.010
0.008
0.006
0.004
0.002
180°
0°
225°
315°
270°


90°
135°
45°
0.030
0.025
0.020
0.015
0.010
0.005
180°
0°
225°
315°
270°


90°
135°
45°
0.05
0.04
0.03
0.02
0.01
180°
0°
225°
315°
270°


90°
135°
45°
0.07
0.06
0.05
0.04
0.03
0.02
0.01
180°
0°
225°
315°
270°

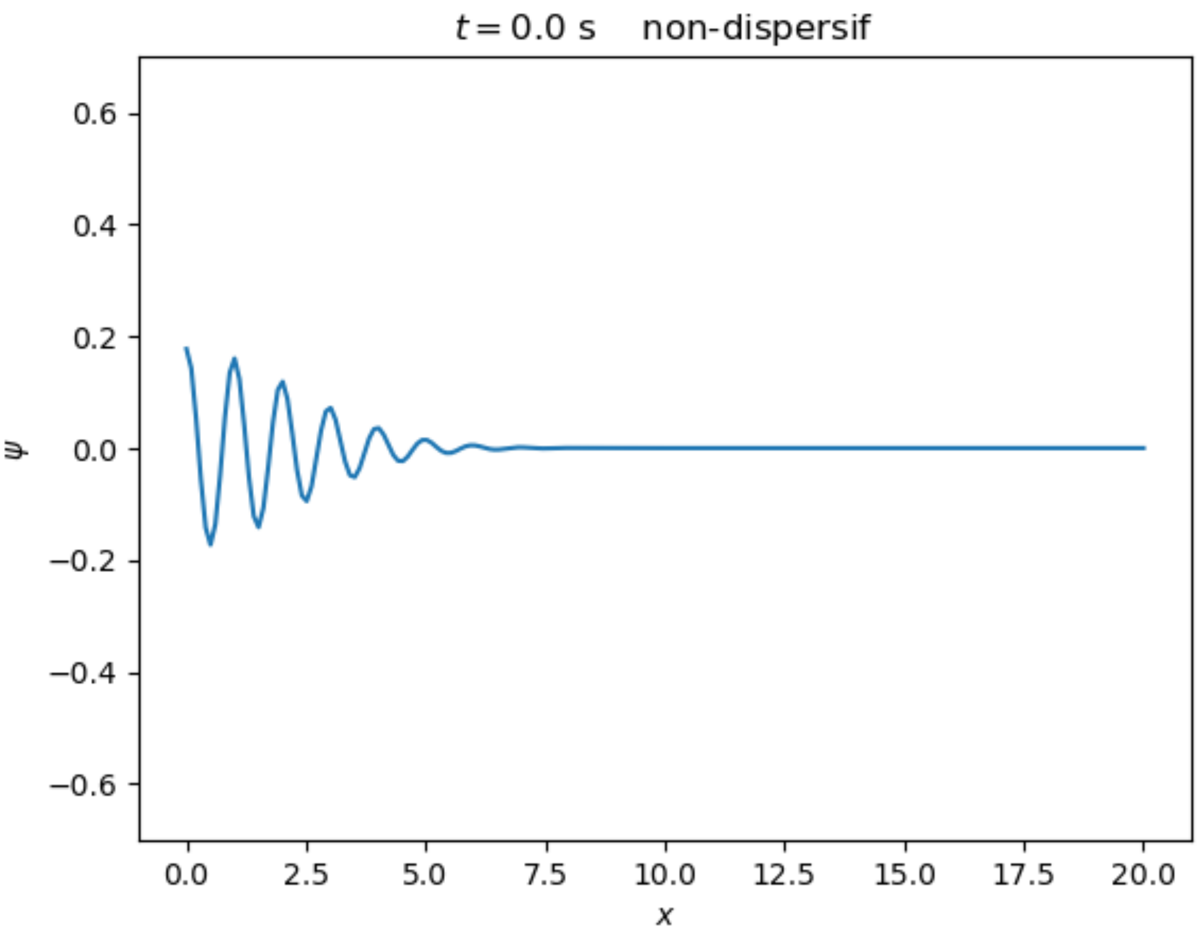


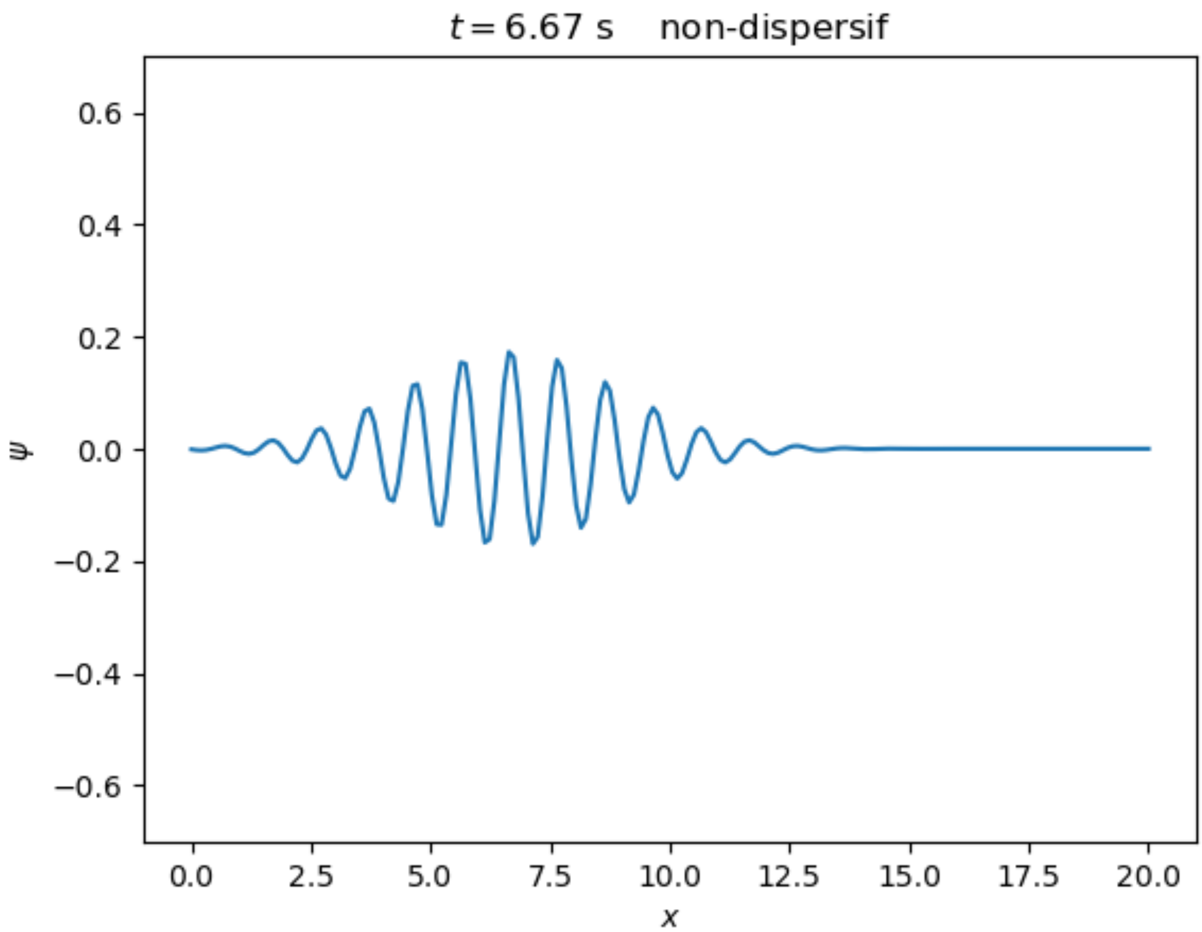
Ex. 2

In [ ]:
```
def g(f, f_0, delta_f):
    return np.exp( -((f-f_0)/delta_f)**2 )

f_0 = 1
delta_f = 0.1
c = 1

t_min = 0
t_max = 10

x_min = 0
x_max = 20

t_domain = np.linspace(t_min, t_max, 200)
x_domain = np.linspace(x_min, x_max, 200)
```

non-dispersif

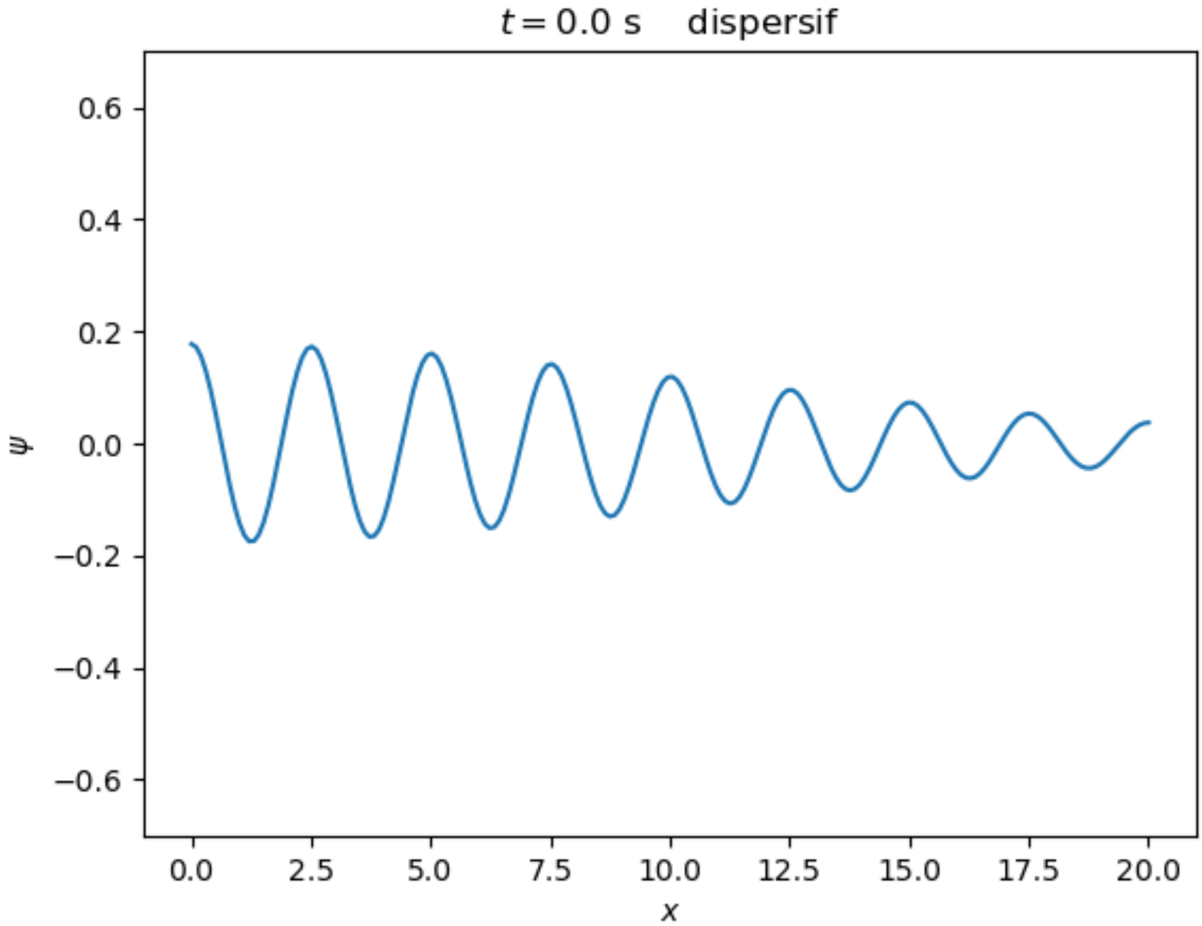
In [ ]:
```
N = 100
f_min, f_max = 0, 2*f_0
f_domain = np.linspace(f_min, f_max, N)

df = f_domain[1]-f_domain[0]
for t in [0., 6.67]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(x_domain)):
        x = x_domain[i]
        for j in range(len(f_domain)):
            f = f_domain[j]
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*(t - x/c))*df


    plt.plot(x_domain, phi_1)
    plt.title(f"$t={t}$ s    non-dispersif")
    plt.xlabel("$x$")
    plt.ylabel("$\psi$")
    plt.ylim((-0.7, 0.7))
    plt.show()
```

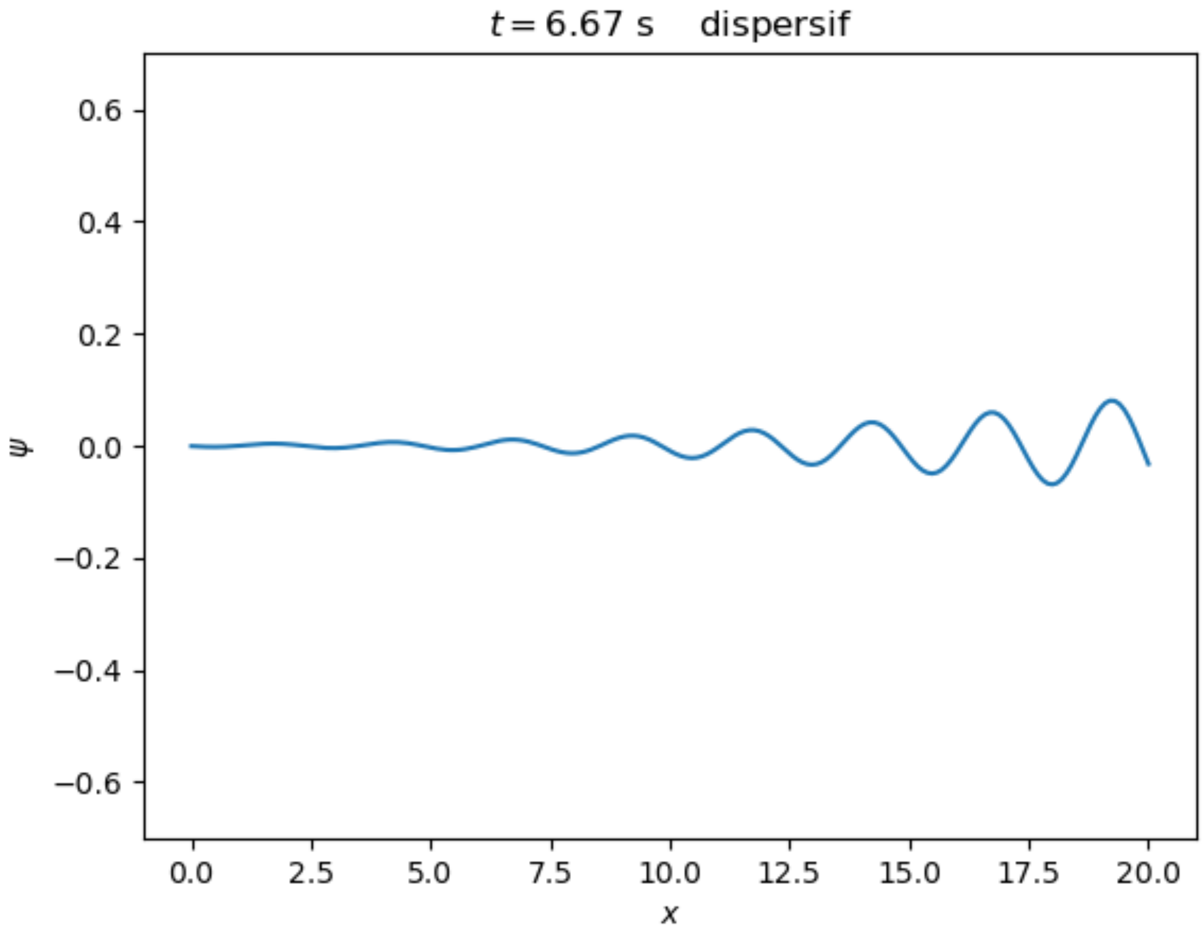




dispersif $\omega = k^2$

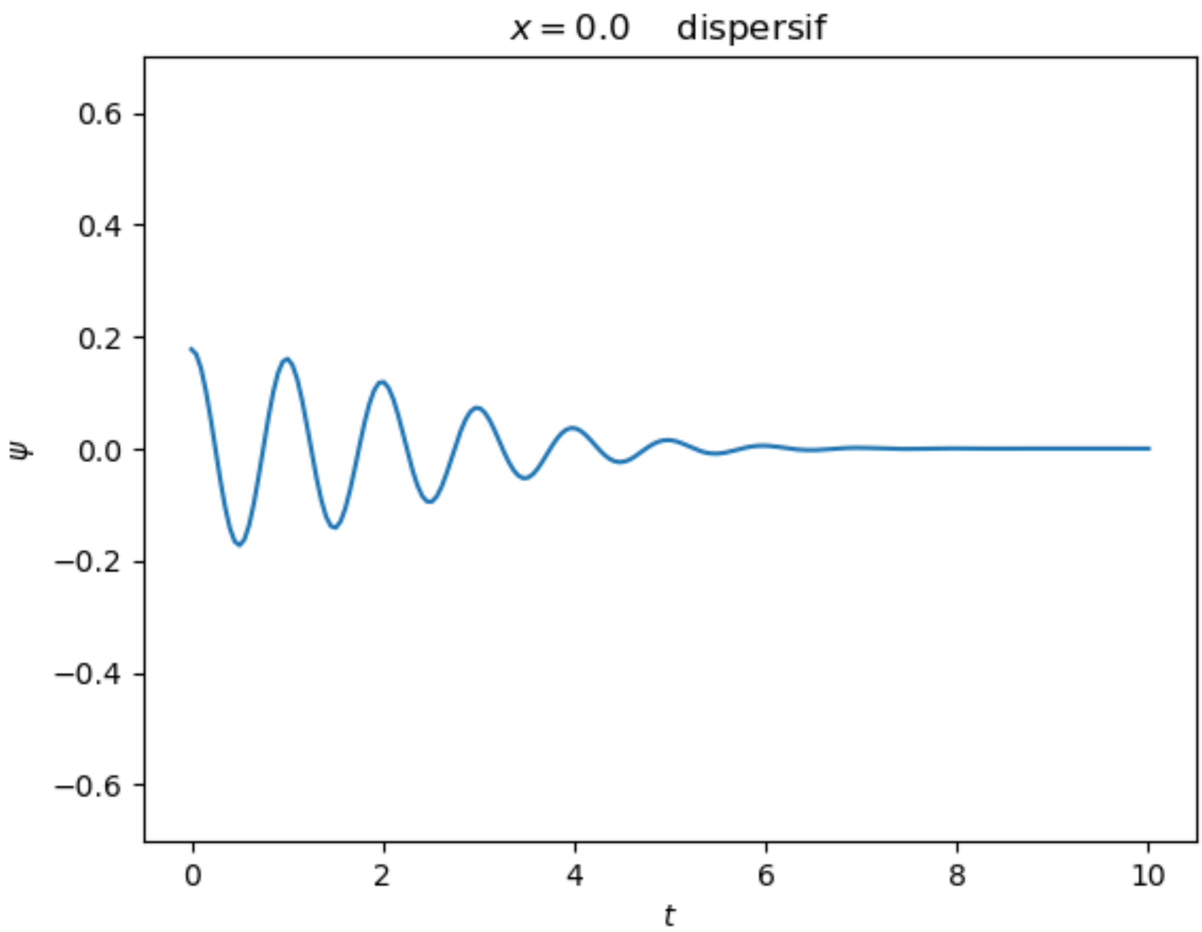
In [ ]:
```
for t in [0., 6.67]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(x_domain)):
        x = x_domain[i]
        for j in range(len(f_domain)):
            f = f_domain[j]
            k = np.sqrt(2*np.pi*f)
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*t - k*x)*df


    plt.plot(x_domain, phi_1)
    plt.title(f"$t={t}$ s    dispersif")
    plt.xlabel("$x$")
    plt.ylabel("$\psi$")
    plt.ylim((-0.7, 0.7))
    plt.show()
```

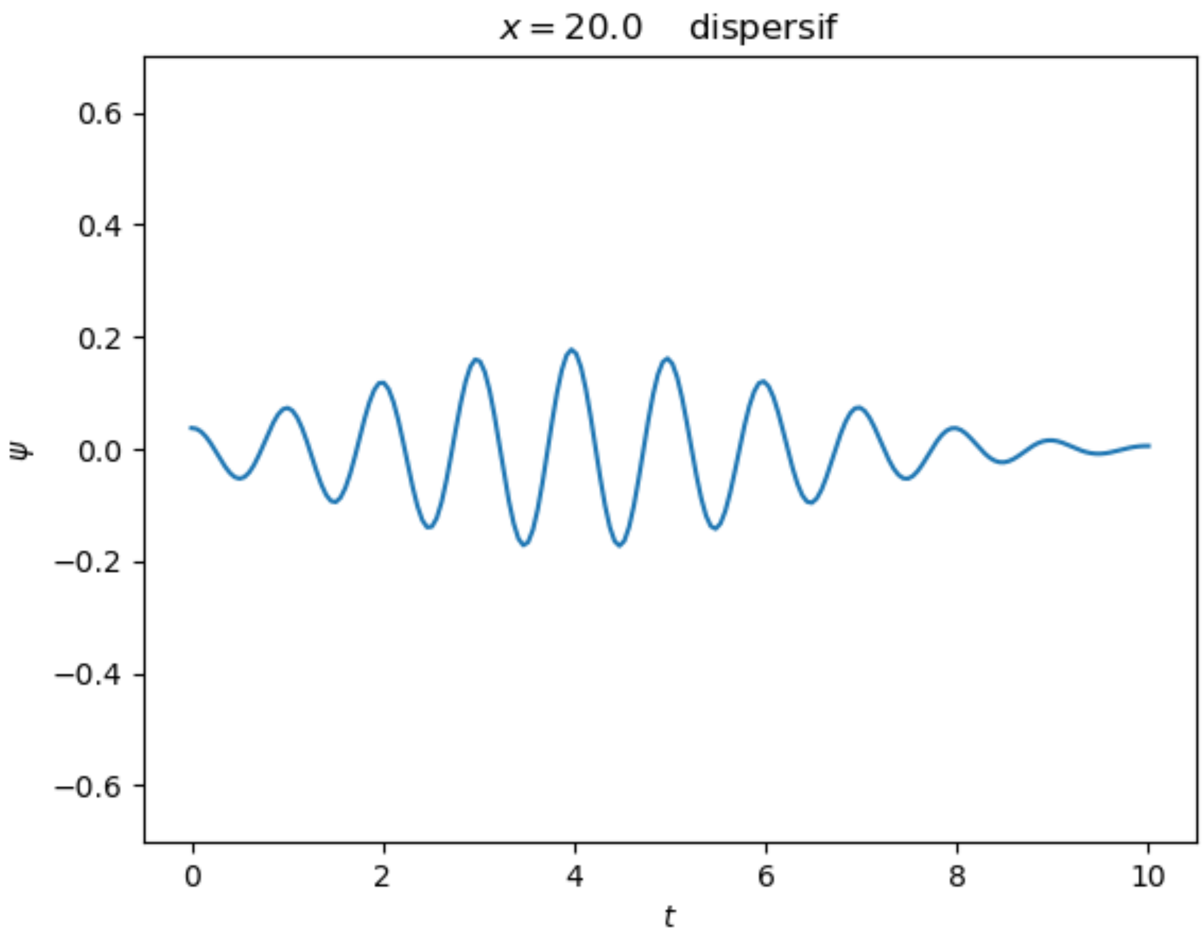

$t = 0.0$ s dispersif
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
ψ
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x


$t = 6.67$ s dispersif
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
ψ
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x

b)

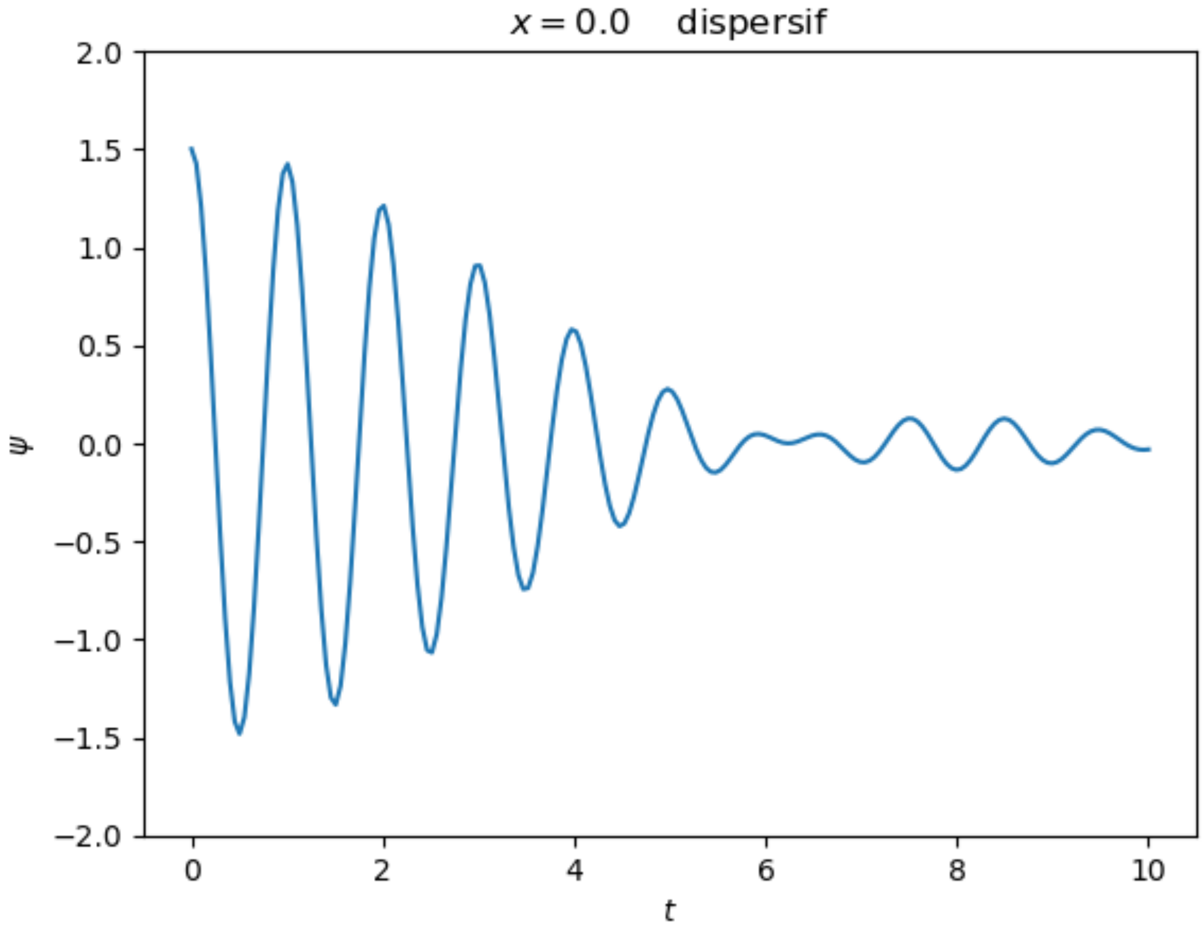
In [ ]:
```
for x in [0., 20.]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(t_domain)):
        t = t_domain[i]
        for j in range(len(f_domain)):
            f = f_domain[j]
            k = np.sqrt(2*np.pi*f)
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*t - k*x)*df


    plt.plot(t_domain, phi_1)
    plt.title(f"$x={x}$    dispersif")
    plt.xlabel("$t$")
    plt.ylabel("$\psi$")
    plt.ylim((-0.7, 0.7))
    plt.show()
```



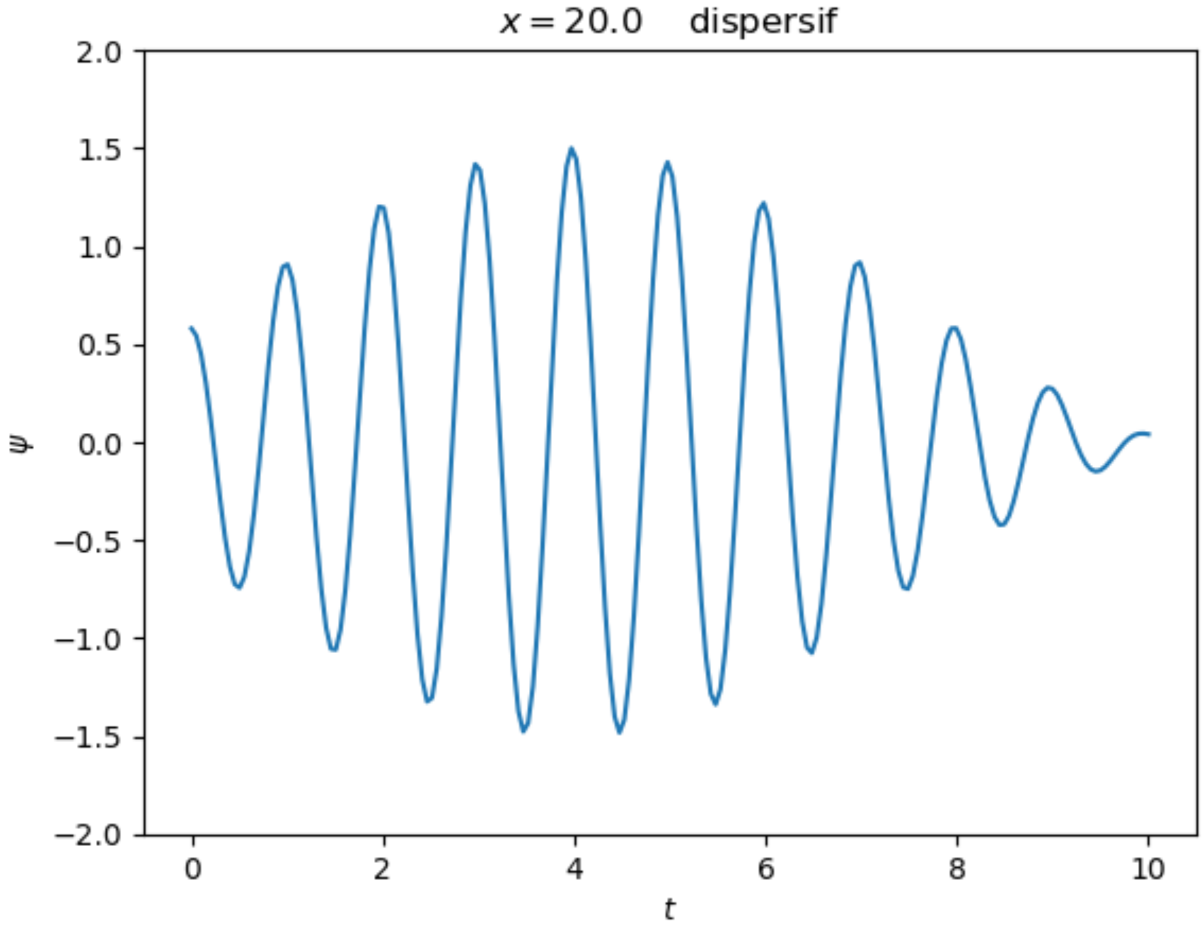


c)

In [ ]:
```
f_domain2 = np.linspace(0.9*f_0, 1.1*f_0, N)

for x in [0., 20.]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(t_domain)):
        t = t_domain[i]
        for j in range(len(f_domain2)):
            f = f_domain2[j]
            k = np.sqrt(2*np.pi*f)
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*t - k*x)*df


    plt.plot(t_domain, phi_1)
    plt.title(f"$x={x}$    dispersif")
    plt.xlabel("$t$")
    plt.ylabel("$\psi$")
    plt.ylim((-2, 2))
    plt.show()
```



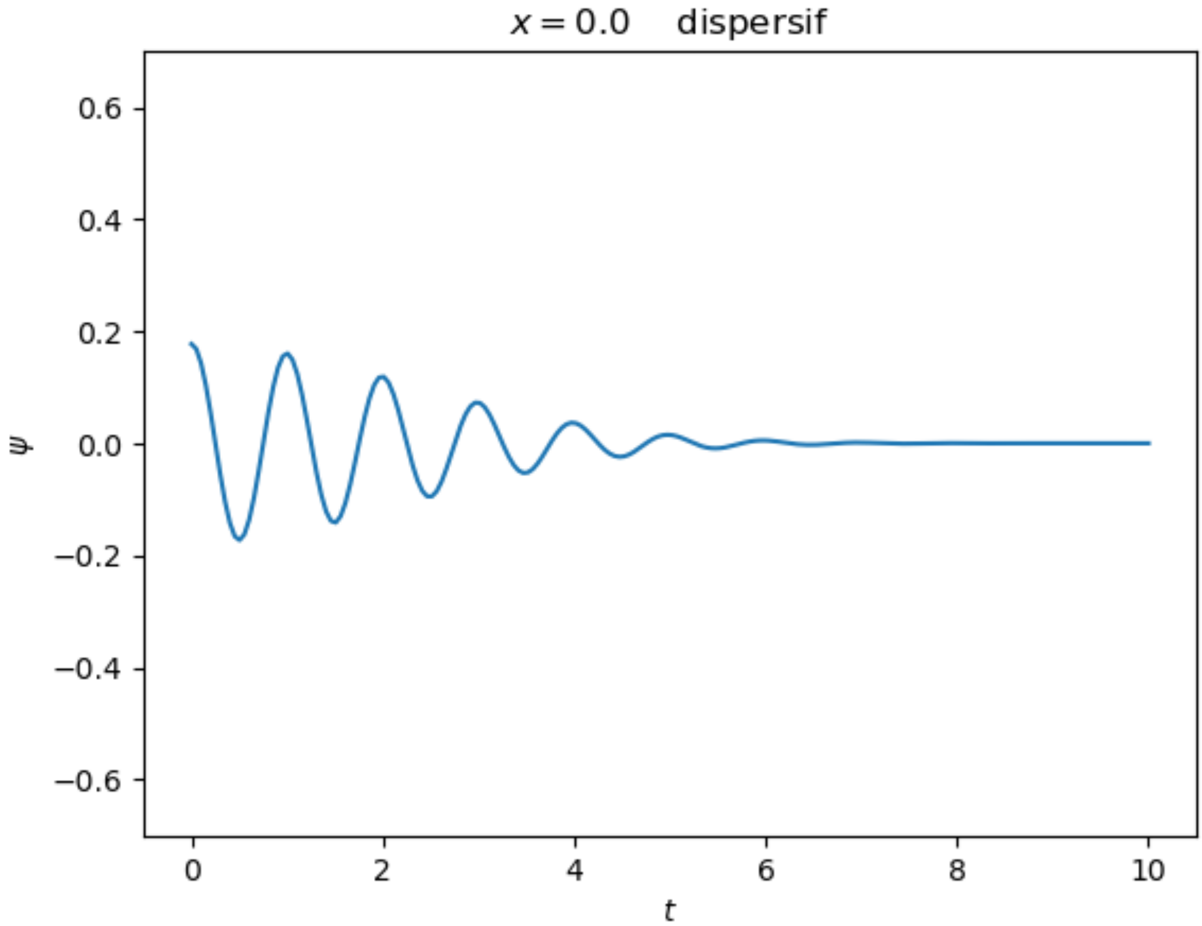
$x = 0.0$ dispersif
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
ψ
0
2
4
6
8
10
t



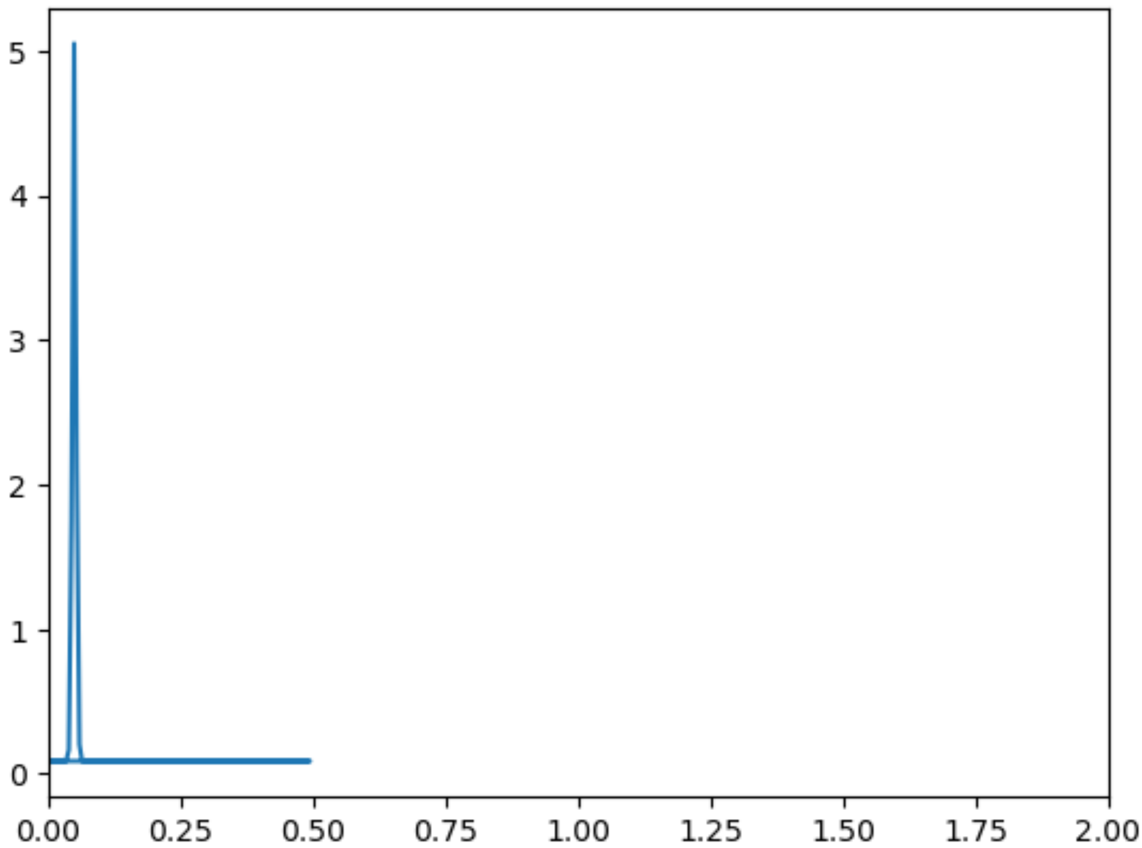
$x = 20.0$ dispersif
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
ψ
0
2
4
6
8
10
t

Questions bonus

In [ ]:
```
for x in [0.]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(t_domain)):
        t = t_domain[i]
        for j in range(len(f_domain)):
            f = f_domain[j]
            k = np.sqrt(2*np.pi*f)
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*t - k*x)*df


    plt.plot(t_domain, phi_1)
    plt.title(f"$x={x}$    dispersif")
    plt.xlabel("$t$")
    plt.ylabel("$\psi$")
    plt.ylim((-0.7, 0.7))
    plt.show()

    # Fourier transform
    sig_1_fft = np.fft.fft(phi_1)
    freq_domain = np.fft.fftfreq(len(t_domain))
    plt.plot(freq_domain, sig_1_fft.real)
    plt.xlim(0, 2*f_0)
    plt.show()
```

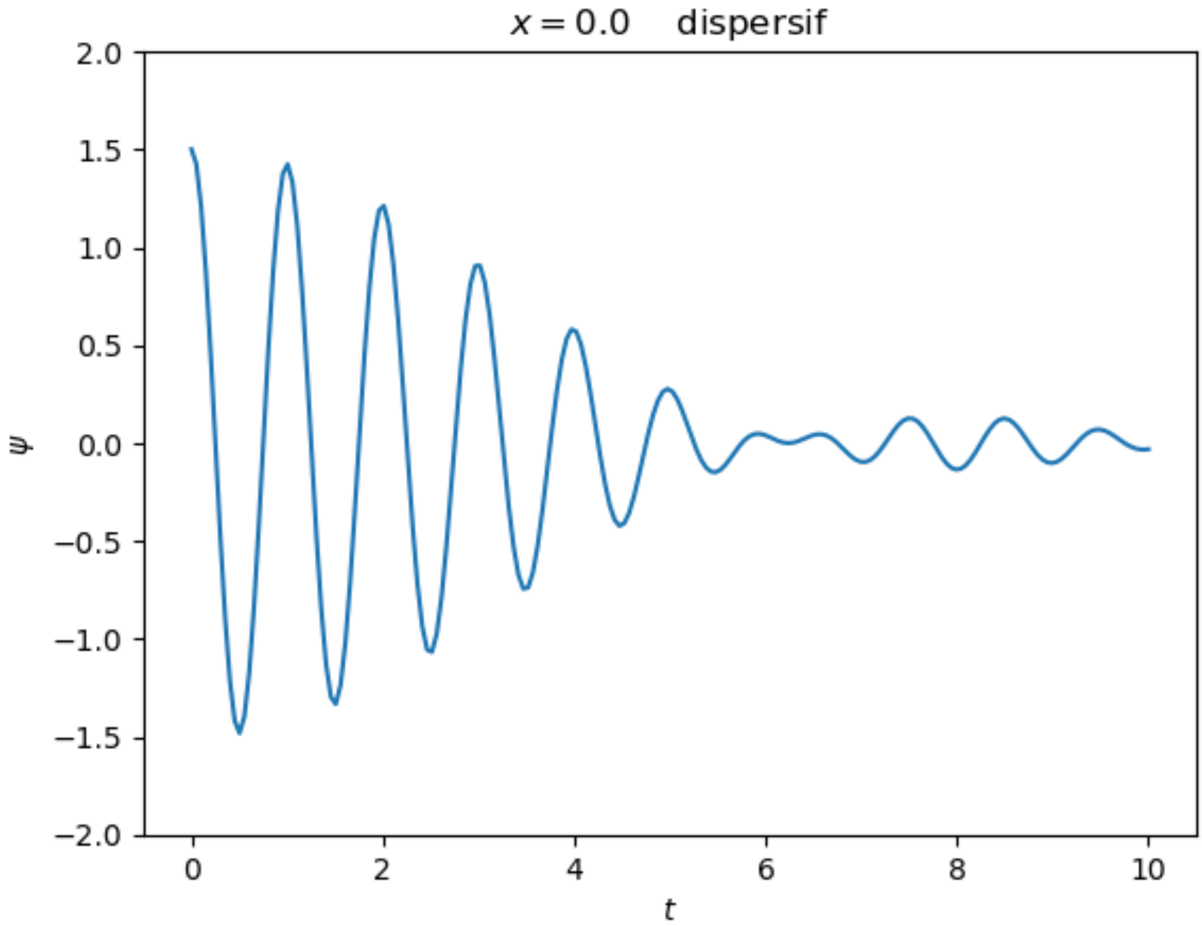


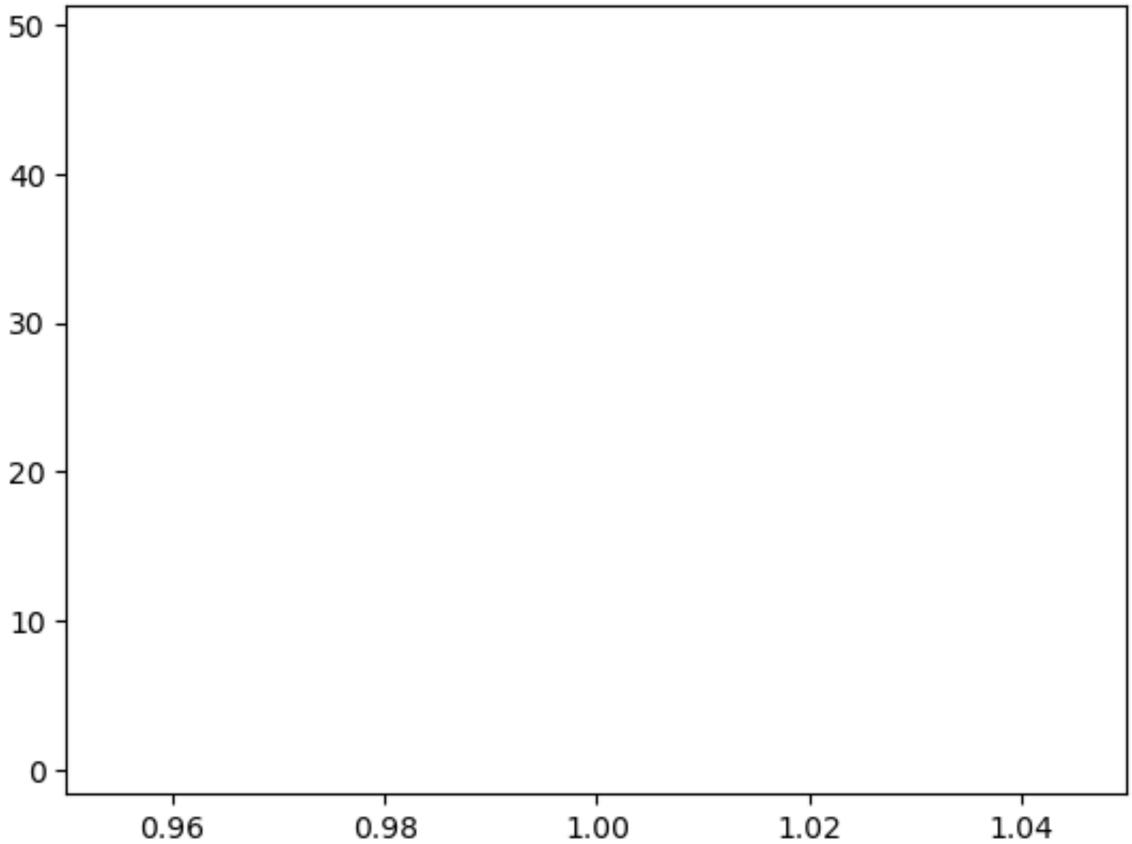
$x = 0.0$ dispersif
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
ψ
0
2
4
6
8
10
t



5
4
3
2
1
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

In [ ]:
```
for x in [0.]:
#t_1 = 0

    phi_1 = np.zeros(len(x_domain))
    for i in range(len(t_domain)):
        t = t_domain[i]
        for j in range(len(f_domain)):
            f = f_domain2[j]
            k = np.sqrt(2*np.pi*f)
            phi_1[i] += g(f, f_0, delta_f)*np.cos(2*np.pi*f*t - k*x)*df


    plt.plot(t_domain, phi_1)
    plt.title(f"$x={x}$    dispersif")
    plt.xlabel("$t$")
    plt.ylabel("$\psi$")
    plt.ylim((-2, 2))
    plt.show()

    # Fourier transform
    sig_1_fft = np.fft.fft(phi_1)
    freq_domain = np.fft.fftfreq(len(t_domain))
    plt.plot(freq_domain, sig_1_fft.real)
    plt.xlim(0.95*f_0, 1.05*f_0)
    plt.show()
```




50
40
30
20
10
0
0.96
0.98
1.00
1.02
1.04