Assignment for the course
The statistical analysis of time series

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1 Theoretical aspects

[1.]

[1.1] Compute the autocovariance function $\gamma_X(k) = \text{cov}(X_t, X_{t+k})$ for $k = 0, 1, \ldots$ Is χstationary? Do we need a restriction on θ?

[1.2] Use the definition of Fourier transform of γ_X and derive the expression of the spectral density, $f_X(\lambda)$, $\lambda \in [-1/2, 1/2]$.

[2.]

[2.1] Compute the autocovariance function $\lambda_Y(k) = \text{cov}(Y_t, Y_{t+k})$ for $k = 0, 1, \ldots$

[2.2] Use the definition of Fourier transform λ_Y and derive the expression of the spectral density $f_Y(\lambda)$, $\lambda \in [-1/2, 1/2]$

[3.]

[3.1] Derive the expression for joint autocovariance function $\lambda_{WY}(k) = \text{cov}(W_{t+k}, Y_t)$.

[3.2] Derive the expression of the cross-spectrum

[3.3] Let f_W be the spectral density of the white noise process. Making use of the result in point 3.2, rearrange the terms and express $f_{WY}(\lambda)$. and provide the form of the term $|A_{WY}(\lambda)|^2$

[4.]

[4.1] Show $\gamma_S(k)$

[4.1] Proof

2 Numerical exercises

Please see *Assignment_TS.rmd* for the analysis (code and comments).