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THE FRENCH AEROSPACE LAB

Domain decomposition

Theory and implementation

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Course Parallel Programming
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Overview

Main ideas

Principles

- Split a finite computing domain Ω (picture, spatial domain, and so.) in some sub-domains Ω_i ;
- Apply a local method on each subdomains ;
- Apply if necessary a global method to find the global result on Ω ;
- The two last steps can be in a direct or iterative method.

Example : JPEG 2000 compression (Wikipedia)

- Color components transformation (RGB to YUV) ;
- Image is split into tiles (subdomains) ;
- Wavelet transformation of each tiles ;
- Quantization of wavelet transformation of each tiles.

Classification of domain decomposition methods

Types of domain decomposition methods

- **Overlapping domain decomposition methods** : The sub-domains share common cells (pixels, elements, and so.) ;
- **Non Overlapping domain decomposition methods** : The sub-domains don't share common cells (pixels, elements, and so.).

Non overlapping domain decomposition methods

Splitting Ω in n subdomains Ω_i, $i = 1, \cdots, n$ verifying

1. $\Omega_i \neq \emptyset, i = 1, \cdots, n$;
2. $\cup_{i=1}^{n} \overline{\Omega_i} = \overline{\Omega}$;
3. $\Omega_i \cap \Omega_j = \emptyset; i \neq j$;
4. $\overline{\Omega_i} \cap \overline{\Omega_j} \neq \emptyset; i \neq j$.

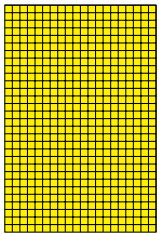
Overlapping method

Non overlapping method

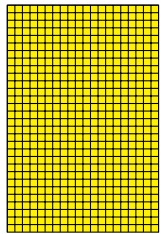
How split a computing domain ?

Let's Ω a rectangle area in $\mathbb{R}^2$.
Some type of splitting :

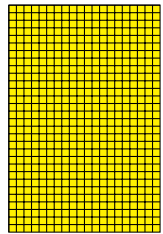
- One direction splitting : Easy to care, but not efficiency for communications ;
- Alternate direction : Split in two, four or height subdomains the initial domain Ω and iterate the splitting for subdomains ; Harder to care but more efficiency for parallelization and communication ;
- Adaptative splitting : Anyway, for parallel efficiency, better to split in the direction where we have a minimal number of cells.



One direction splitting



Alternate direction splitting



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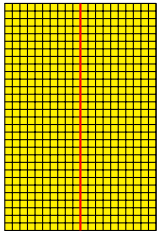

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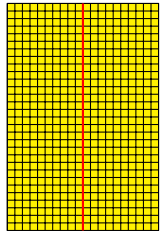

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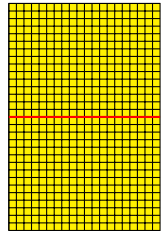

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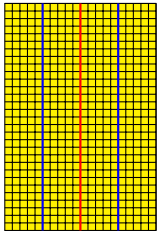
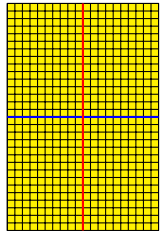
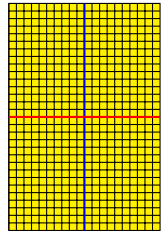


One direction splitting



Alternate direction splitting



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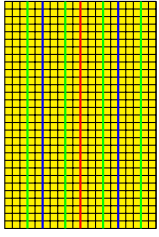
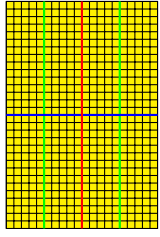
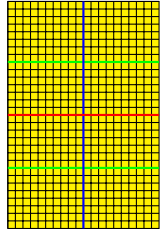


One direction splitting



Alternate direction splitting



Adaptative direction splitting

Domain decomposition for non structured mesh

Non structured mesh

- Mesh where cells can't be located with (multi-)indices ;
- Per example, a triangular mesh

How split a non structured mesh ?

- Minimize the size of the interfaces between sub-domains ;
- Optimal minimization is a NP-problem.
- Use some approximation or heuristics
- Some libraries exists for graph partitioning :
  - scotch (inria) ;
  - metis (Karypis Lab) ;

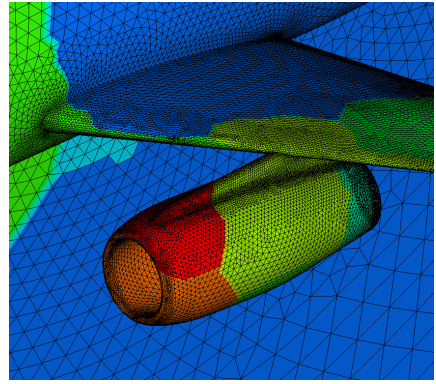


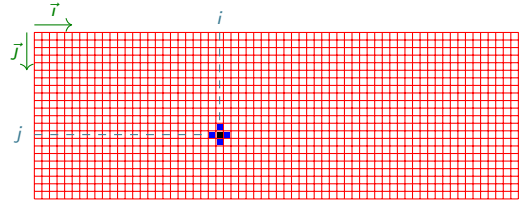
Figure – Fun 3D (NASA)

Overview

Baseline problem

Laplacian filter on a gray-scale image

- A pixel $p_{i,j}$ located by i and j indices on grid
- $v_{i,j}$ is the intensity of the pixel $p_{i,j}$;
- Discrete laplacian scheme
  $u_{i,j} = 4v_{ij} - v_{i-1,j} - v_{i+1,j} - v_{i,j-1} - v_{i,j+1}$
- Consider pixel out of the image as black ($= 0$)



Ghost cells

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Overview