Náhodné veličiny X a Y splňují vztah $X = 2Y$. Platí, že $E(X) = \mu$ a $var(X) = \sigma^2$.

- Určete $E(Y)$ a $var(Y)$.
- Určete $cov(X,Y)$ a $corr(X,Y)$.
- Určete $E(XY)$.
- Jsou veličiny X, Y nezávislé?

$$\mathbb{E}(X) = \mu \,, \quad var(X) = \sigma^2$$

$$Y = \frac{X}{2}$$

$$\mathbb{E}Y = \mathbb{E}(g(x)) = \int_{-\infty}^{+\infty} g(x) \cdot P(X \leq x) = \frac{1}{2} \cdot \underbrace{\int_{-\infty}^{+\infty} x \cdot P(X \leq x)}_{\mathbb{E}(X)} = \frac{\mathbb{E}(X)}{2}$$

$\circledast$ snáze $\mathbb{E}(aX + b) = a\,\mathbb{E}(X) + b$

dk v souboru str_lod_X+Y

$$var(Y) = \frac{var(X)}{2} \quad \dots \quad \mathbb{E}(Y^2) - \mathbb{E}(Y)^2$$

$$\mathbb{E}\left(\frac{E(X)}{2}^2\right) - \mathbb{E}\left(\frac{\mathbb{E}(X)}{2}\right)^2$$

$$\underbrace{\frac{\mathbb{E}(X^2)}{4} - \frac{\mathbb{E}(X)^2}{4}}_{} = \frac{1}{2}\left(\frac{\mathbb{E}(X^2)}{2} - \frac{\mathbb{E}(X)^2}{2}\right) = \frac{1}{2} var(X)$$

$$\frac{1}{4}\left(\mathbb{E}(X^2) - \mathbb{E}(X)^2\right) = \frac{1}{4} var(X)$$

$$\operatorname{cov}(X,Y) = \mathbb{E}\,(X-\mathbb{E}X)(\underbrace{Y-\mathbb{E}Y}_{})$$

$a \qquad \frac{a}{2}$

$$= \mathbb{E}(X-\mathbb{E}X)\cdot\frac{(X-\mathbb{E}X)}{2} = \frac{1}{2}\,\overbrace{\mathbb{E}(X-\mathbb{E}X)^2}^{\operatorname{var} X} = \frac{\operatorname{var} X}{2}$$

$$\operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sqrt{\operatorname{var}(X)\cdot\operatorname{var}(Y)}}$$

Pokud $\operatorname{var}(X) = 0$

jsou X a Y nezávislé

$$\mathbb{E}(XY) = \sum_{X}\sum_{Y} x\cdot y\cdot P(X=x \,\&\, Y=y)$$

$$\sum_{X} x \sum_{Y} y\, P(X=x \,\&\, Y=y)$$

$$\sum_{X} x\cdot\frac{x}{2}\,\underbrace{\sum_{Y} P(X=x \,\&\, Y=y)}_{P(X=x)}$$

$$\sum_{X} x\cdot\frac{x}{2}\,P(X=x) = \sum_{X} x\,P(X=x)\cdot\sum_{X}\frac{x}{2}\,P(X=x)$$

$$= \mathbb{E}X\cdot\mathbb{E}\frac{X}{2} = \mathbb{E}(X)\,\mathbb{E}(Y)$$