Věta o stř. hodnotě součtu náhodných veličin

Pro m.v. X, Y platí:

$$\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$g: X \to \mathbb{R} \qquad Y = g(x)$

$$\mathbb{E}(Y) = \sum_{x \in Im(X)} g(x) \, P(X = x)$$

dk $\quad \bigotimes = Im(x), \; \bigoplus = Im(y)$

$$\mathbb{E}(X + Y) \underset{LOTUS}{=} \sum_{\bigotimes} \sum_{\bigoplus} (x + y) \cdot P(X = x, Y = y)$$

$$= \sum_{\bigotimes} \sum_{\bigoplus} x \cdot P(X = x, Y = y) + \sum_{\bigotimes} \sum_{\bigoplus} y \cdot P(X = x, Y = y)$$

$$P(X = x, Y = y) = P(X = x \cap Y = y)$$

$$= \sum_{\bigotimes} x \underbrace{\sum_{\bigoplus} P(X = x, Y = y)}_{P(X = x)} + \sum_{\bigoplus} y \underbrace{\sum_{\bigotimes} P(X = x, Y = y)}_{P(Y = y)}$$

$$= \underline{\mathbb{E}(X) + \mathbb{E}(Y)}$$

$$\mathbb{E}(a \cdot X + b) = a \cdot \mathbb{E}(X) + b$$

$$g(x) = a \cdot x + b$$

$$\mathbb{E}(g(x)) = \sum_{x \in Im(X)} g(x) \cdot P(X = x)$$

$$= \sum_{x \in Im(X)} (ax + b) \, P(X = x)$$

$$= a \sum_{x \in Im(X)} x \, P(X = x) + b \underbrace{\sum_{x \in Im(X)} P(X = x)}_{= 1}$$

$$= a \, \mathbb{E}(X) + b$$