A NEW ERROR COMPENSATION SCHEME FOR INS VERTICAL CHANNEL

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Abstract: In the inertial navigation system, the altitude error diverges exponentially if external sensors do not compensate it. To suppress divergence of the error, non-inertial aiding sensors that provide vertical information are utilized. With these sensors, the well-known baro-inertial damping loop or the well-known Kalman filter mechanization can be constituted to compensate the error of the vertical channel. In this paper, a new error compensation scheme for INS vertical channel is proposed, especially with a barometer and GPS by fusing the two well established methods. The performance of the proposed scheme is analyzed by computer simulation.

Keywords: Inertial navigation, Kalman filters, Global positioning systems, Error estimation

1. INTRODUCTION

An Inertial Navigation System (INS) is a system that calculates the position, velocity, and attitude of a vehicle with output of inertial sensors. The measurement of the inertial sensors must include noises in it and the cheaper sensors provide the more noisy measurement. These noises are accumulated in the navigation solution of INS, corrupt the accuracy of the solution, and finally make the error of the solution diverge. Specially, the vertical channel error diverges exponentially, so if the error is not compensated with non-inertial sensors, the vertical information of INS can be trusted during a short time (Siouris, 1993). There are many techniques to suppress the divergence of vertical channel error. The most widely used methods are a vertical channel damping loop (baro-inertial damping loop) and Kalman filter mechanization. The vertical channel damping loop is a feedback control loop, which uses output of a non-inertial aiding sensor such as a barometer as a reference signal and it makes the altitude error of INS to converge to the noise level of the aiding sensor. The Kalman filter mechanization is an estimation and compensation scheme that makes use of the INS error model and measurement of non-inertial aiding sensors. In this paper, these conventional methods will be explained and, by fusion of the two methods, a new error compensation scheme for INS vertical channel will be proposed. The design example of the proposed scheme uses a barometer and GPS to compensate the vertical channel error. First, with a barometer, the well-known vertical channel damping loop is constituted, and then with GPS and the loop, a Kalman filter for error compensation is designed.

In the following two sections, the vertical channel damping loop and Kalman filter mechanization is explained. In section IV, a new scheme for error compensation of INS vertical channel is proposed and analysis of the proposed scheme is in the section V. The concluding remarks are presented in section VI.

2. VERTICAL CHANNEL DAMPING LOOP

The vertical channel damping loop is a system that suppresses the divergence of the vertical channel error and makes the height information calculated by INS trustable. It uses the output of non-inertial aiding sensor as reference input, and is classified into 1st order, 2nd order and 3rd order loop according to the number of integrators. The performance of the 3rd order loop is better and the others, so it is most widely used. The system equation of the loop with a aiding sensor is (1) and the structure of it is in fig. 1 (Siouris, 1993).

$$\begin{aligned} \dot{h} &= V_z - K_1(h - h_E) \\ \dot{V}_z &= A_z - K_2(h - h_E) - a + 2w_s^2 h \\ \dot{a} &= K_3(h - h_E) \end{aligned} \tag{1}$$

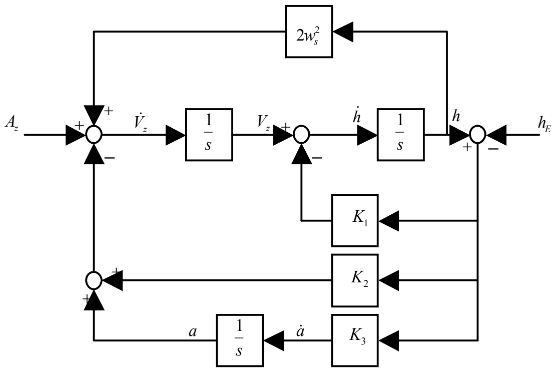

Fig 1. 3rd order damping loop

In equation (1) and fig. 1, A_z is vertical acceleration measurement, V_z is vertical velocity, h is height, h_E is output of an aiding sensor, and w_s is Schuler frequency. a is output of the compensator of the feedback loop and eliminates the steady state error of the above system (Dorf and Bishop, 1995). K_1, K_2, K_3 are feedback control gains and damp or control the instability of exponentially increasing vertical channel error. These gains can be calculated in various ways. In (Widnall and Sinha, 1978), the authors formulated this problem as a stochastic optimal control problem, and then obtained the gains that minimize the mean square error of vertical velocity. In another method, the time constant of vertical channel is properly chosen, and then the gains can be calculated by making the roots of characteristic equation of the system have the same value (Siouris, 1993).

This scheme is used in the case that one external non-inertial sensor such as a barometer is available, and, in the case of two available external sensors, the Kalman filter mechanization that is explained in the following section is used.

3. KALMAN FILTER MECHANIZATION FOR ERROR COMPENSATION

There is another method for vertical channel error compensation: Kalman filter mechanization. In this method, the vertical channel errors can be estimated and compensated with Kalman filter that consists of the error dynamics of INS vertical channel and measurement of aiding sensors. In this paper, the barometer and GPS are considered.

The measurement model of a barometer includes bias error of 1st order Markov process, scale factor error of random constant and white Gaussian noise, and is expressed in (2) (Siouris, 1993; Kim, 1996).

$$\begin{aligned} h_B &= h_T + \delta h_B = h_T + B + Sh_T + v_B \\ \dot{B} &= -\frac{1}{\tau}B + w \\ \dot{S} &= 0 \end{aligned} \tag{2}$$

where, h_B is measurement of a barometer, h_T is true height, δh_B is error component of output of a barometer, B is bias error, S is scale factor error, v_B is measurement noise, τ is correlation time of bias error, and w is driving white Gaussian noise for bias error. With measurement model (2) and with GPS measurement, the Kalman filter for vertical error compensation is designed in equation (3), (4), (5) (Ausman, 1991).

$$\begin{bmatrix} \delta\dot{h} \\ \delta\dot{V}_z \\ \dot{B} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2w_s^2 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta h \\ \delta V_z \\ B \\ S \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta A_z \\ \omega_B \end{bmatrix} \tag{3}$$

$$m_B = h - h_B = \begin{bmatrix} 1 & 0 & -1 & -h \end{bmatrix} \begin{bmatrix} \delta h \\ \delta V_z \\ B \\ S \end{bmatrix} + v_B \tag{4}$$

$$m_{GPS} = h - h_{GPS} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta h \\ \delta V_z \\ B \\ S \end{bmatrix} + v_{GPS} \tag{5}$$

where, h_B and h_{GPS} are measurement of a barometer and GPS, respectively, and v_{GPS} is measurement noise of GPS.

The above structure of Kalman filter mechanization for vertical error compensation is a typical example with the barometer measurement model of equation (2). If the measurement model is modified, then Kalman filters with other structures should be used. For example, according to the properties of a barometer, the bias error and scale factor error can be modeled as Markov process or random constant. Then, in either case the Kalman filter for error compensation has different structure (Ausman, 1991). In the above system, one of the components of the observation model of the barometer is the height value. Therefore, if the estimated height has large error, then the model has considerable uncertainty. It deteriorates the accuracy of the system. The proposed scheme explained in the next section, mitigates the effect of uncertainty.

4. A NEW SCHEME FOR ERROR COMPENSATION

In this section, a new scheme by fusion of the two previously explained methods is proposed. In the newly proposed scheme, a Kalman filter with error dynamics of 3rd order vertical channel damping loop in fig. 1 and GPS measurement is designed, then the estimates of Kalman filter are used for error compensation. In other words, though a barometer and GPS are used as non-inertial aiding sensors, the Kalman filter described in section III does not constituted, but the vertical channel damping loop with a barometer is constituted, the error dynamics of it is derived, and finally Kalman filter with the error dynamics and GPS measurement is designed. In this scheme, the errors of the vertical channel damping loop can be estimated and compensated.

Now, a filter to compensate the error of the system in fig. 1 is designed. After applying the linear perturbation method to the system of fig. 1, the error model which consists of only error terms are derived. The error model is equation (6) and the structure of the model is in fig. 2.

$$
\begin{aligned}
\delta\dot{h} &= \delta V_z - K_1(\delta h - \delta h_B) \\
\delta\dot{V}_z &= \delta A_z - K_2(\delta h - \delta h_B) - \delta a + 2w_s^2\delta h \\
\delta\dot{a} &= K_3(\delta h - \delta h_B)
\end{aligned}
\tag{6}
$$

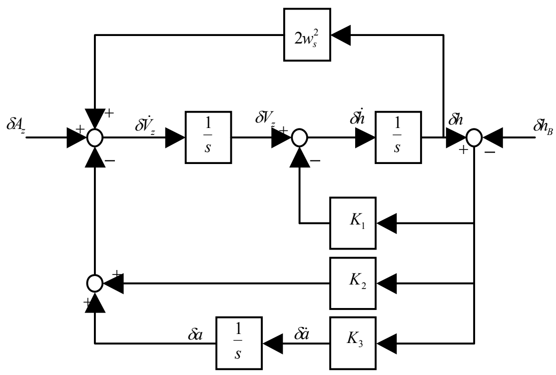

Fig 2. Error model of 3rd order vertical channel camping loop

Designing Kalman filter with equation (6), measurement model (2) of the barometer, and measurement of GPS, the equation (7), (8) are obtained.

System dynamics :

$$
\begin{bmatrix} \delta\dot{h} \\ \delta\dot{V}_z \\ \delta\dot{a} \\ \dot{B} \\ \dot{B}_s \end{bmatrix}
=
\begin{bmatrix}
-K_1 & 1 & 0 & K_1 & K_1 h \\
2w_s^2 - K_2 & 0 & -1 & K_2 & K_2 h \\
K_3 & 0 & 0 & -K_3 & -K_3 h \\
0 & 0 & 0 & -\frac{1}{\tau} & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \delta h \\ \delta V_z \\ \delta a \\ B \\ B_s \end{bmatrix}
+
\begin{bmatrix}
K_1 & 0 & 0 \\
K_2 & 1 & 0 \\
-K_3 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_B \\ \delta A_z \\ w \end{bmatrix}
\tag{7}
$$

Measurement equation :

$$
m_{GPS} = h - h_{GPS} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \delta h \\ \delta V_z \\ \delta a \\ B \\ B_s \end{bmatrix} + v_{GPS}
\tag{8}
$$

After estimating the error of the INS vertical channel damping loop with this Kalman filter model, it can be compensated.

In this model, there is height value in system matrix. But, because the feedback gain K's are multiplied to that, the effect of model uncertainty is reduced and the performance becomes better.

5. SIMULATION

5.1 Simulation condition and method

The INS vertical channel by the proposed method in section IV is constituted, and the performance of it is analyzed by Monte Carlo simulation. The simulation condition is in table 1. The mean square of the vertical channel error of the proposed scheme is compared with that of the conventional Kalman filter mechanization. The individual Monte Carlo simulation repeats 100 iterations, and the time constant of the vertical channel damping loop with a barometer is determined as 5 seconds, consequently the gains of the loop are determined as follows (Siouris, 1993).

$$K_1 = 0.6, \quad K_2 = 0.120003, \quad K_3 = 0.008$$

The time constant of the Markov process is determined as 100 seconds.

Table 1. Simulation conditions

Sensor	Error type	Value
Barometer	Measurement noise (1σ)	3m
	1st order Markov driving white Gaussian noise (1σ)	0.01m/s

	1st order Markov initial bias (1σ)	10m
	Scale factor error (1σ)	0.01
GPS	Measurement noise (1σ)	15m
Accelerometer	Measurement noise (1σ)	$0.005 m/s^2$
Filter initial values	Height (1σ)	25m
	Vertical velocity (1σ)	1m/s
	a (compensator output; 1σ)	0.5

For the error compensation, the feedforward method and the feedback method are adopted. The structures of them are in fig. 3 and 4.

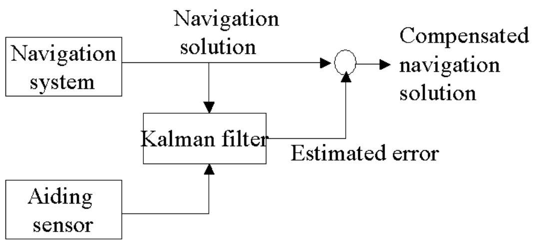

Fig 3. Feedforward method

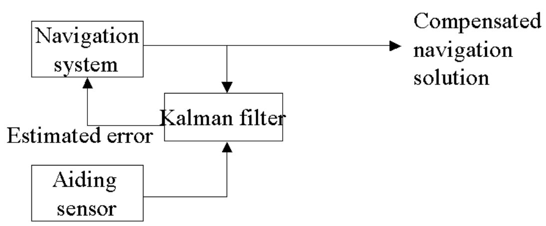

Fig 4. Feedback method

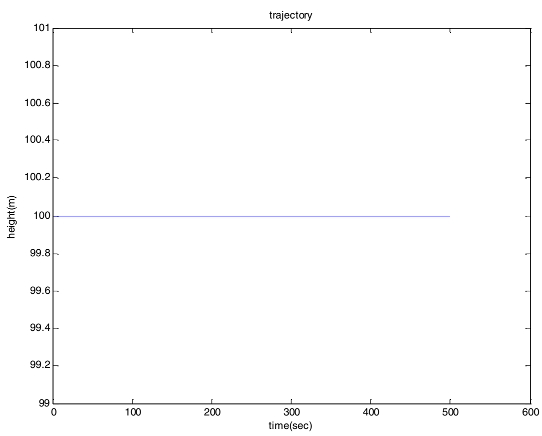

Fig 5. Trajectory I

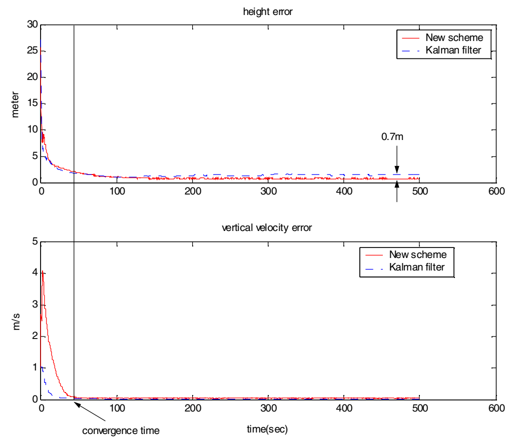

Fig 6. Feedforward method

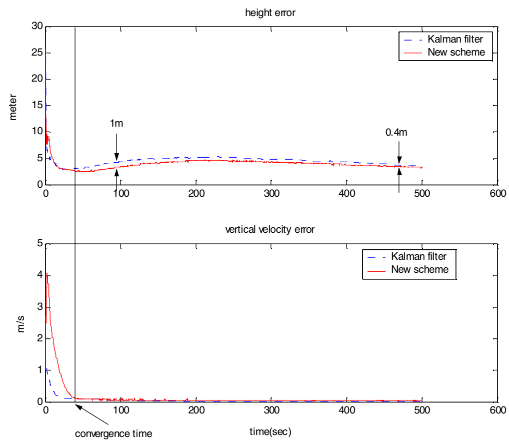

Fig 7. Feedback method

5.2 Level flight

In the case of a fixed height trajectory such as fig. 5, the simulation results by the feedforward method and the feedback method are in fig. 6 and fig. 7, respectively.

5.3 Rapid climbs and dives

In the case of rapid climbs and dives such as fig. 8, the acceleration during climbs and dives is $45m/s^2$(about 4.6g). The simulation results are in fig. 9 and fig. 10.

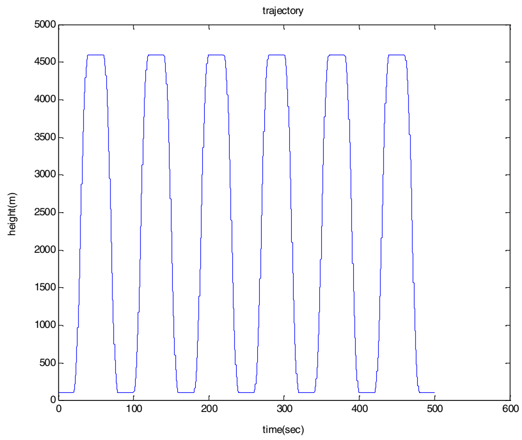

Fig 8. Trajectory II

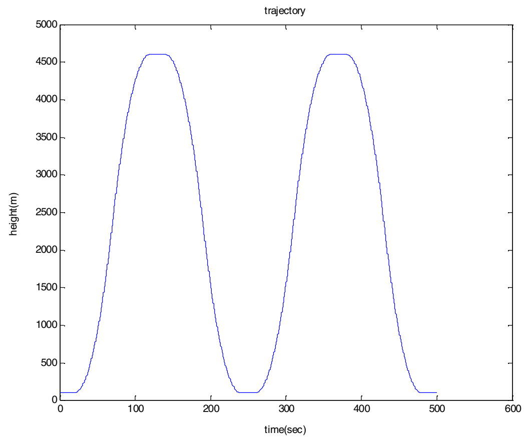

Fig 11. Trajectory III

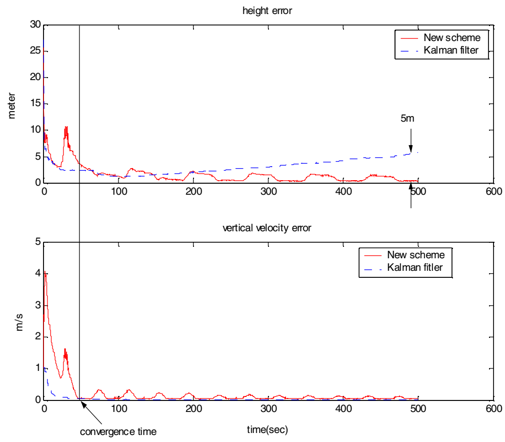

Fig 9. Feedforward method

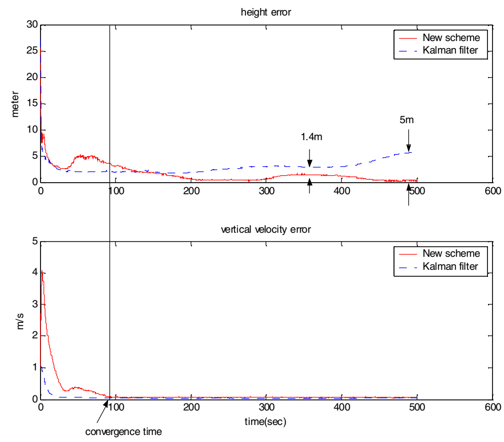

Fig 12. Feedforward method

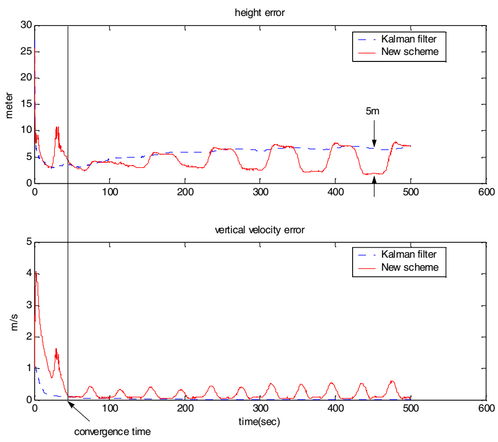

Fig 10. Feedback method

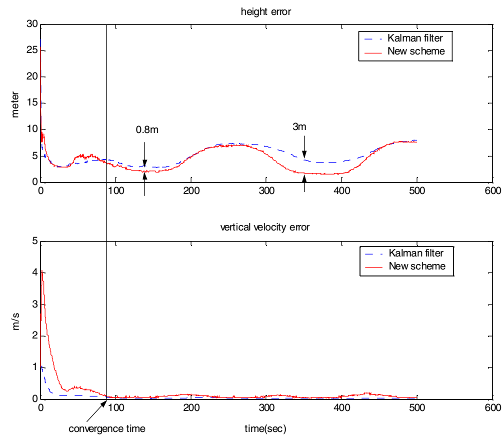

Fig 13. Feedback method

5.4 Slow climbs and dives

In the situation of slowly varied height such as fig. 11, the applied acceleration in the vertical axis for maneuvers is $1.8m/s^2$(about 0.18g) which is much smaller than the former case. In this trajectory, the results are in fig. 12 and fig. 13.

5.5 Analysis of the simulation results

Due to the time constant of the loop, the proposed system takes more time to converge than the previous Kalman filter mechanization does. So, from the start time to about 50~90 seconds, the new scheme aggravates the vertical performance of INS. But after convergence time, the newly proposed scheme reduces the error of altitude and shows good performance. In contrast, the vertical velocity error in the new scheme is larger than that of the conventional Kalman filter mechanization before and after the convergence time. In height estimation, the feedforward method is superior to the feedback method, and the altitude error is dependent on the height itself. In feedforward method, when the height decreases, the error of that reduces and when the height increases, the error of that grows, too. But, in the feedback method, vice versa.

In the new scheme, the Kalman filter is applied to the damping loop that makes the altitude error converge to the noise level of a barometer, and with the help of GPS, the error terms (bias error, scale factor error) of a barometer as well as the vertical errors of the loop can be estimated. In the feedforward method, the states of the system (which is the errors of the loop) propagate through the dynamics of the damping loop. So, it produces more improved result.

6. Conclusion

In this paper, an effective scheme to suppress the exponentially increasing altitude error is proposed. The proposed method is organized by fusing the well-known damping loop and the Kalman fitler, and in the scheme, the error equation of 3rd order damping loop is used as the system dynamics of Kalman filter and the information of GPS is utilized as measurement of the filter. Through computer simulation, the performance of the new scheme is compared with that of the previous Kalman filter mechanization, and the new scheme shows better performance for the altitude error compensation after convergence of the system.

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