

IMU Wind Estimation (Theory)

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Three dimensional wind vector can be estimated with a GPS receiver and the direction cosine matrix in an airframe that is flying with changes in attitude.

The starting point is the identity that relates the speed over ground to the airspeed and the wind:

$$\mathbf{S} = \mathbf{V} + \mathbf{W}$$
$$\mathbf{S} = \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \text{speed over ground vector in earth frame}$$
$$\mathbf{V} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \text{air speed vector in earth frame} \quad \text{equation 1}$$
$$\mathbf{W} = \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} = \text{wind speed vector in earth frame}$$

Whenever the plane is changing attitude, it is possible to use a pair of GPS measurements and direction cosine matrix values to estimate the wind vector, assuming that the wind is about the same for both measurements. Two measurements about 1 second apart will do. The timing is not critical. Suppose we apply the previous equation to both sets of measurements, and take the sum and the difference of the equations. We will find the following two equations:

$$\mathbf{S}_2 - \mathbf{S}_1 = \mathbf{V}_2 - \mathbf{V}_1 \quad \text{equation 2}$$

$$\mathbf{W} = \frac{(\mathbf{S}_2 + \mathbf{S}_1) - (\mathbf{V}_2 + \mathbf{V}_1)}{2} \quad \text{equation 3}$$

We can measure the 3 dimensional velocity over ground vectors with a GPS. If we can somehow compute the air speed vectors, we can compute the wind speed vector.

If the plane is pitching and/or turning, we can use equation 2 and the direction cosine matrix to estimate the airspeed.

We can make the approximation that for an airframe with a tail the air speed vector is parallel with the fuselage. (Actually, many of the following results hold even if that is not true.) In that case, the airspeed vector is equal to the magnitude of the airspeed, times the column of the direction cosine matrix that represents the fuselage. We assume that the majority of the change in speed vector over a one second time interval is due to a change in direction rather than a change in magnitude, so we assume a constant magnitude.

That column of the direction cosine matrix will be rather accurate regarding the pitch of the plane, but there might be a yaw error. So, we represent the airspeed vector in terms of the fuselage column of the direction cosine matrix as follows:

$$\mathbf{V} \approx V \cdot \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \mathbf{F} \quad \text{equation 4}$$

V = magnitude of the airspeed

θ = residual yaw error in the direction cosine matrix

\mathbf{F} = column of the DCM that represents the fuselage

The following is obtained by substituting equation 4 into equation 2 and rearranging:

$$\mathbf{S}_2 - \mathbf{S}_1 \approx V \cdot \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot (\mathbf{F}_2 - \mathbf{F}_1) \quad \text{equation 5}$$

It is easy to show that equation 5 leads to the following equation for computing the airspeed:

$$V \approx \frac{|\mathbf{S}_2 - \mathbf{S}_1|}{|\mathbf{F}_2 - \mathbf{F}_1|} \quad \text{equation 6}$$

By the way, equation 6 is valid even if the airspeed vector is not parallel to the fuselage, as long as they both rotate by approximately the same amount between the two measurements. Also note that the time between the measurements does not enter into the equation, and we have not taken the derivative of anything.

Equation 6 gives us a simple way to estimate airspeed without a pitot. All we have to do is to take the ratio of the magnitude of the change in GPS velocity divided by the magnitude of the change in the “fuselage” column of the direction cosine matrix. We could stop right here and still have a very useful result.

The next thing we need to do is to estimate any residual yaw error. First, we define unit vectors in the directions of the vectors in equation 5:

$$\mathbf{s}_{21} = \frac{\mathbf{S}_2 - \mathbf{S}_1}{|\mathbf{S}_2 - \mathbf{S}_1|} \quad \text{equation 7}$$

$$\mathbf{f}_{21} = \frac{\mathbf{F}_2 - \mathbf{F}_1}{|\mathbf{F}_2 - \mathbf{F}_1|}$$

Equation 5 then becomes:

$$\begin{bmatrix} s_{21x} \\ s_{21y} \\ s_{21z} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} f_{21x} \\ f_{21y} \\ f_{21z} \end{bmatrix} \quad \text{equation 8}$$

Actually, if we take a close look at equation 8, we realize that it can only be approximately satisfied, and that what we should have done in equation 4 was to use a more general representation for the attitude error in the DCM. But we can get an answer that is close enough for our purposes by solving the following equation for the yaw rotation residual error:

$$\begin{bmatrix} S_{21x} \\ S_{21y} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} f_{21x} \\ f_{21y} \end{bmatrix} \quad \text{equation 9}$$

Equation 9 is easy enough to solve for θ , it is equal to the angle between the projections on to the horizontal plane of the vectors in equation 2.

Substituting everything we know at this point back into equation 3, we arrive at the following equations for the estimates of the three components of the wind vector:

$$W_x = \frac{S_{1x} + S_{2x} - V \cdot (\cos(\theta) \cdot (F_{1x} + F_{2x}) - \sin(\theta) \cdot (F_{1y} + F_{2y}))}{2} \quad \text{equation 10}$$

$$W_y = \frac{S_{1y} + S_{2y} - V \cdot (\sin(\theta) \cdot (F_{1x} + F_{2x}) + \cos(\theta) \cdot (F_{1y} + F_{2y}))}{2} \quad \text{equation 11}$$

$$W_z = \frac{S_{1z} + S_{2z} - V \cdot (F_{1z} + F_{2z})}{2} \quad \text{equation 12}$$

Equations 6, 9, 10, 11, and 12 define the method for estimating the wind velocity vector from a pair of GPS and IMU measurements. Implementation will be described in a separate report. The following observations can be made about the technique:

- It reports all 3 components of the wind vector. The Z component should be very useful for sailplane pilots.
- The method reports the wind in real time. Individual wind estimates can either be used for real time decisions and computations, or filtered to provide a more accurate estimate of the wind.
- For the method to work, the plane must change attitude. A simple test to determine whether it can be applied at any instant is whether or not the denominator of equation 6 exceeds a minimum threshold. It is not necessary for the plane to change its yaw angle for it to work, a change in pitch angle will do just as well.