



MTHS24 – Exercise sheet 9

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Afternoon: XXX, YYY



Wednesday, 24 July 2024

Lecture material

Discussed topics:

- Hyperon-nucleon and hyperon-hyperon interactions
- Meson-exchange models
- Chiral effective field theory
- Hyperons in matter
- Brueckner-Goldstone Theory: Brueckner-Hartree-Fock approach

References:

- R. Machleidt, Advances in Nuclear Physics **19**, 189 (1989)
- R. Machleidt, D. R. Entem, Phys. Rept. **503**, 1 (2011)
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- V. Koch, Int. J. Mod. Phys. E **6**, 203 (1997)
- S. Petschauer, J. Haidenbauer, N. Kaiser, U. G. Meißner and W. Weise, Front. in Phys. **8**, 12 (2020)
- B. D. Day, Reviews of Modern Physics **39**, 719 (1967); **50**, 495 (1978)
- R.D. Mattuck, A guide to Feynman Diagrams in the Many-Body problem, Dover, New York, 1992. Editor McGraw-Hill, Inc.

Exercises

9.1 Effective Theory Questions

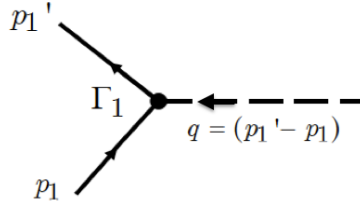
- Explain the Yukawa's idea (for NN interaction)
- Explain what an effective theory is and indicate the four pillars where the power of an effective theory lies in.
- Explain the many-body problem (for NN interaction)

9.2 Gradient Coupling

Consider the pseudo-vector (or gradient coupling) to the nucleon described by the Lagrangian

$$\mathcal{L} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi \cdot \partial_\mu \vec{\phi}^{(\pi)}, \quad (1)$$

and compute the contribution of the following diagram to the one-pion exchange potential (OPEP)



Some hints:

- You should compute $\bar{u}(p_1', s_1) \Gamma_{\pi NN} u(p_1, s_1)$, with $\Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi} \gamma^\mu \gamma_5 \vec{\tau} q_\mu$ for the incoming pion. Note that i is the imaginary unit, (γ^μ, γ_5) are the gamma matrices, $\vec{\tau}$ is the isospin vector, q is the four-momentum carried by the pion ($q_\mu = p_1' - p_1$), $f_{\pi NN}$ is the πNN coupling and m_π is the pion mass.
- Consider the static limit ($q_0 \rightarrow 0$)
- The Dirac spinors $u(p, s)$ in the non-relativistic approach are given by $u(p, s) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$, with χ_s the two-component Pauli spinor.
- The gamma matrices are defined as

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma^5 = \gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

with σ^k the three Pauli matrices (k running from 1 to 3). Also $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\{\gamma_5, \gamma^\mu\} = 0$, with μ and ν running from 0 to 3. The metric tensor is $g_{00} = +1$, $g_{kk} = -1$, $g_{\mu \neq \nu} = 0$.

9.3 Nucleon-nucleon potential with scalar meson exchange

Calculate the nucleon-nucleon potential due to scalar meson exchange. Things to be considered:

- you have to reproduce the expression on the slides;
- the scalar propagator is given by

$$\frac{i}{q^2 - m_s^2},$$

where m_s is the scalar mass and q the four-momentum;

- work in the center-of-mass frame.
If \vec{p}_1 y \vec{p}_2 are the momenta of initial particles 1 y 2, respectively, and \vec{p}'_1 y \vec{p}'_2 are the momenta of the final particles 1 y 2, respectively, then we can define

$$\begin{aligned} \vec{p}_1 &= -\vec{p}_2 = \vec{p}, \\ \vec{p}'_1 &= -\vec{p}'_2 = \vec{p}'. \end{aligned}$$

With these definitions, we define

$$\begin{aligned} \vec{k} &= 1/2(\vec{p} + \vec{p}'), \\ \vec{q} &= \vec{p}' - \vec{p}; \end{aligned}$$

- work in the non-relativistic approximation: $E + M \sim 2M$;
- the angular momentum \vec{L} is defined as $\vec{L} = i (\vec{k} \times \vec{q})$.