MTHS24 – Exercise sheet 11

Morning: Christian Fischer

Afternoon:

Friday, 26 July 2024

Lecture material

Discussed topics:

- Functional methods
- Dynamical Chiral Symmetry Breaking
- Spectra of conventional and exotic hadrons
- (optional: g-2, form factors,...)

References:

- Eichmann et al., "Baryons as relativistic three-quark bound states," PPNP **91** (2016), 1-100 arXiv:1606.09602 [hep-ph]].
- Eichmann et al. "Four-Quark States from Functional Methods," FBS **61** (2020) no.4, 38 arXiv:2008.10240 [hep-ph].

Exercices

11.1 Diquarks

Write down spin, color and flavour wave functions for a scalar and an axialvector diquark built from

(a) two light quarks (what is the resulting isospin ?)

(b) two strange, charm or bottom quarks

(c) a heavy-(not-so-heavy) combination such as bc, bs or cs.

Hint: carefully think about symmetries...

Solution: I.) **Scalar diquarks:**
Spin S=0 $\rightarrow \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ and antisymmetric
Color: From Young-Tableaux we find $3 \bigotimes 3 = 6 \bigoplus \bar{3}$ and $\bar{3}$ is antisymmetric, while 6 is symmetric.
Thus we need an antisymmetric flavour wave function together with $\bar{3}$-color and a symmetric flavour wave function together with 6-color.
We obtain for $\bar{3}$-color:
(a) $\frac{1}{\sqrt{2}}(ud - du)$ and we have I=0.
(b) not possible
(c) $\frac{1}{\sqrt{2}}(bc - cb)$ and analogously for the others.
We obtain for 6-color:
(a) $\{\frac{1}{\sqrt{2}}(ud + du), uu, dd\}$ and we have I=1.
(b) ss, cc, bb
(c) $\frac{1}{\sqrt{2}}(bc + cb)$ and analogously for the others.

II.) Axialvector diquarks:
Spin S=1 $\rightarrow \{\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \uparrow\uparrow, \downarrow\downarrow\}$ and symmetric
Color: Same as above. But now we need a symmetric flavour wave function together with $\bar{3}$-color and an antisymmetric flavour wave function together with 6-color.
Thus, flavour/color combinations are interchanged as compared to scalar diquark.

11.2 Four-quark states

Now think about a four-quark state with two heavy quarks and two light anti-quarks in the two flavour combinations $bb\bar{q}\bar{q}$ and $bc\bar{q}\bar{q}$. Suppose, the quarks and antiquarks are arranged in scalar (S) and axialvector (A) diquarks. Which diquark combinations are possible for the following quantum numbers?

(a) $I(J) = 0(1)$

Solution: $J = 1 \rightarrow$ we need at least one axialvector diquark, i.e. only combinations AA, SA, AS are possible.
color $3 \bigotimes \bar{3}$:
$I = 0 \rightarrow$ light diquark needs to be S $\rightarrow$ heavy diquark needs to be A and indeed, this is possible
color $6 \bigotimes \bar{6}$:
$I = 0 \rightarrow$ light diquark needs to be A $\rightarrow$ heavy diquark needs to be S and indeed, this is possible

(b) $I(J) = 1(1)$

Solution: $J = 1 \rightarrow$ we need at least one axialvector diquark, i.e. only combinations AA, SA, AS are possible.
color $3 \bigotimes \bar{3}$:
$I = 1 \rightarrow$ light diquark needs to be A. Heavy diquark also needs to be A (S is not possible).
color $6 \bigotimes \bar{6}$:
$I = 1 \rightarrow$ light diquark needs to be S $\rightarrow$ heavy diquark needs to be A, but this is not possible.

(c) $I(J) = 0(0)$

Solution: $J = 0 \rightarrow$ we need either SS or AA (from rules of adding angular momenta).
color $3 \bigotimes \bar{3}$:
$I = 0 \rightarrow$ light diquark needs to be S $\rightarrow$ heavy diquark also needs to be S, but this is not possible.
color $6 \bigotimes \bar{6}$:
$I = 0 \rightarrow$ light diquark needs to be A $\rightarrow$ heavy diquark also needs to be A, but this is not possible.

Hint: again carefully think about symmetries...