

## MTHS24 - Exercise sheet 7

Morning: Vincent Mathieu / Arkaitz Rodas Afternoon: Daniel Winney, Whytt Smith



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#### Lecture material

#### Discussed topics:

- Partial waves
- Analyticity
- Unitarity

## References:

- Book 1, inspire
- Book 2, amazon
- A good review, inspire
- A good aper, inspire

## **Exercices**

# 7.1 Two-body phase space

- (a) Particle A with mass M decays into two daughters with masses  $m_1$  and  $m_2$ . Derive the formula for the *break up momentum*,  $\mathbf{p}^* = |\mathbf{p}_1| = |\mathbf{p}_2|$ , in the rest frame of A. Use your result to evaluate  $\mathbf{p}^*$  for the decay  $\Delta(1232) \to p\pi$ .
- (b) Starting from the formula for the 2-body decay rate,

$$\Gamma_{fi} = \frac{1}{2M} \int |\mathcal{M}_{fi}|^2 \frac{\mathsf{d}^3 p_1}{(2\pi)^3 2E_1} \frac{\mathsf{d}^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4 (P - p_1 - p_2) ,$$

perform integrations using  $\delta$  functions to obtain  $\mathrm{d}\Gamma/\mathrm{d}\Omega$  in the centre of mass frame,  $P=\begin{pmatrix} M \\ \vec{0} \end{pmatrix}$ , to obtain the 2-body phase-space factor.

Hint: You may want to use the following property of the delta function:

$$\int_{-\infty}^{+\infty}g(x)\delta(f(x))\mathrm{d}x=\sum_{x_0}\frac{g(x_0)}{|\mathrm{d}f/\mathrm{d}x|_{x_0}}, \text{ where } x_0\in\{x:\,f(x)=0\}.$$