



MTHS24 – Exercise sheet 11

Morning: Christian Fischer

Afternoon:



Friday, 26 July 2024

Lecture material

Discussed topics:

- Functional methods
- Dynamical Chiral Symmetry Breaking
- Spectra of conventional and exotic hadrons
- (optional: $g-2$, form factors,...)

References:

- Eichmann et al., “Baryons as relativistic three-quark bound states,” PPNP **91** (2016), 1-100 [arXiv:1606.09602 \[hep-ph\]](#).
- Eichmann et al. “Four-Quark States from Functional Methods,” FBS **61** (2020) no.4, 38 [arXiv:2008.10240 \[hep-ph\]](#).

Exercises

11.1 Diquarks

Write down spin, color and flavour wave functions for a scalar and an axialvector diquark built from

- (a) two light quarks (what is the resulting isospin ?)
- (b) two strange, charm or bottom quarks
- (c) a heavy-(not-so-heavy) combination such as bc , bs or cs .

Hint: carefully think about symmetries...

Solution: I.) Scalar diquarks:

Spin $S=0 \rightarrow \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ and antisymmetric

Color: From Young-Tableaux we find $3 \otimes 3 = 6 \oplus \bar{3}$ and $\bar{3}$ is antisymmetric, while 6 is symmetric.

Thus we need an antisymmetric flavour wave function together with $\bar{3}$ -color and a symmetric flavour wave function together with 6-color.

We obtain for $\bar{3}$ -color:

- (a) $\frac{1}{\sqrt{2}}(ud - du)$ and we have $I=0$.
- (b) not possible
- (c) $\frac{1}{\sqrt{2}}(bc - cb)$ and analogously for the others.

We obtain for 6-color:

- (a) $\{\frac{1}{\sqrt{2}}(ud + du), uu, dd\}$ and we have $I=1$.
- (b) ss, cc, bb
- (c) $\frac{1}{\sqrt{2}}(bc + cb)$ and analogously for the others.

II.) Axialvector diquarks:

Spin $S=1 \rightarrow \{\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \uparrow\uparrow, \downarrow\downarrow\}$ and symmetric

Color: Same as above. But now we need a symmetric flavour wave function together with $\bar{3}$ -color and an antisymmetric flavour wave function together with 6-color.

Thus, flavour/color combinations are interchanged as compared to scalar diquark.

11.2 Four-quark states

Now think about a four-quark state with two heavy quarks and two light anti-quarks in the two flavour combinations $bb\bar{q}\bar{q}$ and $bc\bar{q}\bar{q}$. Suppose, the quarks and antiquarks are arranged in scalar (S) and axialvector (A) diquarks. Which diquark combinations are possible for the following quantum numbers?

- (a) $I(J) = 0(1)$

Solution: $J = 1 \rightarrow$ we need at least one axialvector diquark, i.e. only combinations AA, SA, AS are possible.

color $3 \otimes \bar{3}$:

$I = 0 \rightarrow$ light diquark needs to be S \rightarrow heavy diquark needs to be A and indeed, this is possible

color $6 \otimes \bar{6}$:

$I = 0 \rightarrow$ light diquark needs to be A \rightarrow heavy diquark needs to be S and indeed, this is possible

- (b) $I(J) = 1(1)$

Solution: $J = 1 \rightarrow$ we need at least one axialvector diquark, i.e. only combinations AA, SA, AS are possible.

color $3 \otimes \bar{3}$:

$I = 1 \rightarrow$ light diquark needs to be A. Heavy diquark also needs to be A (S is not possible).

color $6 \otimes \bar{6}$:

$I = 1 \rightarrow$ light diquark needs to be S \rightarrow heavy diquark needs to be A, but this is not possible.

- (c) $I(J) = 0(0)$

Solution: $J = 0 \rightarrow$ we need either SS or AA (from rules of adding angular momenta).

color $3 \otimes \bar{3}$:

$I = 0 \rightarrow$ light diquark needs to be S \rightarrow heavy diquark also needs to be S, but this is not possible.

color $6 \otimes \bar{6}$:

$I = 0 \rightarrow$ light diquark needs to be A \rightarrow heavy diquark also needs to be A, but this is not possible.

Hint: again carefully think about symmetries...