MTHS24 – Exercise sheet 9

Morning: Laura Tolos / Andrew Jackura
Afternoon: XXX, YYY

Wednesday, 24 July 2024

Lecture material

Discussed topics:

- Partial waves
- Analyticity
- Unitarity

References:

- Book 1, inspire
- Book 2, amazon
- A good review, inspire
- A good aper, inspire

Exercices

9.1 Two-body phase space

(a) Particle A with mass M decays into two daughters with masses m_1 and m_2. Derive the formula for the *break up momentum*, $\mathrm{p}^* = |\mathbf{p}_1| = |\mathbf{p}_2|$, in the rest frame of A. Use your result to evaluate p^* for the decay $\Delta(1232) \to p\pi$.

(b) Starting from the formula for the 2-body decay rate,

$$\Gamma_{fi} = \frac{1}{2M} \int |\mathcal{M}_{fi}|^2 \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(P - p_1 - p_2)\,,$$

perform integrations using δ functions to obtain $\mathrm{d}\Gamma/\mathrm{d}\Omega$ in the centre of mass frame, $P = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}$, to obtain the 2-body phase-space factor.

Hint: You may want to use the following property of the delta function:

$$\int_{-\infty}^{+\infty} g(x)\delta(f(x))\mathrm{d}x = \sum_{x_0} \frac{g(x_0)}{|\mathrm{d}f/\mathrm{d}x|_{x_0}}, \text{ where } x_0 \in \{x : f(x) = 0\}.$$