



## MTHS24 – Exercise sheet 7

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### Lecture material

#### Discussed topics:

- Partial waves
- Analyticity
- Unitarity

#### References:

- Book 1, [inspire](#)
- Book 2, [amazon](#)
- A good review, [inspire](#)
- A good aper, [inspire](#)

### Exercises

#### 7.1 Two-body phase space

- (a) Particle  $A$  with mass  $M$  decays into two daughters with masses  $m_1$  and  $m_2$ . Derive the formula for the *break up momentum*,  $p^* = |\mathbf{p}_1| = |\mathbf{p}_2|$ , in the rest frame of  $A$ . Use your result to evaluate  $p^*$  for the decay  $\Delta(1232) \rightarrow p\pi$ .
- (b) Starting from the formula for the 2-body decay rate,

$$\Gamma_{fi} = \frac{1}{2M} \int |\mathcal{M}_{fi}|^2 \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(P - p_1 - p_2),$$

perform integrations using  $\delta$  functions to obtain  $d\Gamma/d\Omega$  in the centre of mass frame,  $P = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}$ , to obtain the 2-body phase-space factor.

*Hint: You may want to use the following property of the delta function:*

$$\int_{-\infty}^{+\infty} g(x) \delta(f(x)) dx = \sum_{x_0} \frac{g(x_0)}{|df/dx|_{x_0}}, \text{ where } x_0 \in \{x : f(x) = 0\}.$$