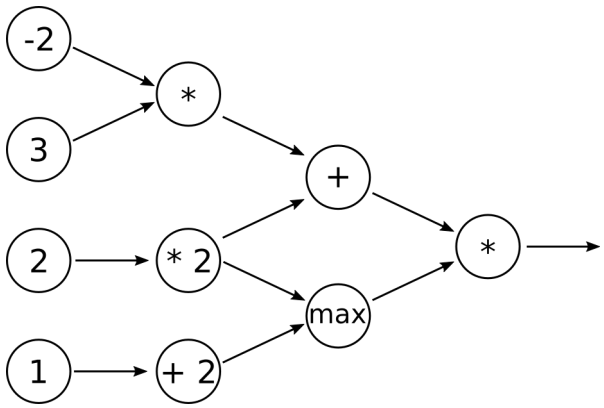
Machine Learning for Image Analysis, SoSe 2023
Exercise Sheet #5: Neural Networks & Backpropagation
Due date: May 30, 2017, before 11 am

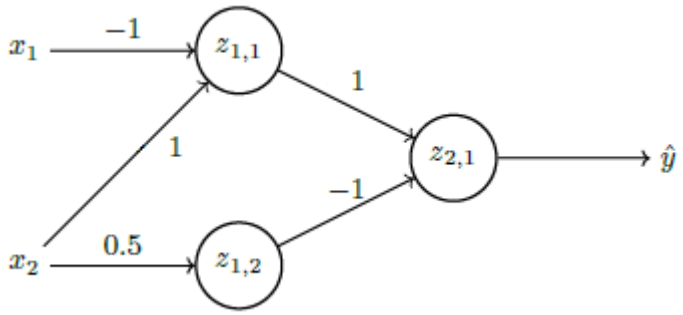
Problem 1 (Backpropagation in Compute Graph) (8 pt).
Given the following computation graph:



(a) (3 pt) Perform a forward pass.

(b) (5 pt) Perform a backward pass.

Problem 2 (Backpropagation in Neural Networks) (22 pt).
Consider the neural network depicted below. Weights are shown at arrows and bias terms are omitted in this exercise.



(a) (4pt) Perform the forward pass for the single input datapoint $x = (1, 2)$ using the ReLU activation function at the hidden layer and the sigmoid activation function for the final output.

(b) (3pt) Given the single training instance $x = (1, 2)$, $y = 1$ we want to perform backpropagation to update all the weights. We are using the log-likelihood objective function function $J = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$, sigmoid activation function, learning rate $\lambda = 1$ and without any regularization. Draw the corresponding compute graph with weights as inputs.

(c) (8pt) Update all the weights once via back-propagation.
Problem 2.1 might be useful here.

(d) (5pt) Perform another forward pass, using the updated weights, and compute $\hat{y}$ and the resulting loss.

(e) (2pt) What result in (d) is expected? You can answer this question even if your calculations might turn out wrong.

Problem 3 (XOR function) (10 pt).
Consider the logical XOR operation on the following data set

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

(a) (1pt) Is there a single-layer perceptron model which has a 100% accuracy on the dataset? Justify your answer.

(b) (1pt) Show that the binary XOR function can be realized as a combination of the binary AND and OR functions plus negations.

(c) (8pt) Design a Neural Network consisting of 3 neurons which realizes the binary XOR function. Provide a full explicit description of the network! (activation function, weights, bias etc.)