COMPSCI/SFWRENG 2FA3

Discrete Mathematics with Applications II

Winter 2020

Assignment 11

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McMaster University

Revised: April 1, 2020

Assignment 11 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 11 as two files, `Assignment_11_`*`YourMacID`*`.tex` and `Assignment_11_`*`YourMacID`*`.pdf`, to the Assignment 11 folder on Avenue under Assessments/Assignments. *`YourMacID`* must be your personal MacID (written without capitalization). The `Assignment_11_`*`YourMacID`*`.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_11.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_11_`*`YourMacID`*`.pdf` is the PDF output produced by executing

```
pdflatex Assignment_11_YourMacID
```

This assignment is due **Sunday, April 12, 2020 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_11_`*`YourMacID`*`.tex` and `Assignment_11_`*`YourMacID`*`.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on April 12.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Problems

1. **[10 points]** Let $\Sigma = \{a, b\}$ and

$$L = \{x \in \Sigma^* \mid \#a(x) \text{ is even and } \#b(x) \text{ is even}\}.$$

Construct a total Turing machine that accepts L.

Name: Hishmat Salehi
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Date: April 10, 2020

Description of the Turing machine:

Let M $= \{Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r\}$ be the TM where:

$Q = \{s, a_1, b_1, q, t, r\}$
$\Sigma = \{a, b\}$
$\Gamma = \Sigma \cup \{\vdash, \sqcup, -\}$
and the transition function δ is specified by the following table:

	$\vdash$	a	b	$-$	$\sqcup$
s	(s, $\vdash$, R)	(a_1, $-$, R)	(b_1, $-$, R)	(s, $-$, R)	(t, $-$, R)
a_1	$-$	(q, $-$, R)	(a_1, b, R)	(a_1, $-$, R)	(r, $-$, R)
b_1	$-$	(b_1, a, R)	(q, $-$, L)	(b_1, $-$, R)	(r, $-$, R)
q	(s, $\vdash$, R)	(q, a, L)	(q, b, L)	(q, $-$, L)	$-$

M accepts $L = \{x \in \Sigma^* \mid \#a(x)$ is even and $\#b(x)$ is even$\}$.

M does the following:

a. Finds the first a or b and replaces it with $-$ and then finds another a or b, if the two symbols are the same it replaces the second symbol with $-$ and moves the pointer all the way to the beginning of the tape.

b. Continues doing that until there is no a's and b's are left and accepts it. (Even number of a's and b's)

c. If there is an a or b left it rejects it. (Odd number of a's or b's)

2. **[10 points]** Let $\Sigma = \{a, b\}$ and

$$L = \{x \in \Sigma^* \mid \#a(x) = \#b(x)\}.$$

Construct a total Turing machine that accepts L.

Hint: Model your Turing machine on the Turing machine described in Example 2 of the 6 Turing Machines and Computability slides that accepts the language $\{a^n b^n c^n \mid n \geq 0\}$.

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Date: April 10, 2020

Description of the Turing machine:

Let M $= \{Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r\}$ be the TM where:

$Q = \{s, a_1, a_2, b_1, b_2, q, t, r\}$

$\Sigma = \{a, b\}$

$\Gamma = \Sigma \cup \{\vdash, \sqcup, -\}$

and the transition function δ is specified by the following table:

	$\vdash$	a	b	$-$	$\sqcup$
s	(s, $\vdash$, R)	(a_1, a, R)	(b_1, b, R)	(s, $-$, R)	(t, $-$, R)
a_1	$-$	(a_1, a, R)	(a_2, $-$, L)	(a_1, $-$, R)	(r, $-$, R)
a_2	$-$	(q, $-$, L)	$-$	(a_2, $-$, L)	$-$
b_1	$-$	(b_2, $-$, L)	(b_1, b, R)	(b_1, $-$, R)	(r, $-$, R)
b_2	$-$	$-$	(q, $-$, L)	(b_2, $-$, L)	$-$
q	(s, $\vdash$, R)	(q, a, L)	(q, b, L)	(q, $-$, L)	$-$

M accepts $L = \{x \in \Sigma^* \mid \#a(x) = \#b(x)\}$.

M does the following:

a. Finds the first a or b and then finds another a or b, if the two symbols are the opposite it replaces both symbols with $-$ and moves the pointer all the way to the beginning of the tape.

b. Continues doing that until there is no a's and b's are left and accepts it. (Same number of a's and b's)

c. If there is an a or b left it rejects it. (Different number of a's and b's)