COMPSCI/SFWRENG 2C03

Data Structures and Algorithms

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Assignment 3

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1. a. 0: $5 \to 2 \to 6$
   1: $4 \to 8 \to 11$
   2: $5 \to 6 \to 0 \to 3$
   3: $10 \to 6 \to 2$
   4: $1 \to 8$
   5: $0 \to 10 \to 2$
   6: $2 \to 3 \to 0$
   7: $8 \to 11$
   8: $1 \to 11 \to 7 \to 4$
   9:
   10: $5 \to 3$
   11: $8 \to 7 \to 1$

   b. Adjacency Matrix:

```
0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1,
1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0,
1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,
```

2.

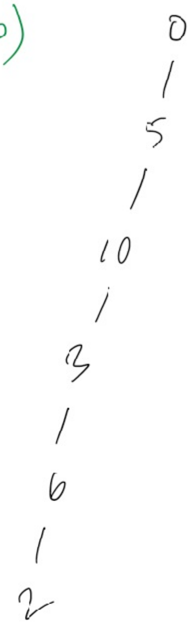
a)

dfs(0)

- dfs(5)
  - check 0
  - dfs(10)
    - check 5
    - dfs(3)
      - check 10
      - dfs(6)
        - dfs(2)
          - check 5
          - check 6
          - check 0
          - check 3
        - 2 done
        - check 3
        - check 0
      - 6 done
      - check 2
    - 3 done
  - 10 done
  - check 2
- 5 done
- check 2
- check 6

0 done

b)

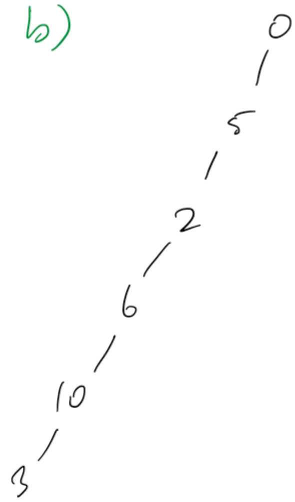


3.

a)

- bfs(0)
  - bfs(5)
    - check 0
    - bfs (10)
      - check 5
      - check 3
    - check 2
  - bfs(2)
    - check 5
    - check 6
    - check 0
    - bfs (3)
      - check 10
      - check 6
      - check 2
  - bfs(6)
    - check 2
    - check 3
    - check 0

b)



4. Consider by contradiction that the edge of maximum weight in the cycle C, edge e, belongs to the MST of the graph. Since MSTs do not contain cycles there is at least one edge in C that is not in the MST. Let's call one of these edges f. Now add f to the MST. There is now a cycle in the MST. Since e has the maximum weight in the cycle C and all edge weights are distinct, it means that weight(f) ¡ weight(e). Removing the edge e after having added the edge f would generate a new MST' with total weight less than the total weight in MST, contradicting its minimality.

5. a. 0: $6 \to 5$
   1:
   2: $0 \to 3$
   3: $10 \to 6$
   4: 1
   5: $10 \to 2$
   6: 2
   7: $8 \to 11$
   8: $1 \to 4$
   9:
   10: 3
   11: 8

   b. Adjacency Matrix:

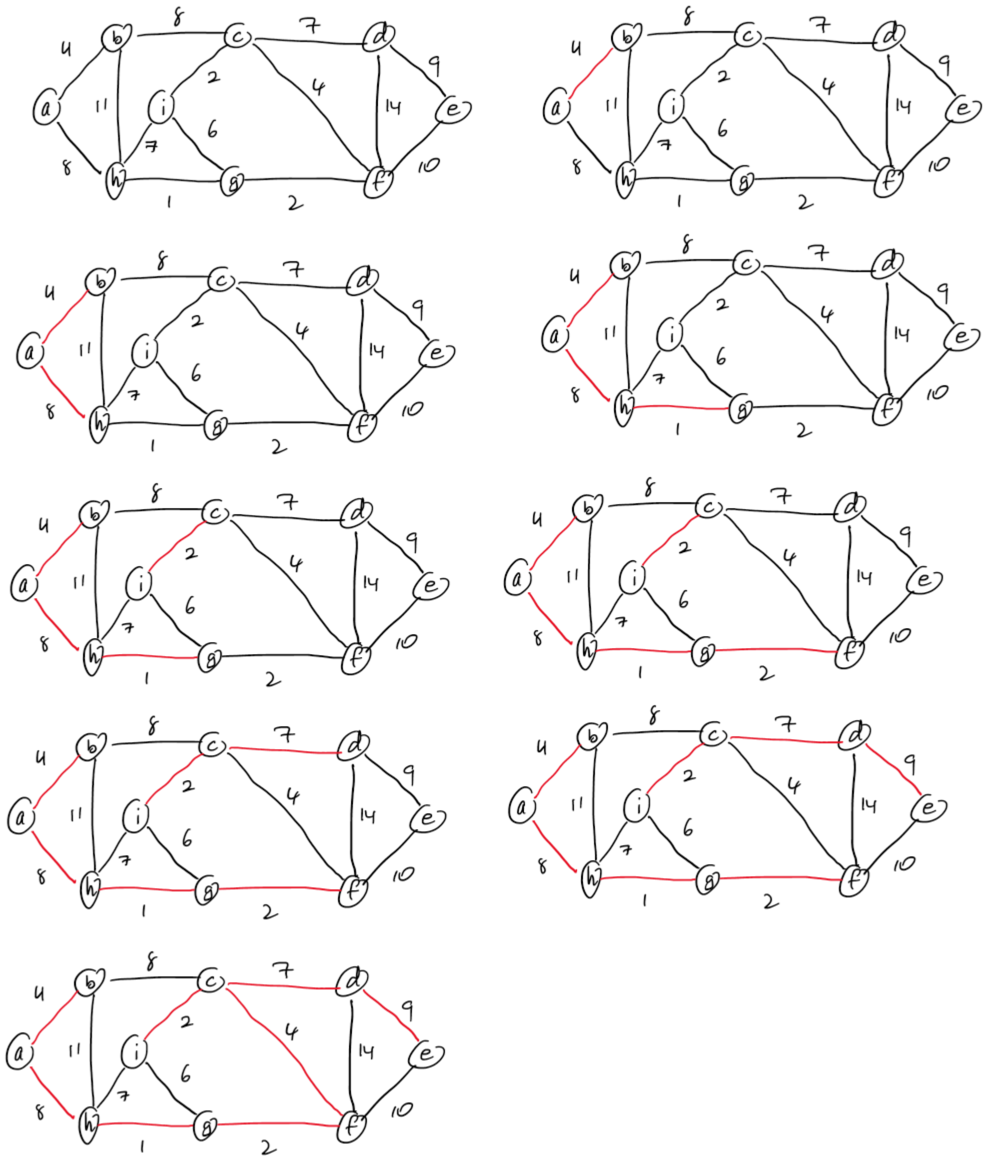
```
0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
```

6. [TODO: Show steps pg. 589] It's strongest component is 0 2 3 5 6 10.

7. Topological order:
   p − n − o − s − m − r − u − y − v − w − z − q − t − x

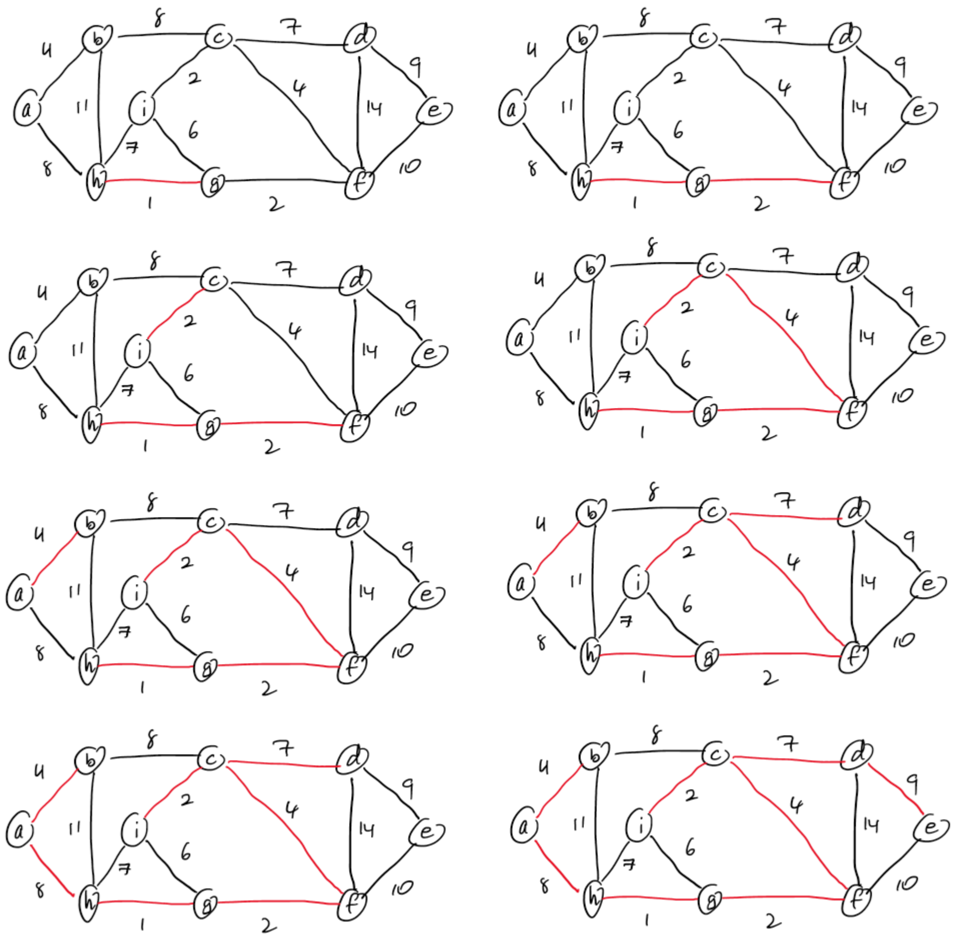
8. Suppose there are two minimum trees, A and B. Let e be the edge in just one of A,B with the smallest cost. Suppose it is in A but not B. Suppose e is the edge PQ. Then B must contain a path from P to Q which is not simply the edge e. So if we add e to B, then we get a cycle. If all the other edges in the cycle were in A, then A would contain a cycle, which it cannot. So the cycle must contain an edge f not in A. Hence, by the definition of e (and the fact that all edge-costs are different) the cost of f must be greater than the cost of e. So if we replace f by e we get a spanning tree with smaller total cost. Contradiction.

9. a. Minimum spanning tree with Greedy Algorithm:

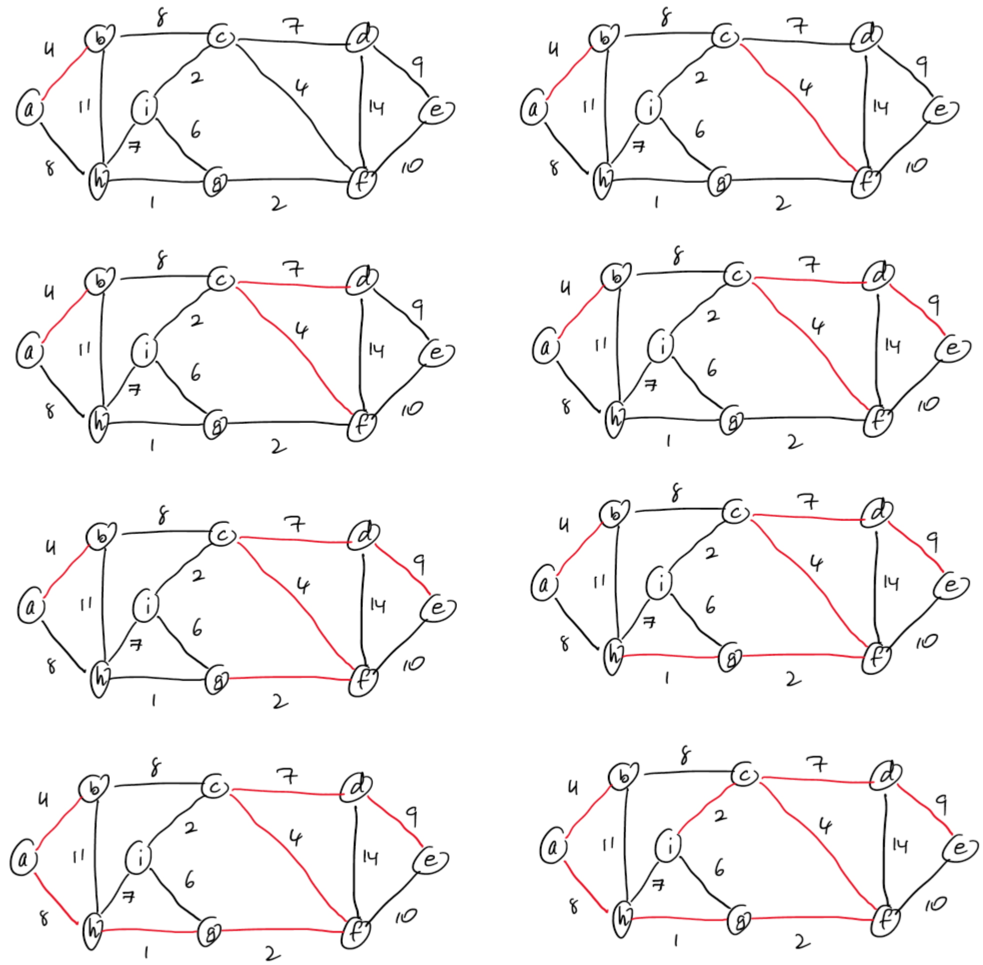


b. Minimum spanning tree with Kruskal's Algorithm:

Monday, April 6, 2020
3:16 AM



c. Minimum spanning tree with Prim's Algorithm:



10. a to a (0.00)
    a to b (2.00) a→b 2.00
    a to c (6.00) a→j 4.00 j→c 2.00
    a to d (6.00) a→b 2.00 b→f 3.00 f→d 1.00
    a to e (8.00) a→l 5.00 l→e 3.00
    a to f (5.00) a→b 2.00 b→f 3.00
    a to h (5.00) a→h 5.00
    a to i (5.00) a→j 4.00 j→i 1.00
    a to j (4.00) a→j 4.00
    a to k (7.00) a→b 2.00 b→f 3.00 f→d 1.00 d→k 1.00
    a to l (5.00) a→l 5.00

11. s to s (0.00)
    s to t (2.00) s→y 7.00 y→x -3.00 x→t -2.00
    s to x (4.00) s→y 7.00 y→x -3.00
    s to y (7.00) s→y 7.00
    s to z (-2.00) s→y 7.00 y→x -3.00 x→t -2.00 t→z -4.00

12.
```
public double diameter(EdgeWeightedDigraph edgeWeightedDigraph) {
        double diameter = Double.NEGATIVE_INFINITY;

        for (int v = 0; v < edgeWeightedDigraph.V(); v++) {
                DijkstraSP dijkstraSP = new DijkstraSP(edgeWeightedDigraph, v);

                for (int v2 = 0; v2 < edgeWeightedDigraph.V(); v2++) {
                        if (dijkstraSP.distTo(v2) > diameter) {
                                diameter = dijkstraSP.distTo(v2);
                        }
                }
        }

        return diameter;
}
```

13. r to r (0.00) r to s (5.00) r-¿s 5.00 r to t (3.00) r-¿t 3.00 r to x (10.00) r-¿t 3.00 t-¿x 7.00 r to y (7.00) r-¿t 3.00 t-¿y 4.00 r to z (5.00) r-¿t 3.00 t-¿z 2.00

14. a. Give a trace for LSD string sort for the keys:
    no is th ti fo al go pe to co to th ai of th pa

input	d=1	d=0	output
no	pa	ai	ai
is	pe	al	al
th	of	co	co
ti	th	fo	fo
fo	th	go	go
al	th	is	is
go	ti	no	no
pe	ai	of	of
to	al	pa	pa
co	no	pe	pe
to	fo	th	th
th	go	th	th
ai	to	th	th
of	co	ti	ti
th	to	to	to
pa	is	to	to

    b. Give a trace for MSD string sort for the keys:
    no is th ti fo al go pe to co to th ai of th pa

input	--	--			output
no	al	ai	ai	ai	ai
is	ai	al	al	al	al
th	co	--	co	co	co

ti	fo	co	fo	fo	fo
fo	go	fo	go	go	go
al	is	go	is	is	is
go	no	is	no	no	no
pe	of	no	of	of	of
to	pe	of	--	pa	pa
co	pa	pe	pa	pe	pe
to	th	pa	pe	--	th
th	ti	th	--	th	th
ai	to	ti	th	th	th
of	to	to	ti	th	ti
th	th	to	to	ti	to
pa	th	th	to	to	to
	--	th	th	to	
			th	--	

c. Give a trace for MSD string sort for the keys:
now is the time for all good people to come to the aid of

input	---	---			output
now	all	aid	aid	aid	aid
is	aid	all	all	all	all
the	come	---	come	come	come
time	for	come	for	for	for
for	good	for	good	good	good
all	is	good	is	is	is
good	now	is	now	now	now
people	of	now	of	of	of
to	people	of	people	people	people
come	the	people	---	---	the
to	time	the	the	the	the
the	to	time	the	the	time
aid	to	to	time	---	to
of	the	to	to	time	to
	----	the	to	to	
			---	to	

15.

16. Failure Function:

P = abbababaaabab

j	0	1	2	3	4	5	6	7	8	9	10	11	12
P[j]	a	b	b	a	b	a	b	a	a	a	b	a	b
f(j)	0	0	0	1	2	1	2	1	1	1	2	1	2

```
Solution using Knuth-Morris-Pratt algorithm:
abaababaababbababaaababaabaababb
123
abbababaaabab
  45
  abbababaaabab
   678
   abbababaaabab
-----------------------------
a b a a b a b a a b a b b a b a b a a a b a b a a b a a b a b b
                9 1011
                a b b a b a b a a a b a b
-----------------------------
a b a a b a b a a b a b b a b a b a a a b a b a a b a a b a b b
                    12131415161718192021222324
                    a b b a b a b a a a b a b

Final Answer:
abaababaababbababaaababaabaababb
          abbababaaabab
```

The algorithm performs 24 character comparisons, which are indicated with numerical labels.

17. Solution using Boyer-Moore algorithm:

```
i j 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
    b a c a a b a c c a b a c a b a a b b
          3 2 1
0 3 a b a c a b
                    4
3 5       a b a c a b
                     10 9 8 7 6 5
9 5                   a b a c a b
```

The algorithm performs 10 character comparisons, which are indicated with numerical labels.

18. Solution using Rabin-Karp algorithm:

```
text:    b a c a a b a c c a b a c a b a a b b
pattern:                   a b a c a b
```

19. Prefix-free codes: code 3 and code 4
    Uniquely decodable codes: code 3 and code 4

    Decoding of 1000000000000:
    code 3: Not recognized by the alphabet
    code 4: ADDDD

20.

Bits required: $93 + 142 + 8 = 243$

21. a.
```
T  O  B  E  O  R  N  O  T  T  O  B  E
54 4F 42 45 4F 52 4E 4F 54 81    83    80

Codeword table
key   value
TO     81
OB     82
BE     83
EO     84
OR     85
RN     86
NO     87
OT     88
TT     89
TOB    8A
```

b.
```
Y  A  B  B  A  D  A  B  B  A  D  A  B  B  A  D  O  O
59 41 42 42 41 44 82    84    86    83    85    4F 4F 80

Codeword table
key   value
YA     81
AB     82
BB     83
BA     84
AD     85
DA     86
ABB    87
BAD    88
DAB    89
BBA    8A
ADO    8B
OO     8C
```

c.
```
A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A
41 81    82       83          84             85                80

Codeword table
key   value
AA     81
AAA    82
AAAA   83
AAAAA  84
AAAAAA 85
```