COMPSCI/SFWRENG 2C03

Data Structures and Algorithms

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Assignment 3

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1. a. 0: 5 → 2 → 6
   1: 4 → 8 → 11
   2: 5 → 6 → 0 → 3
   3: 10 → 6 → 2
   4: 1 → 8
   5: 0 → 10 → 2
   6: 2 → 3 → 0
   7: 8 → 11
   8: 1 → 11 → 7 → 4
   9:
   10: 5 → 3
   11: 8 → 7 → 1

   b. Adjacency Matrix:

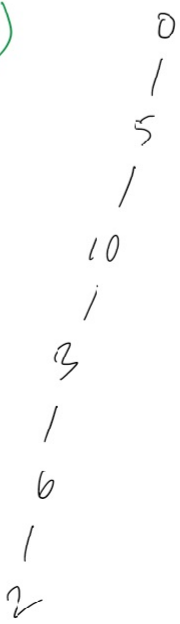
```
0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1,
1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0,
1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,
```

2.

a)

```
dfs(0)
  dfs(5)
    check 0
    dfs(10)
      check 5
      dfs(3)
        check 10
        dfs(6)
          dfs(2)
            check 5
            check 6
            check 0
            check 3
          2 done
          check 3
          check 0
        6 done
        check 2
      3 done
    10 done
    check 2
  5 done
  check 2
  check 6
0 done
```

b)

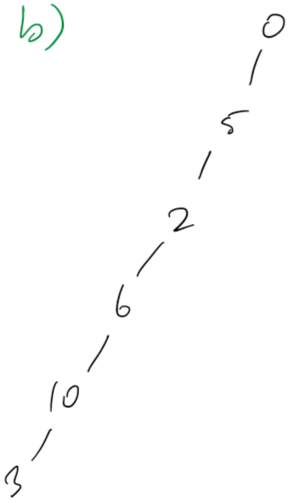


3.

a)

```
bfs(0)
    bfs(5)
        check 0
        bfs(10)
            check 5
            check 3
        check 2
    bfs(2)
        check 5
        check 6
        check 0
        bfs(3)
            check 10
            check 6
            check 2
    bfs(6)
        check 2
        check 3
        check 0
```

b)



4. Consider by contradiction that the edge of maximum weight in the cycle C, edge e, belongs to the MST of the graph. Since MST does not contain any cycles, there is at least one edge in C that is not in the MST. Let's call one of these edges g. Now add g to the MST. There is now a cycle in the MST. Since e has the maximum weight in the cycle C and all edge weights are distinct, it means that weight(g) < weight(e). Removing the edge e after having added the edge g would generate a new MST prime with total weight, less than the total weight in MST, therfore contradicting its minimality.

5. a. 0: 6 → 5
   1:
   2: 0 → 3
   3: 10 → 6
   4: 1
   5: 10 → 2
   6: 2
   7: 8 → 11
   8: 1 → 4
   9:
   10: 3
   11: 8

   b. Adjacency Matrix:

```
0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
```

6. Edges:
   0 → 6, 5
   2 → 0, 3
   3 → 10, 6
   5 → 10, 2
   6 → 2
   10 → 3

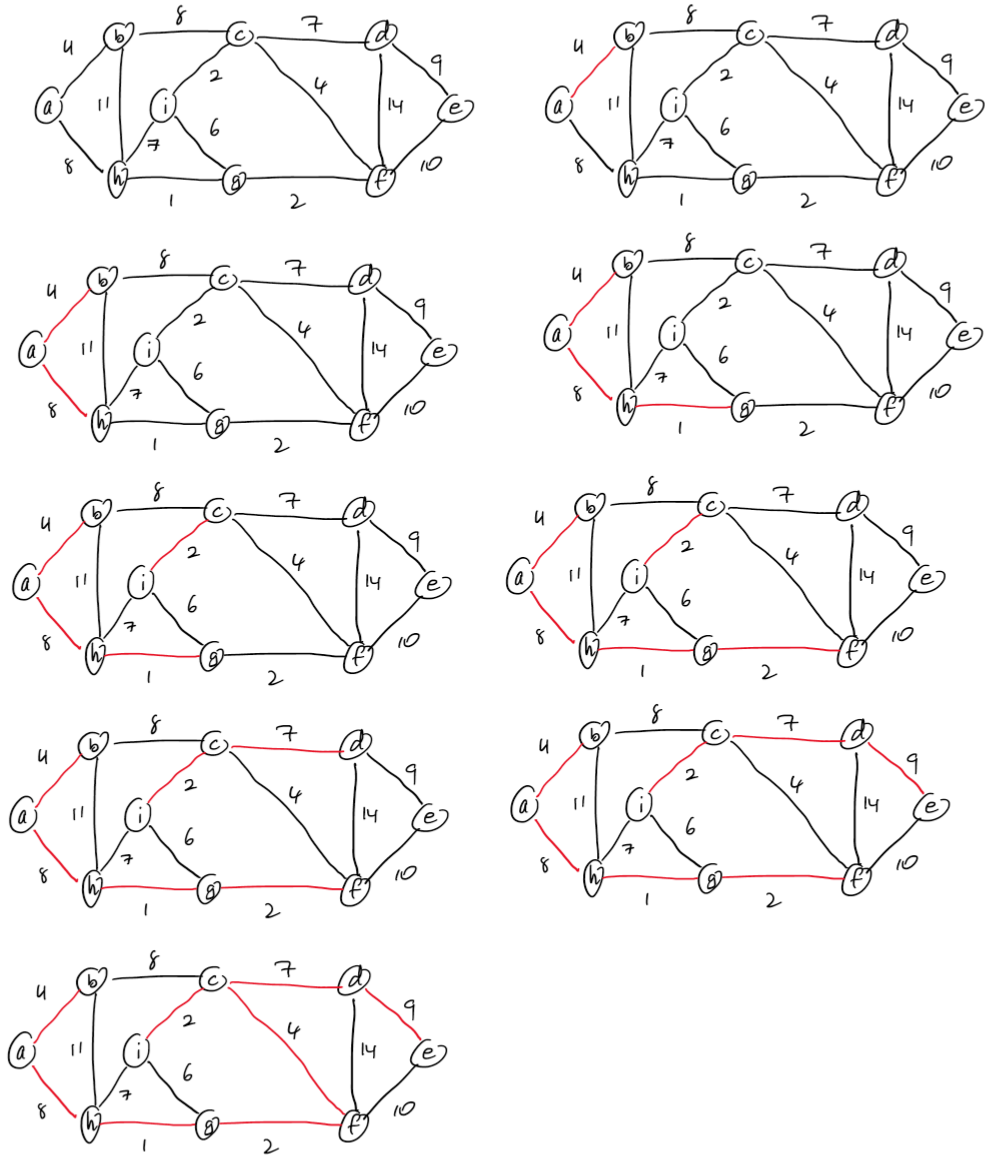
   0 → 5 → 2 → 3 → 10 → 3 → 6 → 2 → 0

   Therefore, It's strongest component is 0 2 3 5 6 10.

7. Topological order:
   p − n − o − s − m − r − u − y − v − w − z − q − t − x

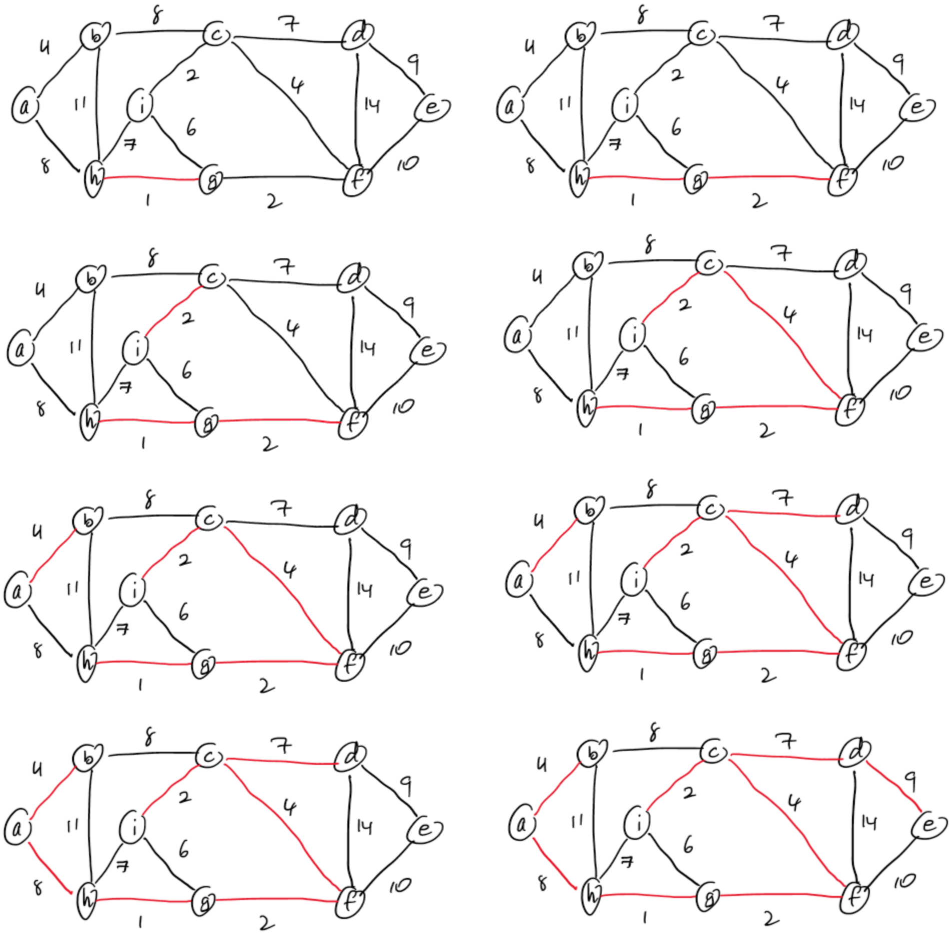
8. Suppose by contradiction there are two minimum trees, E and F. Let e be the edge in just one of E,F with the smallest cost. Suppose it is in E but not F. Suppose e is the edge MN. Then F must contain a path from M to n which is not simply the edge e. So if we add e to F, then we get a cycle. If all the other edges in the cycle were in E, then E would contain a cycle, which it cannot. So the cycle must contain an edge f not in E. Hence, by the definition of e (and the fact that all edge-costs are different) the cost of f must be greater than the cost of e. So if we replace f by e we get a spanning tree with smaller total cost, which derives a contradiction.

9. a. Minimum spanning tree with Greedy Algorithm:

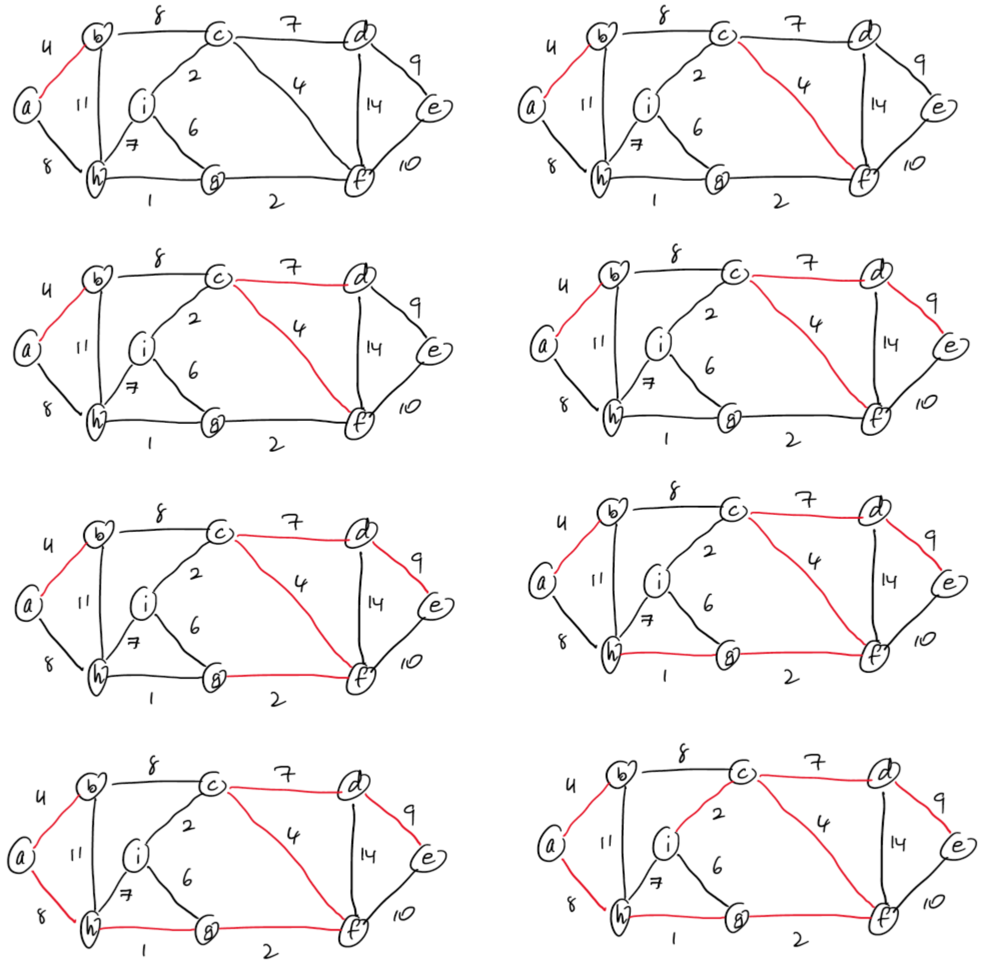


b. Minimum spanning tree with Kruskal's Algorithm:

Monday, April 6, 2020
3:16 AM



c. Minimum spanning tree with Prim's Algorithm:



10. a to a (0.00)
a to b (2.00) a→b 2.00
a to c (6.00) a→j 4.00 j→c 2.00
a to d (6.00) a→b 2.00 b→f 3.00 f→d 1.00
a to e (8.00) a→l 5.00 l→e 3.00
a to f (5.00) a→b 2.00 b→f 3.00
a to h (5.00) a→h 5.00
a to i (5.00) a→j 4.00 j→i 1.00
a to j (4.00) a→j 4.00
a to k (7.00) a→b 2.00 b→f 3.00 f→d 1.00 d→k 1.00
a to l (5.00) a→l 5.00

11. s to s (0.00)
s to t (2.00) s→y 7.00 y→x -3.00 x→t -2.00
s to x (4.00) s→y 7.00 y→x -3.00
s to y (7.00) s→y 7.00
s to z (-2.00) s→y 7.00 y→x -3.00 x→t -2.00 t→z -4.00

12.
```
public double diameter(EdgeWeightedDigraph edgeWeightedDigraph) {
        double diameter = Double.NEGATIVE_INFINITY;

        for (int v = 0; v < edgeWeightedDigraph.V(); v++) {
                DijkstraSP dijkstraSP = new DijkstraSP(edgeWeightedDigraph, v);

                for (int v2 = 0; v2 < edgeWeightedDigraph.V(); v2++) {
                        if (dijkstraSP.distTo(v2) > diameter) {
                                diameter = dijkstraSP.distTo(v2);
                        }
                }
        }

        return diameter;
}
```

13. r to r (0.00)
r to s (5.00) r→s 5.00
r to t (3.00) r→t 3.00
r to x (10.00) r→t 3.00 t→x 7.00
r to y (7.00) r→t 3.00 t→y 4.00
r to z (5.00) r→t 3.00 t→z 2.00

14.

a. Give a trace for LSD string sort for the keys:
no is th ti fo al go pe to co to th ai of th pa

input	d=1	d=0	output
no	pa	ai	ai
is	pe	al	al
th	of	co	co
ti	th	fo	fo
fo	th	go	go
al	th	is	is
go	ti	no	no
pe	ai	of	of
to	al	pa	pa
co	no	pe	pe
to	fo	th	th
th	go	th	th
ai	to	th	th
of	co	ti	ti
th	to	to	to
pa	is	to	to

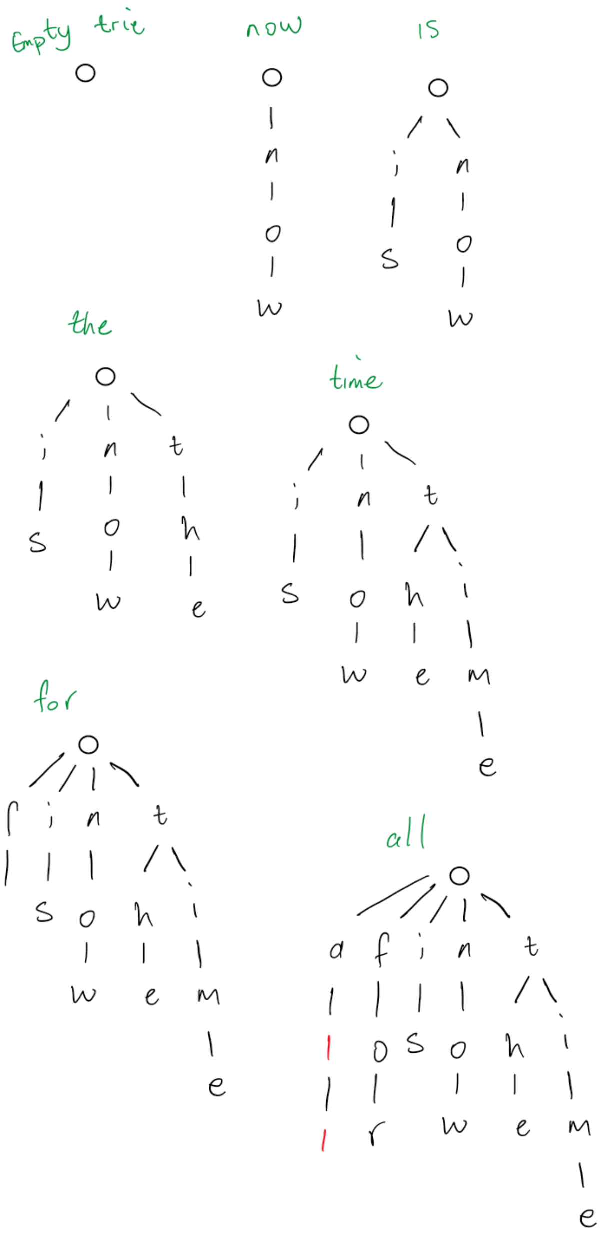
b. Give a trace for MSD string sort for the keys:
no is th ti fo al go pe to co to th ai of th pa

```
input   --      --                      output
no      al      ai      ai      ai      ai
is      ai      al      al      al      al
th      co      --      co      co      co
ti      fo      co      fo      fo      fo
fo      go      fo      go      go      go
al      is      go      is      is      is
go      no      is      no      no      no
pe      of      no      of      of      of
to      pe      of      --      pa      pa
co      pa      pe      pa      pe      pe
to      th      pa      pe      --      th
th      ti      th      --      th      th
ai      to      ti      th      th      th
of      to      to      ti      th      ti
th      th      to      to      ti      to
pa      th      th      to      to      to
        --      th      th      to
                        th      --
```

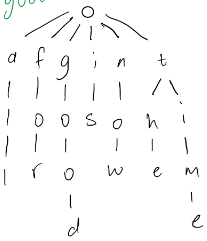
c. Give a trace for MSD string sort for the keys:
now is the time for all good people to come to the aid of

```
input       ---        ---                                  output
now         all         aid         aid         aid          aid
is          aid         all         all         all          all
the         come        ---         come        come         come
time        for         come        for         for          for
for         good        for         good        good         good
all         is          good        is          is           is
good        now         is          now         now          now
people      of          now         of          of           of
to          people      of          people      people       people
come        the         people      ---         ---          the
to          time        the         the         the          the
the         to          time        the         the          time
aid         to          to          time        ---          to
of          the         to          to          time         to
            ----        the         to          to
                                    ---         to
```

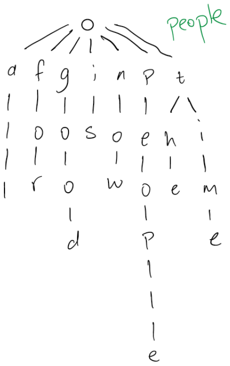
15.



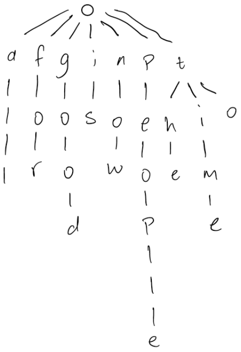
good



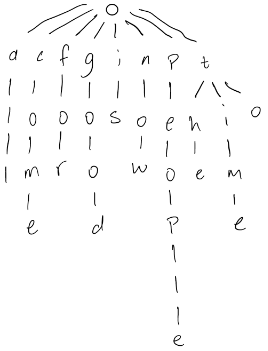
people



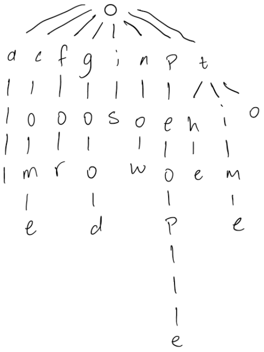
to



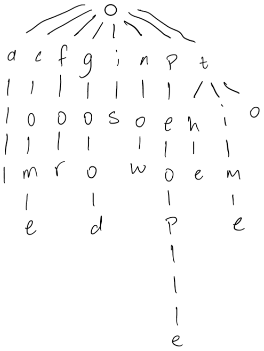
come



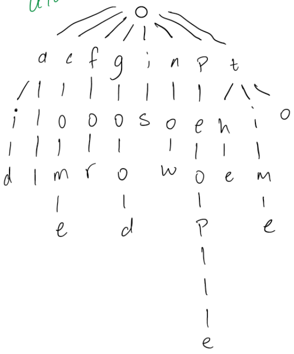
to



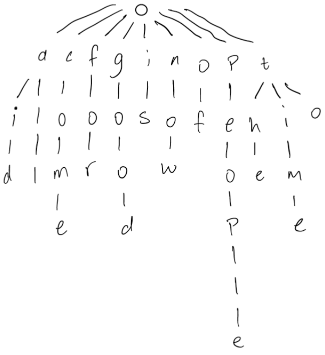
the



aid



of



16. Failure Function:

```
P = abbababaaabab
j     0 1 2 3 4 5 6 7 8 9 10 11 12
P[j]  a b b a b a b a a a b  a  b
f(j)  0 0 0 1 2 1 2 1 1 1 2  1  2
```

```
Solution using Knuth-Morris-Pratt algorithm:
abaababaababbababaaababaabaababb
123
abbababaaabab
  45
  abbababaaabab
   678
   abbababaaabab
-----------------------------
a b a a b a b a a b a b b a b a b a a a b a b a a b a a b a b b
                9 1011
                a b b a b a b a a a b a b
-----------------------------
a b a a b a b a a b a b b a b a b a a a b a b a a b a a b a b b
                    12131415161718192021222324
                    a b b a b a b a a a b a b

Final Answer:
abaababaababbababaaababaabaababb
          abbababaaabab
```

The algorithm performs 24 character comparisons, which are indicated with numerical labels.

17. Solution using Boyer-Moore algorithm:

```
i j 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
    b a c a a b a c c a b a c a b a a b b
          3 2 1
0 3 a b a c a b
                    4
3 5       a b a c a b
                     10 9 8 7 6 5
9 5                   a b a c a b
```

The algorithm performs 10 character comparisons, which are indicated with numerical labels.

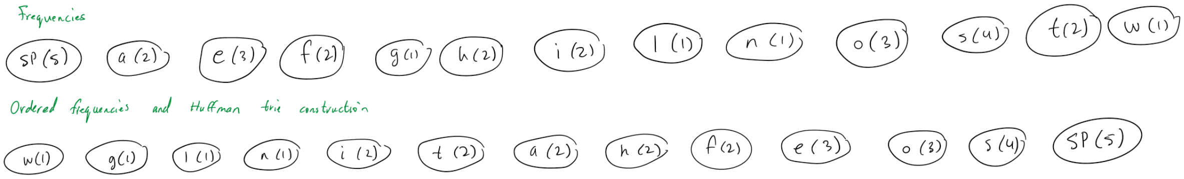
18. Solution using Rabin-Karp algorithm:

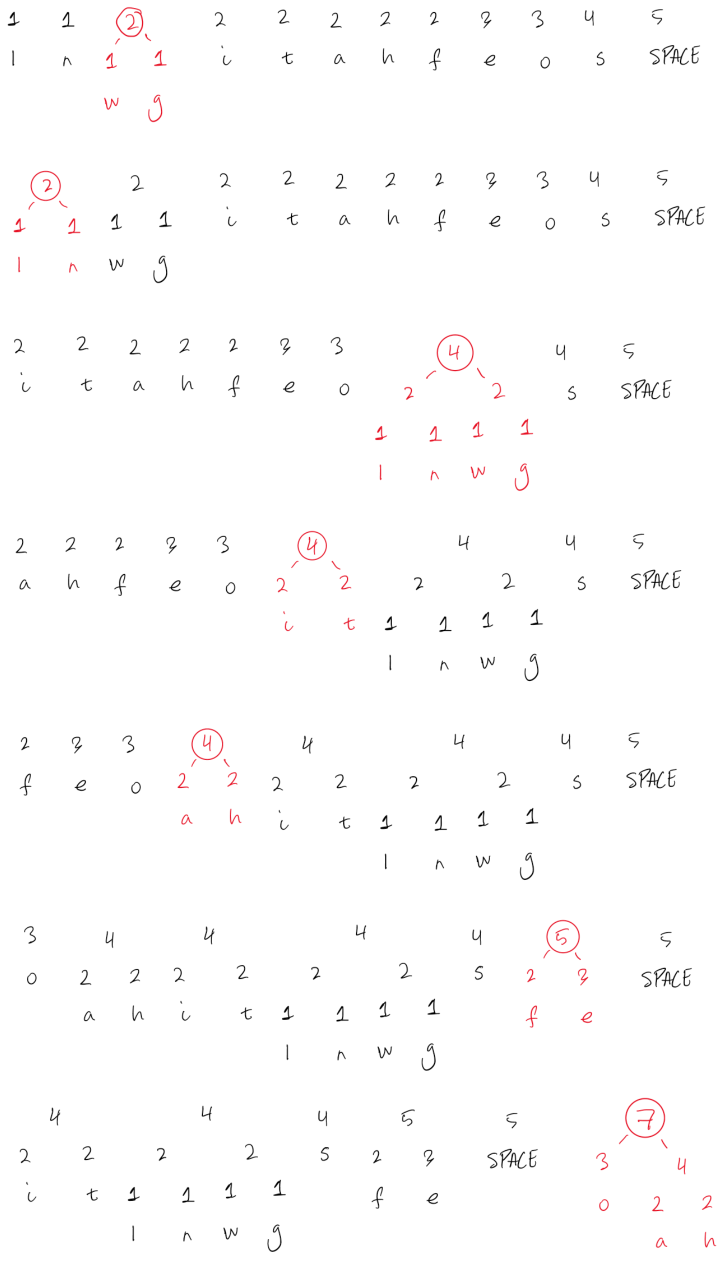
```
text:    b a c a a b a c c a b a c a b a a b b
pattern:                   a b a c a b
```

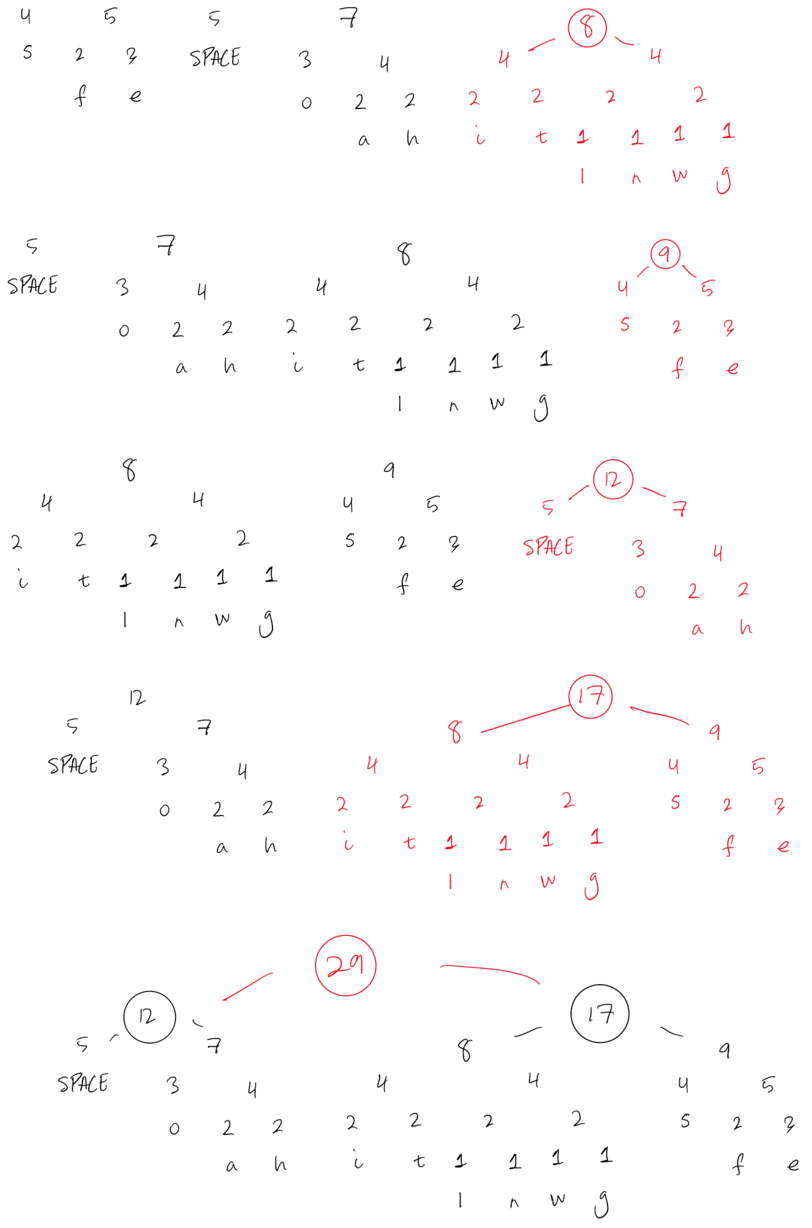
19. Prefix-free codes: code 3 and code 4
    Uniquely decodable codes: code 3 and code 4

    Decoding of 1000000000000:
    code 3: Not recognized by the alphabet
    code 4: ADDDD

20.




1 1 2 2 2 2 2 2 3 3 4 5
l n 1 1 i t a h f e o s SPACE
w g
2 2 2 2 2 2 2 3 3 4 5
1 1 1 1 i t a h f e o s SPACE
l n w g
2 2 2 2 2 3 3 4 4 5
i t a h f e o 2 2 s SPACE
1 1 1 1
l n w g
2 2 2 3 3 4 4 4 5
a h f e o 2 2 2 2 s SPACE
i t 1 1 1 1
l n w g
2 3 3 4 4 4 4 5
f e o 2 2 2 2 2 2 s SPACE
a h i t 1 1 1 1
l n w g
3 4 4 4 4 5 5
o 2 2 2 2 2 2 s 2 3 SPACE
a h i t 1 1 1 1 f e
l n w g
4 4 4 5 5 7
2 2 2 2 s 2 3 SPACE 3 4
i t 1 1 1 1 f e o 2 2
l n w g a h



Bits required: 93 + 142 + 8 = 243

21. a.

T	O	B	E	O	R	N	O	T	T	O	B	E	
54	4F	42	45	4F	52	4E	4F	54	81		83		80

Codeword table

key	value
TO	81
OB	82
BE	83
EO	84

```
OR     85
RN     86
NO     87
OT     88
TT     89
TOB    8A
```

b.

```
Y  A  B  B  A  D  A  B  B  A  D  A  B  B  A  D  O  O
59 41 42 42 41 44 82    84    86    83    85    4F 4F 80

Codeword table
key   value
YA     81
AB     82
BB     83
BA     84
AD     85
DA     86
ABB    87
BAD    88
DAB    89
BBA    8A
ADO    8B
OO     8C
```

c.

```
A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A  A
41 81    82       83          84             85                80

Codeword table
key   value
AA     81
AAA    82
AAAA   83
AAAAA  84
AAAAAA 85
```