GR Tensor

Task I

A spherically symmetric black hole can be described by Schwarzschild metric,

$$ds^2 = -\mathrm{A}(r)\ dt^2 + \mathrm{B}(r)\ dr^2 + r^2\ d\theta^2 + r^2 \sin(\theta)^2\ d\phi^2$$

which describes the geometry of a spherically symmetric spacetime configuration. Find the general relativistic solution of an empty space.

Task II

A rotating black hole can be described by Kerr metric,

$$ds^2 = \frac{(r^2 + a^2 \cos(\theta)^2)\ dr^2}{r^2 - 2\,m\,r + a^2} + (r^2 + a^2 \cos(\theta)^2)\ d\theta^2 + \sin(\theta)^2 \left(r^2 + a^2 + \frac{2\,m\,r\,a^2 \sin(\theta)^2}{r^2 + a^2 \cos(\theta)^2}\right) d\phi^2 - \frac{4\,m\,a\,r \sin(\theta)^2\ d\phi\ dt}{r^2 + a^2 \cos(\theta)^2} + \left(-1 + \frac{2\,m\,r}{r^2 + a^2 \cos(\theta)^2}\right) dt^2$$

which describes the geometry of an axially symmetric spacetime configuration. Verify that the above metric is satisfying the general relativity field equations.

Task III

Given that Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$ds^2 = dt^2 - a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \right\},$$

which describes the geometry of a homogeneous and isotropic spacetime configuration, where $k = 0, \pm 1$ is the sign of spatial curvature. Use Einstein's field equations to find the scale factor *a(t)*.