Task 2

1.3

Q(1 - a)

p	$p \wedge T$
T	T
F	F

The equivalence $p \wedge T = p$, is valid because the second column $p \wedge T$ is identical to p.

Q(9)

a)

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b)

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

d)

p	q	$p \rightarrow q$	$p \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

c)

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e)

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

e)

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Q(18)

It is easy to see from the definitions of conditional statement and negation that each of these propositions is false in the case in which p is true and q is false, and true in the other three cases. Therefore the two propositions are logically equivalent.

Q(21)

This is essentially the same as Exercise 17. The proposition $\neg(p \leftrightarrow q)$ is true when $p \leftrightarrow q$ is false. Since $p \leftrightarrow q$ is true when p and q have the same truth value, it is false when p and q have different truth values

1.4

Q(12)

a) Since $0 + 1 > 2 \cdot 0$, we know that $Q(0)$ is true.

b) Since $(-1) + 1 > 2 \cdot (-1)$, we know that $Q(-1)$ is true.

c) Since $1 + 1$,> $2 \cdot 1$, we know that $Q(1)$ is false. d) From part (a) we know that there is at least one x that makes $Q(x)$ true, so $\exists x\, Q(x)$ is true.

e) From part (c) we know that there is at least one x that makes $Q(x)$ false, so $\forall x$ $Q(x)$ is false.

f) From part (c) we know that there is at least one x that makes $Q(x)$ false, so $\exists x \neg Q(x)$ is true.

g) From part (a) we know that there is at least one x that makes $Q(x)$ true, so $\forall x \neg Q(x)$ is false

Q(19)

a) We want to assert that P(x) is true for some x in the universe, so either P(l) is true or P(2) is true or P(3) is true or P(4) is true or P(5) is true. Thus the answer is P(l) V P(2) V P(3) V P(4) V P(5).

b) P(l) /\ P(2) /\ P(3) /\ P(4) /\ P(5)

c) This is just the negation of part (a): •(P(l) V P(2) V P(3) V P(4) V P(5))

d) This is just the negation of part (b): •(P(l) /\ P(2) /\ P(3) /\ P(4) /\ P(5))

e) The formal translation is as follows: (((1 =I- 3) -+ P(l)) /\ ((2 =I- 3) -+ P(2)) /\ ((3 =I- 3) -+ P(3)) /\ ((4 =I- 3) -+ P(4)) /\ ((5 =I- 3) -+ P(5))) V (•P(l) V -.P(2) V -.P(3) V -.P(4) V ·P(5))..

Q(43)

A conditional statement is true if the hypothesis is false. Thus it is very easy for the second of these propositions to be true-just have P(x) be something that is not always true, such as "The integer x is a multiple of 2.'' On the other hand, it is certainly not always true that if a number is a multiple of 2, then it is also a multiple of 4, so if we let Q(x) be "The integer xis a multiple of 4,'' then \ix(P(:r) _, Q(x)) will be false. Thus these two propositions can have different truth values. Of course, for some choices of P and Q, they will have the same truth values, such as when P and Q are true all the time.