Task 4

Q(1)

This is modus ponens. The first statement is $p \to q$, where p is "Socrates is human" and q is "Socrates is mortal." The second statement is p. The third is q. Modus ponens is valid. We can therefore conclude that the conclusion of the argument (third statement) is true, because the hypotheses (the first two statements) are true.

Q(3)

b) This is the simplification rule. We are concluding from p A q that p must be true, where p is "Jerry is a mathematics major" and q is "Jerry is a computer science major."

d) This is modus tollens. We are concluding from $p \to q$ and $-q$ that $-p$ must be true, where p is "it will snow today'' and q is ''the university will close today."

Q(5)

Let w be the proposition "Randy works hard," let d be the proposition "Randy is a dull boy," and let j be the proposition "Randy will get the job." We are given premises w, $w \to d$, and $d \to -j$. We want to conclude $-j$. We set up the proof in two columns, with reasons

Step	
1. w	Reason
2. $w \to d$	Hypothesis
3. d	Hypothesis
4. $d \to -j$	Hypothesis Modus ponens using (2) and (3)
5. $-j$	Hypothesis Modus ponens using (3) and (4)

Q(15)

b) This is invalid. After applying universal instantiation, it contains the fallacy of affirming the conclusion.

d) This is valid by universal instantiation and modus tollens.

Q(19)

b) This reasoning is valid; it is modus tollens.

1.7

Q(1)

We must show that whenever we have two odd integers, their sum is even. Suppose that a and b are two odd integers. Then there exist integers s and t such that $a = 2s + 1$ and $b = 2t + 1$. Adding, we obtain $a+ b = (2s + 1) + (2t + 1) = 2(s + t + 1)$. Since this represents $a+ b$ as 2 times the integer $s + t + 1$, we conclude that $a + b$ is even, as desired.

Q(8)

Let $n = m2$. If $m = 0$, then $n + 2 = 2$, which is not a perfect square, so we can assume that $m \geq 1$. The smallest perfect square greater than n is $(m + 1)2$, and we have $(m + 1)2 = m2 + 2m + 1 = n + 2m + 1 > n + 2 \cdot 1 + 1 > n + 2$. Therefore $n + 2$ cannot be a perfect square.

Q(16)

We give a proof by contraposition. If it is not true than m is even or n is even, then m and n are both odd. By Exercise 6, this tells us that mn is odd, and our proof is complete.

Q(29)

This proposition is true. We give a proof by contradiction. Suppose that m is neither 1 nor -1. Then mn has a factor (namely lml) larger than 1. On the other hand, $mn = 1$, and 1 clearly has no such factor. Therefore we conclude that $m = 1$ or $m = -1$. It is then immediate that $n = 1$ in the first case and $n = -1$ in the second case, since $mn = 1$ implies that $n = 1/m$