UNIVERSITY OF RUHUNA
DEPARTMENT OF MATHEMATICS

BACHELOR OF SCIENCE (GENERAL) DEGREE (LEVEL I)
MATHEMATICS
MAT 121β : ALGEBRA

Tutorial No: 1 **Semester II, 2019**

Submit answer sheets on : 06/01/2020

---

1. A, B, C, D and E are the matrices given by :

$$A = \begin{pmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix}, \quad E = \begin{pmatrix} 5 & -3 & 1 \\ 4 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix},$$

$$C = \begin{pmatrix} -2 & 6 \\ -5 & 2 \end{pmatrix} \quad , \quad D = \begin{pmatrix} -3 & 2 & 9 & -5 \end{pmatrix}$$

Find

20

(a) AB (b) C^3 (c) DD^T
(d) $B - 2I$ (e) $B + 3E$ (f) $E - B$

---

2. For each of the following find x and y.

(a) $\begin{pmatrix} 2x+y & -7 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 2 & -7 \\ 4 & 4x+y \end{pmatrix}$

(b) $\begin{pmatrix} 2x+y & 3 & 10 \\ y+1 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 10 \\ 3 & -2 & 0 \end{pmatrix}$ $\rightarrow$ 15

(c) $\begin{pmatrix} 6 & -4 & -6 & x-y \end{pmatrix} = -2\begin{pmatrix} -3 & 2 & 2x+2y & 13 \end{pmatrix}$

---

3. (a) Solve for x and y : $\begin{pmatrix} x & y \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} 7 & 6 \\ 2 & 3 \end{pmatrix}$.

(b) Find x and y if $\begin{pmatrix} x+y & -2 \\ x-y & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 12 & -8 \end{pmatrix}$.

(c) Suppose $A = \begin{pmatrix} a & 0 & -1 \\ 2 & 3 & b \end{pmatrix}$.

Find a and b so that $AA^T = \begin{pmatrix} 26 & -11 \\ -11 & 14 \end{pmatrix}$ 25

---

4. Verify that matrices A and B given below are inverses of each other.

(a) $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$

---

5. (a) Let $A = \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$. 25

Find the matrix C such that $CA = B$.

(b) Assume that A is a square matrix that satisfies $A^2 - 3A + I = 0$. Show that $A^{-1} = 3I - A$.

---

6. Find the determinant of following matrices using the definition.

(a) $\begin{pmatrix} -2 & 3 & -2 \\ -4 & -2 & 1 \\ 2 & 4 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -1 & 4 \\ 2 & 5 & 1 \\ 2 & 0 & 6 \end{pmatrix}$ 15

(c) $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & -1 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$

****

MAT 121 β : Algebra

Tutorial No : 1

$\boxed{1}$

$$A = \begin{pmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix}.$$

$$C = \begin{pmatrix} -2 & 6 \\ -5 & 2 \end{pmatrix} \qquad D = \begin{pmatrix} -3 & 2 & 9 & -5 \end{pmatrix}$$

$$E = \begin{pmatrix} 5 & -3 & 1 \\ 4 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix}$$

(a) $$AB = \begin{pmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{pmatrix}_{2\times 3} \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix}_{3\times 3}$$

$$= \begin{pmatrix} 1+2 & -2-3+4 & 0+4-6 \\ 0-1 & 6-2 & -8+3 \end{pmatrix}.$$

$$= \begin{pmatrix} 3 & -1 & -2 \\ -1 & 4 & -5 \end{pmatrix}$$

(b) $$C^2 = \begin{pmatrix} -2 & 6 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} -2 & 6 \\ -5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4-30 & -12+12 \\ 10-10 & -30+4 \end{pmatrix}$$

$$= \begin{pmatrix} -26 & 0 \\ 0 & -26 \end{pmatrix}$$

$$C^3 = C^2 . C$$

$$= \begin{pmatrix} -26 & 0 \\ 0 & -26 \end{pmatrix} \begin{pmatrix} -2 & 6 \\ -5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & -156 \\ 130 & -52 \end{pmatrix}$$

(c) $DD^T = \begin{pmatrix} -3 & 2 & 9 & -5 \end{pmatrix}_{1\times 4} \begin{pmatrix} -3 \\ 2 \\ 9 \\ -5 \end{pmatrix}_{4\times 1}$

$= \begin{pmatrix} 9 + 4 + 81 + 25 \end{pmatrix}$

$= 119$

10

(d) $B - 2I = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1-2 & 2 & 0 \\ 0 & -3-2 & 4 \\ -1 & -2 & 3-2 \end{pmatrix}$

$= \begin{pmatrix} -3 & 2 & 0 \\ 0 & -5 & 4 \\ -1 & -2 & 1 \end{pmatrix}$

(e) $B + 3E = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix} + 3\begin{pmatrix} 5 & -3 & 1 \\ 4 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix} + \begin{pmatrix} 15 & -9 & 3 \\ 12 & 6 & 0 \\ 3 & 9 & 9 \end{pmatrix}$

$= \begin{pmatrix} 14 & -7 & 3 \\ 12 & 3 & 4 \\ 2 & 7 & 12 \end{pmatrix}$

10

(f) $E - B = \begin{pmatrix} 5 & -3 & 1 \\ 4 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 6 & -5 & 1 \\ 4 & 5 & -4 \\ 2 & 5 & 0 \end{pmatrix}$

2] (a) $\begin{pmatrix} 2x+y & -7 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 2 & -7 \\ 4 & 4x+y \end{pmatrix}$

$2x + y = 2$ — 1

$4x + y = 6$ — 2

2 − 1 $\quad 2x = 4$

$\boxed{x = 2}$

1 ⇒ $y = 2 - 2x$

$= 2 - 2(2)$

$\boxed{y = -2}$

(b) $\begin{pmatrix} 2x+y & 3 & 10 \\ y+1 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 10 \\ 3 & -2 & 0 \end{pmatrix}$

$2x + y = 2$ — 1

$y + 1 = 3$ — 2

15

1 − 2 $\quad 2x - 1 = -1$

$2x = 0$

$\boxed{x = 0}$

1 ⇒ $\boxed{y = 2}$

(c) $\begin{pmatrix} 6 & -4 & -6 & x-y \end{pmatrix} = -2\begin{pmatrix} -3 & 2 & 2x+2y & 13 \end{pmatrix}$

$-6 = -2(2x + 2y)$

$3 = 2x + 2y$

$\cancel{2 \times 13 =} \quad 2x + 2y = 3 \rightarrow$ 1

$x - y = -2(13)$

$x - y = -26 \rightarrow$ 2

1 + 2 × 2

$4x = 3 - 26(2)$

$\boxed{x = -\frac{49}{4}}$

2 ⇒ $y = x + 26$

$y = -\frac{49}{4} + 26$

$\boxed{y = \frac{55}{4}}$

3] (a) $\begin{pmatrix} x & y \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} x & y \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x & y \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} x^2 + y & xy \\ x & y \end{pmatrix}$

$\begin{pmatrix} x^2 + y & xy \\ x & y \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 2 & 3 \end{pmatrix}$

$x = 2 \quad , \quad y = 3$

(b) $\begin{pmatrix} x + y & -2 \\ x - y & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 12 & -8 \end{pmatrix}$

$\begin{pmatrix} 2(x+y) & -(x+y) + 4 \\ 2(x-y) & -(x-y) - 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 12 & -8 \end{pmatrix}$

$2x + 2y = 8 \longrightarrow ①$

$2x - 2y = 12 \longrightarrow ②$

① + ② $\quad 4x = 20$

$\boxed{x = 5}$

① ⇒ $\quad 2(5) + 2y = 8$

$2y = 8 - 10$

$\boxed{y = -1}$

(3) (c) $A = \begin{pmatrix} a & 0 & -1 \\ 2 & 3 & b \end{pmatrix}$

$AA^T = \begin{pmatrix} a & 0 & -1 \\ 2 & 3 & b \end{pmatrix} \begin{pmatrix} a & 2 \\ 0 & 3 \\ -1 & b \end{pmatrix}$

$\begin{pmatrix} 26 & -11 \\ -11 & 14 \end{pmatrix} = \begin{pmatrix} a^2 + 1 & 2a - b \\ 2a - b & 4 + 9 + b^2 \end{pmatrix}$

$a^2 + 1 = 26$

$a^2 = 25$

$a = \pm 5 \longrightarrow ①$

$2a - b = -11 \longrightarrow ②$

$$13 + b^2 = 14$$
$$b^2 = 1$$
$$b = \pm 1 \longrightarrow (3)$$

25

But $2a - b = -11$

$\underline{\cancel{2(5) - 1}}$ $\therefore a = -5$ and $b = 1$.

4] (a) $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$

$$AB = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$$
$$= \begin{pmatrix} -8+9 & 6-6 \\ -12+12 & 9-8 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

These pair of matrices are inverses.

$$\Rightarrow AB = BA = I$$

(b) $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$AB = I$$

$$BA = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AB = BA = I$$

These pair of matrices are inverses of each other.

5] (a) $A = \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$

$$CA = B$$
$$(CA)A^{-1} = BA^{-1}$$
$$C(AA^{-1}) = BA^{-1}$$
$$CI = BA^{-1}$$
$$C = BA^{-1}$$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$A = \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-1-2}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$C = BA^{-1}$$
$$= \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} -1/3 & -1/3 \\ -2/3 & 1/3 \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{2}{3} + \frac{2}{3} & -\frac{2}{3} - \frac{1}{3} \\ \frac{2}{3} - \frac{4}{3} & \frac{2}{3} + \frac{2}{3} \end{pmatrix}$$

25

$$= \begin{pmatrix} 0 & -1 \\ -\frac{2}{3} & \frac{4}{3} \end{pmatrix}$$

(b) $A^2 - 3A + I = 0$

$$A^2A^{-1} - 3\underbrace{AA^{-1}} + IA^{-1} = 0$$
$$A.\underbrace{A.A^{-1}} - 3I + A^{-1} = 0$$
$$\underbrace{AI}_{A} - 3I + A^{-1} = 0$$
$$A^{-1} = 3I - A$$

6] (a)

$$\begin{array}{ccc} (+) & (-) & + \end{array}$$
$$\begin{pmatrix} -2 & 3 & -2 \\ -4 & -2 & 1 \\ 2 & 4 & 2 \end{pmatrix} = A$$

$$|A| = -2\begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} -4 & 1 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} -4 & -2 \\ 2 & 4 \end{vmatrix}$$

$$= -2(-4-4) - 3(-8-2) - 2(-16+4)$$

$$= 16 + 30 + 24$$

$$= 70$$

(b) $$B = \begin{pmatrix} 3 & -1 & 4 \\ 2 & 5 & 1 \\ 2 & 0 & 6 \end{pmatrix}$$

15

$$|B| = 3\left(\begin{vmatrix} 5 & 1 \\ 0 & 6 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 1 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 2 & 5 \\ 2 & 0 \end{vmatrix}\right.$$

$$= 3(30) + (12-2) + 4(0-10)$$

$$= 90 + 10 - 40$$

$$= 60$$

(c)

$$\begin{array}{cccc} + & - & + & - \end{array}$$
$$C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & -1 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$|C| = 1\begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{vmatrix} - 0 + 1\begin{vmatrix} 2 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} \cancel{+20} + -(-1)\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} +$$

$$2\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - \left\{ 2\begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix}\right.$$

$$\left. -2\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} + -1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \right\}$$

$|c| = (6 + 1 - 3) + (2 - 4 + 1) + -2(-3) + 2(-3) + (2-1)$

$= 4 - 1 + 1$

$= 4$