UNIVERSITY OF RUHUNA
DEPARTMENT OF MATHEMATICS

BACHELOR OF SCIENCE (GENERAL) DEGREE (LEVEL I)
MATHEMATICS
MAT 121β : ALGEBRA

Tutorial No:02 **Semester II, 2020**

Submit answer sheets on : 13/01/2020

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1. Using propeties of the determinats, show that;

(a) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

(b) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

(c) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(d) $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

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2. (a) Assume that $\det(A) = 4$, where $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Find,

(i) $\det(3A)$ (ii) $\det(2A^{-1})$ (iii) $\det((2A)^{-1})$

(iv) $\det \begin{bmatrix} b & a & c \\ 2e & 2d & 2f \\ h & g & i \end{bmatrix}$ (v) $\det \begin{bmatrix} 2a+3d & 2b+3e & 2c+3f \\ d & e & f \\ 5g-7d & 5h-7e & 5i-7f \end{bmatrix}$

(b) If A and B are 2×2 matrices with $\det(A) = 3$ and $\det(B) = 2$. Show that

$$det((3A^{-1})(B^T)) = 6$$

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3. (a) Find the ajoint of the given matrix. Hence, find the inverse.

(i) $\begin{pmatrix} 1 & 0 & -3 \\ 2 & -2 & 1 \\ 0 & -1 & 3 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$

(b) Find all the values of x so that the following matrix A is a singuler matrix.

$$\begin{pmatrix} x & x^2 & 1 \\ 2 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

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4. (a) State the Carmer's rule

(b) Solve the following system of equations;

(i) $4x + 8y + z = -6$
$2x - 3y + 2z = 0$
$x + 7y - 3z = -8$

(ii) $-7x - 6y - 12z = -0$
$5x + 5y + 7z = 0$
$x + 4z = 0$

(c) For what value of λ and μ the following simultaneous equations.

$2x + 3y + 5z = 9$
$7x + 3y - 2z = 8$
$2x + 3y + \lambda z = \mu$

Find ;
(i) No Solution.
(ii) Unique solution.
(iii) Infinitely many solution.

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5. (a) Find the area of the triangle bounded by the points $(-1, 3)$, $(0, -5)$ and $(2, 8)$.

(b) Find the equation of line passing through $(1, -1)$ and $(4, 1)$ using determinants.

(c) Find the volume of the tetrahedron with vertices at $(1, -7, 2)$, $(-4, -7, 1)$, $(1, -1, 2)$. (2, 2, 3)

(d) What is the relation of a, b and c so that the points $A = (1, 0, 1), B = (1, 1, 0), C = (0, 1, 1)$ and $D = (a, b, c)$ are coplanar?

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Algebra
MAT 121/3
Tutorial No #2

01) (a) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ba \\ 1 & ab & ca+cb \end{vmatrix}$$

$$\downarrow C_3 \longrightarrow C_3 + C_2$$

$$= \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & bc+ba+bc \\ 1 & ab & ca+cb+bc \end{vmatrix}$$

$$= (ab+ac+bc) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$$= 0$$

(b.) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

$$= (abc)(abc) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= (abc)^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\downarrow \; C_2 \rightarrow C_2 + C_1$$

$$= (abc)^2 \begin{vmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$\downarrow \; C_3 \rightarrow C_3 + C_1$$

$$= (abc)^2 \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

$$= (abc)^2 \left(-1 \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} - 0 + 0 \right)$$

$$= 4(abc)^2$$

$$= 4a^2b^2c^2 \;//$$

(c) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\Big\downarrow \; R_1 \longrightarrow R_1 + R_2 + R_3$$

$$= \begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\Big\downarrow \; C_1 \longrightarrow C_1 - C_2$$

$$= (a+b+c) \begin{vmatrix} 0 & 1 & 1 \\ 2b-(b-c-a) & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

$$\Big\downarrow \; C_2 \longrightarrow C_2 - C_3$$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b+c+a & b-c-a-2b & 2b \\ 0 & 2c-(c-a-b) & c-a-b \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -b-c-a & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$

$$= (a+b+c)^3 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

$$= (a+b+c)^3 \times 1 \underbrace{\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}}_{1}$$

$$= (a+b+c)^3$$

(d) $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$\downarrow R_2 \to R_2 - 2R_1$$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$\downarrow R_3 \to R_3 - 3R_1$$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

$$\downarrow R_3 \to R_3 - 3R_2$$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

$$= a \begin{vmatrix} a & a+b \\ 0 & a \end{vmatrix}$$

$$= a^3 //$$

(Q2) (i) $\cdot \det(3A) = 3^3 \det(A)$

$= 27 \times 4$

$= 108 //$

(ii) $\det(2A^{-1}) = 2^3 \det(A^{-1})$

$= 2^3 \times \dfrac{1}{\det(A)}$

$= \dfrac{8}{4}$

$= 2 //$

(iii) $\det((2A)^{-1}) = \dfrac{1}{\det(2A)}$

$= \dfrac{1}{2^3 \det(A)}$

$= \dfrac{1}{8 \times 4}$

$= \dfrac{1}{32} //$

(iv) $\det \begin{bmatrix} b & a & c \\ 2e & 2d & 2f \\ h & g & i \end{bmatrix}$

$$= -\det \begin{bmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{bmatrix}$$

$$= -2\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= -2 \quad \det(A)$$

$$= -2 \times 4$$

$$= -8$$

(v) $\det \begin{bmatrix} 2a+3d & 2b+3e & 2c+3f \\ d & e & f \\ 5g-7d & 5h-7e & 5i-7f \end{bmatrix}$

$$\downarrow \begin{matrix} R_1 \longrightarrow R_1 - 3R_2 \\ R_3 \longrightarrow R_3 + 7R_2 \end{matrix}$$

$$= \det \begin{bmatrix} 2a & 2b & 2c \\ d & e & f \\ 5g & 5h & 5i \end{bmatrix}$$

$$= 2 \times \det \begin{bmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{bmatrix}$$

$$= 2 \times 5 \quad \det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= 10 \quad \times 4$$

$$= 40$$

(b) $\det((3A^{-1})(B^{T})) = \det(3A^{-1})\,\det(B^{T})$

$= 3^{2}\det(A^{-1})\,\det(B^{T})$

$= 3^{2} \times \frac{1}{\det(A)}\,\det(B)$

$= 3^{2} \times \frac{1}{\cancel{3}} \times 2$

$= 3 \times 2$

$= 6$

(03) (a) (i) Let $C = [C_{ij}]_{3\times 3}$ is the cofactor matrix of A

$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} = -5$

$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = -6$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -2 \\ 0 & -1 \end{vmatrix} = -2$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -3 \\ -1 & -3 \end{vmatrix} = 3$

$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -3 \\ 0 & 3 \end{vmatrix} = 3$

$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 1$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -3 \\ -2 & 1 \end{vmatrix} = -6$

$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = -7$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix} = -2$

$C = \begin{pmatrix} -5 & -6 & -2 \\ 3 & 3 & 1 \\ -6 & -7 & -2 \end{pmatrix}$

$$\operatorname{adj}(A) = C^T = \begin{pmatrix} -5 & 3 & -6 \\ -6 & 3 & -7 \\ -2 & 1 & -2 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 0 & -3 \\ 2 & -2 & 1 \\ 0 & -1 & 3 \end{pmatrix} = 1 \times \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} - 0 - 3 \begin{vmatrix} 2 & -2 \\ 0 & -1 \end{vmatrix}$$

$$= 1 \times (-6+1) \quad -3 \cdot \times (-2)$$

$$= -5 + 6$$

$$= 1$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$= \frac{1}{1} \begin{pmatrix} -5 & 3 & -6 \\ -6 & 3 & -7 \\ -2 & 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 3 & -6 \\ -6 & 3 & -7 \\ -2 & 1 & -2 \end{pmatrix}$$

(iii) $B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$

$C_{11} = \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = 1$

$C_{12} = -\begin{vmatrix} 0 & 4 \\ 3 & 1 \end{vmatrix} = 12$

$C_{21} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0$

$C_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$

$C_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix} = -2$

$C_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = -4$

$C_{13} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} = -3$

$C_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$C_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\det(B) = \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} + 2 \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix}$

$= 1 - 6$

$= -5$

$$\operatorname{adj}(B) = C^T = \begin{pmatrix} 1 & 0 & -2 \\ 12 & -5 & -4 \\ -3 & 0 & 1 \end{pmatrix}$$

$$B^{-1} = \frac{1}{\det(B)} \operatorname{Adj}(B) = \frac{1}{5} \begin{pmatrix} -1 & 0 & 2 \\ -12 & 5 & 4 \\ 3 & 0 & -1 \end{pmatrix}$$

(b) $A = \begin{pmatrix} x & x^2 & 1 \\ 2 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

$$\det A = 0$$

$$x \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} - x^2 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix} = 0$$

$$x(3+1) - x^2(2-0) + (-2) = 0$$

$$4x - 2x^2 - 2 = 0$$

$$-2x^2 + 4x - 2 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1 \quad //$$

(04) (a) Carmer's Rule is a method that uses determinants to solve systems of equations that have the same number of equations as variables.

(b) (i) $A = \begin{pmatrix} 4 & 8 & 1 \\ 2 & -3 & 2 \\ 1 & 7 & -3 \end{pmatrix}$ $b = \begin{pmatrix} -6 \\ 0 \\ -8 \end{pmatrix}$

$\det A = \begin{vmatrix} 4 & 8 & 1 \\ 2 & -3 & 2 \\ 1 & 7 & -3 \end{vmatrix}$

$= 4 \begin{vmatrix} -3 & 2 \\ 7 & -3 \end{vmatrix} - 8 \begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 2 & -3 \\ 1 & 7 \end{vmatrix}$

$= 4(9-14) - 8(-6-2) + (14+3)$

$= -20 + 64 + 17$

$|A| = 61$

$A_x = \begin{pmatrix} -6 & 8 & 1 \\ 0 & -3 & 2 \\ -8 & 7 & -3 \end{pmatrix}$

$|A_x| = -6 \begin{vmatrix} -3 & 2 \\ 7 & -3 \end{vmatrix} - 8 \begin{vmatrix} 0 & 2 \\ -8 & -3 \end{vmatrix} + 1 \begin{vmatrix} 0 & -3 \\ -8 & 7 \end{vmatrix}$

$= 30 - 128 - 24$

$= -122$

$x = \frac{|A_x|}{|A|} = \frac{-122}{61} = -2$

$$A_y = \begin{pmatrix} 4 & -6 & 1 \\ 2 & 0 & 2 \\ 1 & -8 & -3 \end{pmatrix}$$

$$|A_y| = 4\begin{vmatrix} 0 & 2 \\ -8 & -3 \end{vmatrix} + 6\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 1 & -8 \end{vmatrix}$$

$$= +64 - 48 - 16$$

$$= 0$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{61} = 0$$

$$A_z = \begin{pmatrix} 4 & 8 & -6 \\ 2 & -3 & 0 \\ 1 & 7 & -8 \end{pmatrix}$$

$$|A_z| = 4\begin{vmatrix} -3 & 0 \\ 7 & -8 \end{vmatrix} - 8\begin{vmatrix} 2 & 0 \\ 1 & -8 \end{vmatrix} - 6\begin{vmatrix} 2 & -3 \\ 1 & 7 \end{vmatrix}$$

$$= +96 + 128 - 102$$

$$= 122$$

$$z = \frac{|A_z|}{|A|} = \frac{122}{61} = 2$$

(4) $B = \begin{pmatrix} -7 & -6 & -12 \\ 5 & 5 & 7 \\ 1 & 0 & 4 \end{pmatrix}$ $b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$|B| = -7 \begin{vmatrix} 5 & 7 \\ 0 & 4 \end{vmatrix} + 6 \begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix} - 12 \begin{vmatrix} 5 & 5 \\ 1 & 0 \end{vmatrix}$

$= -140 + 78 + 60$

$= -2$

$B_x = \begin{pmatrix} 0 & -6 & -12 \\ 0 & 5 & 7 \\ 0 & 0 & 4 \end{pmatrix}$ $B_y = \begin{pmatrix} -7 & 0 & -12 \\ 5 & 0 & 7 \\ 1 & 0 & 4 \end{pmatrix}$

$|B_x| = 0$ $|B_y| = 0$

$x = \dfrac{|B_x|}{|B|} = \dfrac{0}{-2} = 0$

$B_z = \begin{pmatrix} -7 & -6 & -12 \\ 5 & 5 & 7 \\ 1 & 0 & 4 \end{pmatrix}$

$|B_z| = 0$

∴ System has unique solution.

Then the solution must be trivial solution.

(C) (i) No Solution
(ii) Unique solution.
(iii) Infinitely many solution.

$x = \frac{\Delta x}{\Delta}$ $y = \frac{\Delta y}{\Delta}$ $z = \frac{\Delta z}{\Delta}$

(i) No Solution, $\Delta = 0$, at least one of $\Delta x, \Delta y, \Delta z$ must be not zero

$$\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} = 0$$

$\Delta = 0$

$$2\begin{vmatrix} 3 & -2 \\ 3 & \lambda \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ 2 & \lambda \end{vmatrix} + 5\begin{vmatrix} 7 & 3 \\ 2 & 3 \end{vmatrix} = 0$$

$$2(3\lambda + 6) - 3(7\lambda + 4)$$
$$+5(21 - 6) = 0$$
$$6\lambda + \cancel{12} - 21\lambda - \cancel{12} + 105 - 30 = 0$$
$$-15\lambda + 75 = 0$$
$$\lambda = 5$$

$\Delta x \neq 0 \Rightarrow \begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ \mu & 3 & \lambda \end{vmatrix} \neq 0$

$$9(3\lambda + 6) - 3(8\lambda + 2\mu) + 5(24 - 3\mu) \neq 0$$
$$27\lambda + 54 - 24\lambda - 6\mu + 120 - 15\mu \neq 0$$
$$3\lambda + 174 - 21\mu \neq 0$$
$$15 + 174 - 21\mu \neq 0$$
$$\mu \neq 9$$

$\Delta y \neq 0 \quad \rightarrow \quad \begin{vmatrix} 2 & 9 & 5 \\ 7 & 8 & -2 \\ 2 & \mu & \lambda \end{vmatrix} \neq 0$

$2(8\lambda + 2\mu) - 9(7\lambda + 4) + 5(7\mu - 16) \neq 0$

$\cancel{3\lambda} + 4\mu - 315 - 36 + 35\mu - \cancel{30} \neq 0$

$39\mu - 351 \neq 0$

$\mu \neq 9$

$\Delta z \neq 0 \quad \Rightarrow \quad \begin{vmatrix} 2 & 3 & 9 \\ 7 & 3 & 8 \\ 2 & 3 & \mu \end{vmatrix} \neq 0$

$2(3\mu - 24) - 3(7\mu - 16) + 9(21 - 6) \neq 0$

$-15\mu + 135 \neq 0$

$\mu \neq 9$

(ii) Unique solution $\Delta \neq 0$

$$\Delta = -15\lambda + 75 \neq 0$$

$$-\lambda + 3 \neq 0$$

$$\lambda \neq 3 \quad , \mu \in \mathbb{R}$$

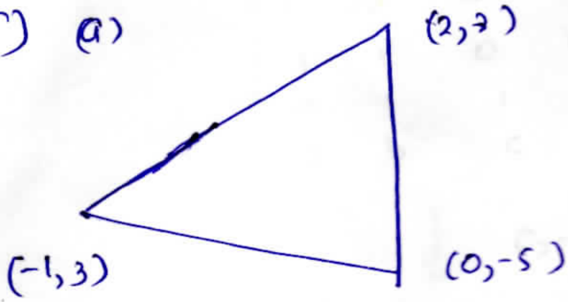
(iii) Infinite many solution.

$\Delta = 0 \quad , \quad \Delta_x = 0 \; , \; \Delta_y = 0 \; , \; \Delta_z = 0$

$\Delta = 0 \Rightarrow \lambda = 3 \qquad \Delta_x = 0 \; , \quad \Delta_y = 0 \qquad \Delta_z = 0$

$\mu = 10$

(05) (a)



$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

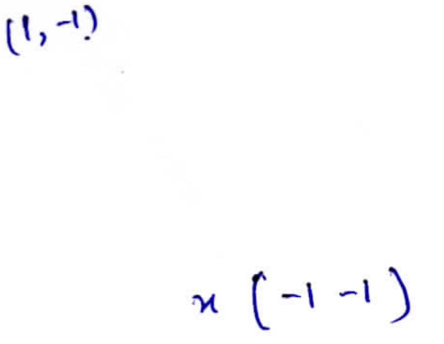
$$= \frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ 0 & -5 & 1 \\ 2 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left[-1(-5-7) - 3(0-2) + 1(0+10) \right]$$

$$= \frac{1}{2} \times 29$$

$$= 14.5$$

(b)



$$\det \begin{pmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{pmatrix} = 0$$

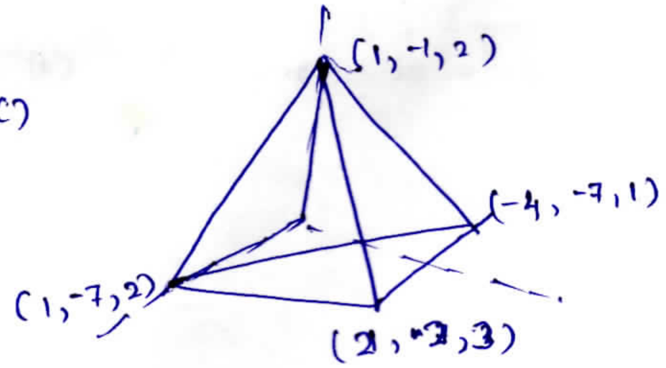
$$\begin{vmatrix} x & y & 1 \\ 1 & -1 & 1 \\ 4 & 1 & 1 \end{vmatrix} = 0$$

$$x(-1-1) - y(1-4) + 1(1+4) = 0$$

$$-2x + 3y + 5 = 0$$

equation of line $2x - 3y = 5$

(c)



$$\text{Volume} = \pm \frac{1}{6} \det \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{bmatrix}$$

$$\text{Volume} = \pm \frac{1}{6} \begin{vmatrix} 1 & -7 & 2 & 1 \\ 2 & 2 & 3 & 1 \\ -4 & -7 & 1 & 1 \\ 1 & -1 & 2 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{6} (-24) = +4$$

$$= 4$$

(d) A, B, C, D are coplanar,
If and only if

$$\det \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ a & b & c & 1 \end{vmatrix} = 0$$

$\downarrow R_1 \to R_3 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ a & b & c & 1 \end{vmatrix}$$

$$\Big\downarrow \quad C_2 \longrightarrow C_2 + C_1$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ a & a+b & c & 1 \end{vmatrix}$$

$$1 \times \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ a+b & c & 1 \end{vmatrix} = 0$$

$$-2(1-c) + 1\,(c - (a+b)) = 0$$

$$2 - 2c + c - a - b = 0$$

$$a + b + c = 2$$