UNIVERSITY OF RUHUNA
DEPARTMENT OF MATHEMATICS

BACHELOR OF SCIENCE (GENERAL) DEGREE (LEVEL II)
MATHEMATICS
MAT 121β : ALGEBRA

Tutorial No:05 **Semester II, 2020**

Submit answer sheets on or before : 10/02/2020

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1. (a) If $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 4\}$ determine
(i) $A \times (B \cup C)$
(ii) $B \times (A \cup C)$
(iii) $(A \cup B) \times C$
(iv) $(A \times B) \cap (A \times C)$

(b) If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$;
find A and B.

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2. (a) If R is a relation 'is greater than' from A to B, where $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 6\}$.
Find;
(i) The set of ordered pairs in R.
(ii) Domain of R.
(iii) Range of R.

(b) Find the domain and range of the relation R given by

$$R = \{(x, y) : y = x + \frac{6}{x};\ where\ x, y \in \mathbb{N}\ and\ x < 6\}$$

(c) If $A = \{2, 4, 6, 9\}$ and $B = \{4, 6, 18, 27, 54\}$, $a \epsilon A$, $b \epsilon B$, find the set of ordered pairs such that $'a'$ is factor of $'b'$ and $a < b$.

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3. (a) Define the relation R on $\mathbb{R}$ by $_xR_y$ if $xy > 0$. Is R an equivalence relation?

(b) Let assume that R be a relation on the set $\mathbb{R}$ real numbers defined by $_xR_y$ if and only if $x - y$ is an integer. Prove that R is an equivalence relation.

(c) Show that the relation R is an equivalence relation in the set $A = \{1, 2, 3, 4, 5\}$ given by the relation $R = \{(a, b) : |a - b|\ is\ even\}$.

(d) Show that the relation $R = \{(a, b) : a - b\ is\ divisible\ by\ 3\}$. a, b belongs to $\mathbb{Z}$ is an equivalence relation.

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4. (a) If R is a relation in $\mathbb{N} \times \mathbb{N}$ defined by $_{(a,b)}R_{(c,d)}$ if and only if $a + d = b + c$, where $\mathbb{N}$ is the set of natural numbers. Show that R ia an equivalence relation.

[2015]

(b) Let R be a relation on $\mathbb{R} \times \mathbb{R}$ defined by $_{(a,b)}R_{(c,d)}$ if $a^2 + b^2 = c^2 + d^2$. Show that R is an equivalence relation.

[2016]

(c) Let R be a relation on $\mathbb{R} \times \mathbb{R}$ defined by $_{(a,b)}R_{(c,d)}$ if $a + d = b + c$. Show that R is an equivalence relation.

[2018]

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Tutorial No: 05

Algebra

MAT 121β

(01)(a) (i) $B \cup C = \{2, 3, 4\}$ $A = \{1, 2\}$

$A \times (B \cup C) = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4)\}$

(ii) $(A \cup C) = \{1, 2, 3, 4\}$ $B = \{2, 3\}$

$B \times (A \cup C) = \{(2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

(iii) $(A \cup B) = \{1, 2, 3\}$ $C = \{3, 4\}$

$(A \cup B) \times C = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

(iv) $(A \times B) = \{(1,2), (1,3), (2,2), (2,3)\}$

$(A \times C) = \{(1,3), (1,4), (2,3), (2,4)\}$

$(A \times B) \cap (A \times C) = \{(1,3)\}$

(b) $(A \times B) = \{(a,1), (b,3), (a,3), (b,1), (a,2), (b,2)\}$

$(A \times B) = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$A = \{a, b\}$ $B = \{1, 2, 3\}$

(02)

(a) (i) $R = \{(3,1), (3,2), (4,1), (4,2), (5,1), (5,2)\}$

(ii) Domain of $R = \{3, 4, 5\}$

(iii) Range of $R = \{1, 2\}$

(b) $x < 6 \longrightarrow x = 1, 2, 3, 4, 5$

$$\left.\begin{array}{ll} x=1 \longrightarrow & y = 1 + \frac{6}{1} = 7 \\ x=2 \longrightarrow & y = 2 + \frac{6}{2} = 5 \\ x=3 \longrightarrow & y = 3 + \frac{6}{3} = 5 \end{array}\right\} \in \mathbb{N}$$

$x=4 \longrightarrow y = 4 + \frac{6}{4} \notin \mathbb{N}$

$x=5 \longrightarrow y = 5 + \frac{6}{5} \notin \mathbb{N}$

Thus $R = \{(1,7), (2,5), (3,5)\}$

Domain of $R = \{(1, 2, 3)\}$

Range of $R = \{7, 5\}$

(c) $A = \{2, 4, 6, 9\}$

$B = \{4, 6, 18, 27, 54\}$

2 is a factor of 4 and $2 < 4$.

So $(2,4)$ is one such ordered pair.

Similarly, $(2,6)$, $(2,18)$, $(2,54)$ are other such ordered pairs.

Thus the required set of ordered pairs is

$\{(2,4), (2,6), (2,18), (2,54), (6,18), (6,54), (9,18), (9,27), (9,54)\}$

(03)

(a) NO. Since $0.0 = 0$ is not greater than 9

So R is not relflexive.

$\therefore$ R is not equivalence relation.

(b) (i) R is reflexive:

Consider $x \in \mathbb{R}$, then $x - x = 0 \in \mathbb{Z}$

Therefore xRx.

(ii) R is Symmetric:

Consider $x \in \mathbb{R}$, $y \in \mathbb{R}$ and xRy,

then $x - y$ is an integer

$\Rightarrow y - x = -(x-y) \in \mathbb{Z}$

$\Rightarrow yRx$

(iii) R is transitive;

Consider $x \in \mathbb{R}, y \in \mathbb{R}$, xRy and yRz,

then $x-y$ and $y-z$ are integers.

$\Rightarrow (x-y)+(y-z) \in \mathbb{Z}$

$\Rightarrow x-z \in \mathbb{Z}$

$\Rightarrow xRz$

Consequently, by (i), (ii) and (iii), R is an equivalence relation.

(C) $R = \{(a,b) : |a-b| \text{ is even}\}$. Where $a, b \in A$

(1) R is reflexive:

from the given relation,

$|a-a| = |0| = 0$

And 0 is always even.

Thus, $|a-a|$ is even.

Therefore, $(a,a) \in R$

Hence R is reflexive.

(ii) Symmetric property

From the given relation,

$\Rightarrow |a-b| = |b-a|$

$\Rightarrow |a-b| = |-(b-a)|$

$\Rightarrow |b-a|$

Therefore, if $(a,b) \in R$, then $(b,a) \in R$

Hence R is symmetric.

(iii) Transitive Property

If $|a-b|$ is even, then $(a-b)$ is even.

Similarly, if $|b-c|$ is even, then $(b-c)$ is also even.

Sum of even numbers is also even.

$\Rightarrow a-b+b-c$
$\Rightarrow a-c$

Then $a-c$ is also even.

Therefore, if $(a,b) \in R$ and $(b,a) \in R$, then (a,c) also belongs to R.

Hence R is transitive.

Consequently, by (i), (ii) and (iii), R is an equivalence relation.

(03) (d) $R = \{(a,b);\ a-b$ is divisible by $3\}$. a, b belongs to $\mathbb{Z}$.

(i) R is reflective:

From given relation.

$\Rightarrow (a,a) : a-a = 0$

0 is divisible by all numbers, so it is divisible by 3.

Therefore $(a,a) \in R$

$\therefore R$ is reflexive relation.

(ii) R is symmetric:

From given relation.

$(a,b) : a-b$ is divisible by 3

also $(b,a) : b-a$ is divisible by 3

Therefore $(b,a) \in R$

$\therefore R$ is symmetric relation.

(iii) R is transitive

from given relation

$(a,b) : a-b = 3k$, is divisible by 3.
k is constant.

$(b,c) : b-c = 3p$ is divisible by 3.
p is constant.

adding the above two equations,

$$a-b+b-c = 3K + 3P$$
$$= 3\underbrace{(K+P)}_{q}$$
$$= 3q \quad , q \text{ is constant}$$

Therefore (a,c) also belongs to R

$\therefore$ R is transitive relation.

Consequently, by (i), (ii) and (iii), R is an equivalence relation.

(04)
(a) $R \subseteq \mathbb{N} \times \mathbb{N}$; $(a,b) \, R \, (c,d) \iff a+d = b+c$

To show that R is an equivalence relation.

(i) R is reflexive:

$(a,b) \, R \, (a,b)$ as $a+b = a+b$ for all $a,b \in \mathbb{N}$

(ii) R is symmetric:

If $(a,b) \, R \, (c,d)$, then $a+d = b+c$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d) \, R \, (a,b)$

(iii) R is transitive.

If $(a,b) \, R \, (c,d)$ and $(c,d) \, R \, (e,f)$

then $a+d = b+c$ and $c+f = d+e$

$\Rightarrow a+d = b+c$ and $c+f = d+e$

$\Rightarrow a-b = e-f$

$\Rightarrow a+f = b+e$

$\Rightarrow (a,b) \, R \, (e,f)$

Consequently, by (i), (ii) and (iii), R is an equivalence relation.

(b)

$$a^2+b^2 = c^2+d^2 \Leftrightarrow (a,b) \, R \, (c,d)$$

(i) R is reflexive:

$a^2+b^2 = a^2+b^2 \Rightarrow (a,b) \, R \, (a,b)$

(ii) R is symmetric:

$(a,b) \, R \, (c,d) \Rightarrow a^2+b^2 = c^2+d^2$

$\Rightarrow c^2+d^2 = a^2+b^2$

$\Rightarrow (c,d) \, R \, (a,b)$

(iii) R is transitive:

$(a,b) \, R \, (c,d)$ and $(c,d) \, R \, (e,f)$

$\Rightarrow a^2+b^2 = c^2+d^2$ and $c^2+d^2 = e^2+f^2$

$\Rightarrow a^2+b^2 = e^2+f^2$

$\Rightarrow (a,b) \, R \, (e,f)$

Consequently by (i), (ii) and (iii), R is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.

(c) $R \subseteq \mathbb{R} \times \mathbb{R}$ $(a,b) R (c,d) \iff a+d = b+c$.

To show that R is an equivalence relation.

(i) R is reflexive:

$(a,b) R (a,b)$ as $a+b = a+b$ for all $a,b \in \mathbb{N}$

(ii) R is symmetric

If $(a,b) R (c,d)$, then $a+d = b+c$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d) R (a,b)$

(iii) R is transitive

If $(a,b) R (c,d)$ and $(c,d) R (e,f)$

then $a+d = b+c$ and $c+f = d+e$

$\Rightarrow \underline{a-b = c-d}$ and $\underline{c-d = e-f}$

$\Rightarrow a-b = e-f$

$\Rightarrow a+f = b+e$

$\Rightarrow (a,b) R (e,f)$

Consequently by (i), (ii) and (iii), R is an equivalence relation.