Question (02)

$$f(x) = e^x - 3x^2 \quad @ \quad [0, 1]$$
$$t = 1 \times 10^{-2}$$

(i) Here,

$$f(0) = e^0 - 3(0)^2 = 1$$
$$f(1) = e^1 - 3(1)^2 = -0.2817$$

As $f(0) \cdot f(1) < 0$

According to the Intermediate Value Theorem this continuous function has a root in $[0, 1]$

(ii)

$$n \geq \frac{\log(b-a) - \log(\varepsilon)}{\log(2)}$$

$$n \geq \frac{\log(1) - \log(1 \times 10^{-2})}{\log(2)}$$

$$n \geq 6.6435$$
$$\underline{n \approx 7}$$

@ least 7 iteration should be used

(iii) $a_1 = 0 \qquad b_1 = 1$

$$c_1 = \frac{1-0}{2} = 0.5$$

$$f(c_1) = f(0.5) = 0.898721 > 0.01$$
$$f(a_1) \cdot f(c_1) = 0.898721 > 0$$

$a_2 = 0.5 \qquad b_2 = 1$

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n	a	b	C_n	$f(C_n)$	$f(a)\cdot f(C_n)$
1	0	1	0.5	0.8987	0.8987 > 0
2	0.5	1	0.75	0.4295	0.3860 > 0
3	0.75	1	0.875	0.1020	0.0438 > 0
4	0.875	1	0.9375	-0.0831	-0.0085 < 0
5	0.875	0.9375	0.90625	0.01116	0.0014 > 0
6	0.90625	0.9375	0.92187	-0.0356	-0.0004 < 0
7	0.90625	0.92187	0.91406	-0.01209	-0.00013 < 0

Here,

$$|C(n+1) - C(n)| < \varepsilon$$

$$|C(6) - C(5)| = |0.92187 - 0.90625|$$
$$= 0.01567 > \varepsilon\ (0.01)$$

$$|C(7) - C(6)| = |0.91406 - 0.92187|$$
$$= 0.00781 < \varepsilon\ (0.01)$$

Therefore,

the root $= \underline{\underline{x^* = 0.91406}}$

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(iv) Advantages

- Simple and easy to implement.
- One function evaluation per iteration
- The size of the interval containg the zero is reduce by 50% after each iteration
- The number of iteration can be determine before the process.
- No knowlege of derivative is needed.
- The function does not have to be differentiable

Disadvantages

- Slow to converge
- Good intermediate approximations may be discarded.

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Question 04

(i) Above Q-03 part (i)

(ii) Let $x = \sqrt{10}$

$x^2 = 10 \Rightarrow x^2 - 10 = 0$

$x_{(n+1)} = x_{(n)} - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x^2 - 10$

$f'(x) = 2x$

$$x_{(n+1)} = x_n - \left(\frac{x_n^2 - 10}{2x_n}\right)$$

n	$x_{(n)}$	$x_{(n+1)}$	$\lvert x_{n+1} - x_n \rvert$
0	3	3.1666666667	0.1666666667
1	3.1666666667	3.1622807018	0.0043859646
2	3.1622807018	3.1622776602	0.0000030416
3	3.1622776602	3.1622776602	0.0000000000

Here $\lvert x_4 - x_3 \rvert = 0.0000000000 < 1 \times 10^{-8}$

The root of $\sqrt{10} = 3.1622776602$

(iii) If $y = (-\sqrt{10} - 1)$

$= (-3.1622776602 - 1)$

$\underline{\underline{y = -4.1622776602}}$

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Question (06)

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

@ the interval $[-1, 0]$

n	$x(n-1)$	$x(n)$	$x(n+1)$	$\lvert (x(n+1)) - x_n \rvert$
1	-1	0	-0.02036199	0.02036199
2	0	-0.02036199	-0.04069126	0.02032927
3	-0.02036199	-0.04069126	-0.04065926	0.00003199
4	-0.04065926	-0.0406592	-0.04065929	0.00000003

$$\lvert x_{(5)} - x_{(4)} \rvert = 0.00000003 < 1 \times 10^{-6}$$

The root $= \underline{\underline{-0.04065929}}$

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Advantages

- It converges at faster than a linear rate, so that it is more rapidly convergent than the bisection method.

- It does not require use of the derivative of the function.

- It requires only one evaluation per iteration, as compared Newton's method.

Disadvantages.

- May not converge.

- There is no guaranteed error bound for the computed iterates.

- Likely to have difficulty if $f'(\alpha) = 0$.

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