Problem Set 1

I am Loading the Diabetes dataset from sklearn and Split it into training (80%) and testing (20%) parts.

For the first question I split the data with a different way that, thaught in the class. But for the rest, I used the same technique.

```
import matplotlib.pyplot as plt
import numpy as np
from sklearn import datasets, linear_model
from sklearn.metrics import mean_squared_error, r2_score
import pandas as pd
# import seaborn
import seaborn as sns
%matplotlib inline

# Load the diabetes dataset
diabetes_X, diabetes_y = datasets.load_diabetes(return_X_y=True)

# Split the data into training/testing sets
diabetes_X_train = diabetes_X[:int(-.2*diabetes_X.shape[0])]
print((diabetes_X_train.shape))
diabetes_X_test = diabetes_X[int(-.2*diabetes_X.shape[0]):]
print(len(diabetes_X_test))

# # Split the targets into training/testing sets
diabetes_y_train = diabetes_y[:int(-.2*diabetes_X.shape[0])]
diabetes_y_test = diabetes_y[int(-.2*diabetes_X.shape[0]):]
print((diabetes_y_train.shape))
print((diabetes_y_test.shape))
```

```
(354, 10)
88
(354,)
(88,)
```

Using DataFrame for the data. And showing the result for example for X_test.

```
diabetes_X_train = pd.DataFrame(diabetes_X_train, columns = ['A','B','C','D','E','F','G','H',
diabetes_X_test = pd.DataFrame(diabetes_X_test, columns = ['A','B','C','D','E','F','G','H','I

diabetes_y_train = pd.DataFrame(diabetes_y_train, columns = ['label'])
diabetes_y_test = pd.DataFrame(diabetes_y_test, columns = ['label'])
diabetes_X_test
```

	A	B	C	D	E	F	G	H	
0	-0.023677	0.050680	0.045529	0.021872	0.109883	0.088873	0.000779	0.034309	C
1	-0.074533	0.050680	-0.009439	0.014987	-0.037344	-0.021669	-0.013948	-0.002592	-C
2	-0.005515	0.050680	-0.033151	-0.015999	0.008063	0.016222	0.015505	-0.002592	-C
3	-0.060003	0.050680	0.049840	0.018429	-0.016704	-0.030124	-0.017629	-0.002592	C
4	-0.020045	-0.044642	-0.084886	-0.026328	-0.035968	-0.034194	0.041277	-0.051671	-C
...	...	...	...	...	...	...	...	...	
83	0.041708	0.050680	0.019662	0.059744	-0.005697	-0.002566	-0.028674	-0.002592	C
84	-0.005515	0.050680	-0.015906	-0.067642	0.049341	0.079165	-0.028674	0.034309	-C
85	0.041708	0.050680	-0.015906	0.017282	-0.037344	-0.013840	-0.024993	-0.011080	-C
86	-0.045472	-0.044642	0.039062	0.001215	0.016318	0.015283	-0.028674	0.026560	C
87	-0.045472	-0.044642	-0.073030	-0.081414	0.083740	0.027809	0.173816	-0.039493	-C

88 rows × 10 columns

Add column of ones

Now add a column of ones to X for easier matrix manipulation of our hypothesis and cost function later on.

```
diabetes_X_test["Intercept"] = 1
diabetes_X_train["Intercept"] = 1
np.set_printoptions(formatter={'float_kind':'{:f}'.format})
cross_tab = np.matmul(np.matrix.transpose(diabetes_X_train.values), diabetes_X_train.values)
cross_tab
```

```
array([[0.831145, 0.142308, 0.159918, 0.257529, 0.225069, 0.188774,
        -0.034304, 0.157329, 0.207618, 0.232650, 0.260063],
       [0.142308, 0.800452, 0.061975, 0.173611, -0.000857, 0.096661,
        -0.309510, 0.258148, 0.079271, 0.183943, -0.075050],
       [0.159918, 0.061975, 0.757806, 0.307323, 0.222875, 0.237738,
        -0.291233, 0.357209, 0.352310, 0.308502, -0.267624],
       [0.257529, 0.173611, 0.307323, 0.796524, 0.213421, 0.167943,
        -0.126681, 0.211341, 0.303888, 0.311877, 0.062917],
       [0.225069, -0.000857, 0.222875, 0.213421, 0.794976, 0.707667,
        0.044769, 0.433351, 0.409985, 0.276294, 0.077527],
       [0.188774, 0.096661, 0.237738, 0.167943, 0.707667, 0.791005,
        -0.173801, 0.536587, 0.255903, 0.247785, 0.095267],
       [-0.034304, -0.309510, -0.291233, -0.126681, 0.044769, -0.173801,
        0.835434, -0.624283, -0.306339, -0.233202, 0.156045],
       [0.157329, 0.258148, 0.357209, 0.211341, 0.433351, 0.536587,
        -0.624283, 0.841694, 0.497741, 0.366450, 0.014093],
       [0.207618, 0.079271, 0.352310, 0.303888, 0.409985, 0.255903,
        -0.306339, 0.497741, 0.780338, 0.388566, -0.066984],
       [0.232650, 0.183943, 0.308502, 0.311877, 0.276294, 0.247785,
        -0.233202, 0.366450, 0.388566, 0.788533, -0.228247],
```

```
       [0.260063, -0.075050, -0.267624, 0.062917, 0.077527, 0.095267,
        0.156045, 0.014093, -0.066984, -0.228247, 354.000000]])
```

```
X = diabetes_X_train[["Intercept", 'A','B','C','D','E','F','G','H','I','J']].values
y = diabetes_y_train[["label"]].values

Xt = diabetes_X_test[["Intercept", 'A','B','C','D','E','F','G','H','I','J']].values
yt = diabetes_y_test[["label"]].values
```

Now I want to calculate through the direct mode.

$$\Theta = (X^T X)^{-1}(X^T Y)$$

```
XT_X = np.matmul(XT, X)
XT_X
```

```
array([[354.000000, 0.260063, -0.075050, -0.267624, 0.062917, 0.077527,
        0.095267, 0.156045, 0.014093, -0.066984, -0.228247],
       [0.260063, 0.831145, 0.142308, 0.159918, 0.257529, 0.225069,
        0.188774, -0.034304, 0.157329, 0.207618, 0.232650],
       [-0.075050, 0.142308, 0.800452, 0.061975, 0.173611, -0.000857,
        0.096661, -0.309510, 0.258148, 0.079271, 0.183943],
       [-0.267624, 0.159918, 0.061975, 0.757806, 0.307323, 0.222875,
        0.237738, -0.291233, 0.357209, 0.352310, 0.308502],
       [0.062917, 0.257529, 0.173611, 0.307323, 0.796524, 0.213421,
        0.167943, -0.126681, 0.211341, 0.303888, 0.311877],
       [0.077527, 0.225069, -0.000857, 0.222875, 0.213421, 0.794976,
        0.707667, 0.044769, 0.433351, 0.409985, 0.276294],
       [0.095267, 0.188774, 0.096661, 0.237738, 0.167943, 0.707667,
        0.791005, -0.173801, 0.536587, 0.255903, 0.247785],
       [0.156045, -0.034304, -0.309510, -0.291233, -0.126681, 0.044769,
        -0.173801, 0.835434, -0.624283, -0.306339, -0.233202],
       [0.014093, 0.157329, 0.258148, 0.357209, 0.211341, 0.433351,
        0.536587, -0.624283, 0.841694, 0.497741, 0.366450],
       [-0.066984, 0.207618, 0.079271, 0.352310, 0.303888, 0.409985,
        0.255903, -0.306339, 0.497741, 0.780338, 0.388566],
       [-0.228247, 0.232650, 0.183943, 0.308502, 0.311877, 0.276294,
        0.247785, -0.233202, 0.366450, 0.388566, 0.788533]])
```

```
XT = np.matrix.transpose(X)
```

```
XT_X_inv = np.linalg.inv(XT_X)
XT_X_inv
```

```
array([[0.002830, -0.001089, 0.000465, 0.001458, -0.000845, 0.010475,
        -0.007880, -0.005826, -0.003642, -0.003331, 0.001212],
       [-0.001089, 1.461256, -0.208485, -0.053855, -0.262176, 0.313990,
        -0.478855, -0.319551, 0.052926, -0.319057, -0.179472],
```

```
        [0.000465, -0.208485, 1.626795, 0.262824, -0.300050, 0.211181,
         -0.057438, 0.406755, -0.336723, 0.279871, -0.219074],
        [0.001458, -0.053855, 0.262824, 2.041989, -0.460931, 1.139085,
         -1.162852, 0.016235, -0.073180, -0.784282, -0.270025],
        [-0.000845, -0.262176, -0.300050, -0.460931, 1.799079, -0.546602,
         0.272957, 0.228012, 0.426018, -0.229631, -0.295771],
        [0.010475, 0.313990, 0.211181, 1.139085, -0.546602, 75.543011,
         -59.480977, -33.771744, -10.433314, -27.267103, 0.150532],
        [-0.007880, -0.478855, -0.057438, -1.162852, 0.272957, -59.480977,
         50.376152, 24.381456, 3.932913, 22.421990, -0.155066],
        [-0.005826, -0.319551, 0.406755, 0.016235, 0.228012, -33.771744,
         24.381456, 19.504282, 9.549627, 11.356589, -0.192973],
        [-0.003642, 0.052926, -0.336723, -0.073180, 0.426018, -10.433314,
         3.932913, 9.549627, 10.558928, 1.276978, -0.370138],
        [-0.003331, -0.319057, 0.279871, -0.784282, -0.229631, -27.267103,
         22.421990, 11.356589, 1.276978, 12.668577, -0.543656],
        [0.001212, -0.179472, -0.219074, -0.270025, -0.295771, 0.150532,
         -0.155066, -0.192973, -0.370138, -0.543656, 1.974032]])
```

```
XT_y = np.matmul(XT, y)
XT_y
```

```
array([[53581.000000],
       [268.755930],
       [9.783420],
       [684.525956],
       [549.189316],
       [302.778582],
       [262.373044],
       [-480.551509],
       [583.020195],
       [705.915304],
       [467.168531]])
```

As we see from the formula, we want to calculate the θ function.

```
thetas = np.matmul(XT_X_inv, XT_y)
thetas
```

```
array([[151.859501],
       [-3.742844],
       [-247.762264],
       [520.395997],
       [298.512841],
       [-637.839376],
       [342.218503],
       [35.404933],
       [168.083838],
       [678.601534],
       [87.564379]])
```

```
import statsmodels.api as sm

regressor = sm.OLS(y, X).fit()
print(regressor.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.508
Model:                            OLS   Adj. R-squared:                  0.493
Method:                 Least Squares   F-statistic:                     35.38
Date:                Sat, 22 Oct 2022   Prob (F-statistic):           4.21e-47
Time:                        16:57:36   Log-Likelihood:                -1910.4
No. Observations:                 354   AIC:                             3843.
Df Residuals:                     343   BIC:                             3885.
Df Model:                          10
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        151.8595      2.886     52.625      0.000     146.184     157.535
x1            -3.7428     65.575     -0.057      0.955    -132.723     125.237
x2          -247.7623     69.190     -3.581      0.000    -383.852    -111.672
x3           520.3960     77.518      6.713      0.000     367.925     672.867
x4           298.5128     72.762      4.103      0.000     155.398     441.628
x5          -637.8394    471.492     -1.353      0.177   -1565.219     289.540
x6           342.2185    385.025      0.889      0.375    -415.090    1099.527
x7            35.4049    239.575      0.148      0.883    -435.817     506.627
x8           168.0838    176.273      0.954      0.341    -178.629     514.797
x9           678.6015    193.082      3.515      0.000     298.829    1058.375
x10           87.5644     76.217      1.149      0.251     -62.348     237.477
==============================================================================
Omnibus:                        1.555   Durbin-Watson:                   1.998
Prob(Omnibus):                  0.459   Jarque-Bera (JB):                1.436
Skew:                           0.038   Prob(JB):                        0.488
Kurtosis:                       2.697   Cond. No.                         230.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
```

```
yT_y = cross_tab[-1:, -1:]
n = cross_tab[:1, :1]
y_bar_square = np.square(cross_tab[:1, -1:])

SST = yT_y - (y_bar_square / n)
SST
```

```
array([[353.918627]])
```

```
n = cross_tab[:1, :1]
y_bar_square = np.square(cross_tab[:1, -1:])
```

```
SSR = np.sum(np.multiply(thetas, XT_y)) - (y_bar_square / n)
SSR
```

```
array([[9151108.332670]])
```

```
r_square = SSR / SST
r_square
```

```
array([[25856.532100]])
```

```
# predict using coefficients
yhat = Xt.dot(thetas)
yhat
```

```
[108.938506],
[138.307753],
[110.872973],
[95.737802],
[159.503009],
[75.560608],
[254.527133],
[57.215126],
[99.570307],
[101.808954],
[261.249383],
[168.813523],
[64.200434],
[183.107140],
[169.903161],
[187.008544],
[184.805878],
[90.376296],
[148.745925],
[251.588736],
[198.127711],
[279.870444],
[51.412531],
[175.317737],
[201.501720],
[170.209760],
[155.296086],
[152.875063],
[232.747954],
[122.876688],
[163.183031],
[172.878832],
[225.283822],
[152.366554],
[99.613128],
[84.108354],
[142.051055],
```

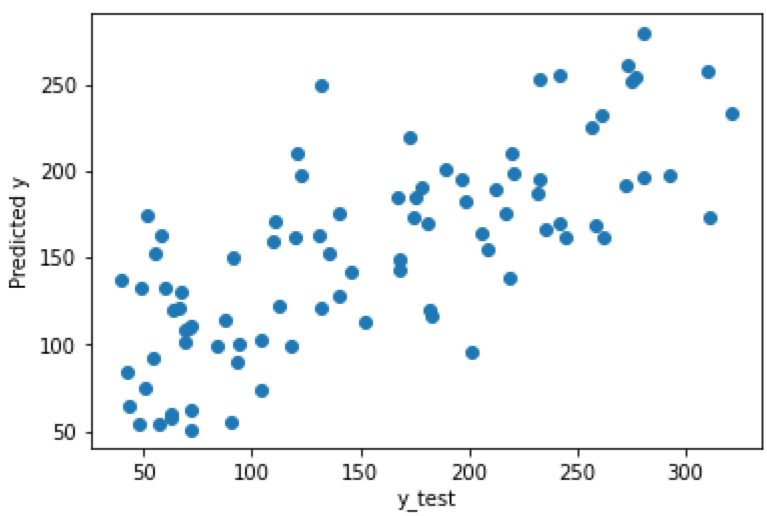
```
       [189.883285],
       [195.331731],
       [150.020750],
       [171.088852],
       [113.075900],
       [161.346473],
       [130.976803],
       [257.196880],
       [100.364386],
       [116.383785],
       [121.479385],
       [219.426698],
       [61.972948],
       [132.733000],
       [120.518188],
       [54.183853],
       [190.315965],
       [102.825689],
       [121.907933],
       [209.997799],
       [53.806932]])
```

```
print('MSE:',mean_squared_error(yhat,yt))
# R2 score = 1 - (MSE / (variance of data))
print('R2 score:',r2_score(yhat,yt))
```

```
MSE: 2910.206933266525
R2 score: 0.07446875020469845
```

```
plt.scatter(yt,yhat)
plt.xlabel('y_test')
plt.ylabel('Predicted y')
```

```
Text(0, 0.5, 'Predicted y')
```




```
import sklearn
sklearn.metrics.r2_score(yt,yhat)
```

```
0.5502492259658616
```

Gradient Descent

Now, I am solving linear regression problem by minimizing the cost:

$C = \frac{1}{N}\left\|\mathbf{y} - \mathbf{X}\mathbf{w}\right\|^2$

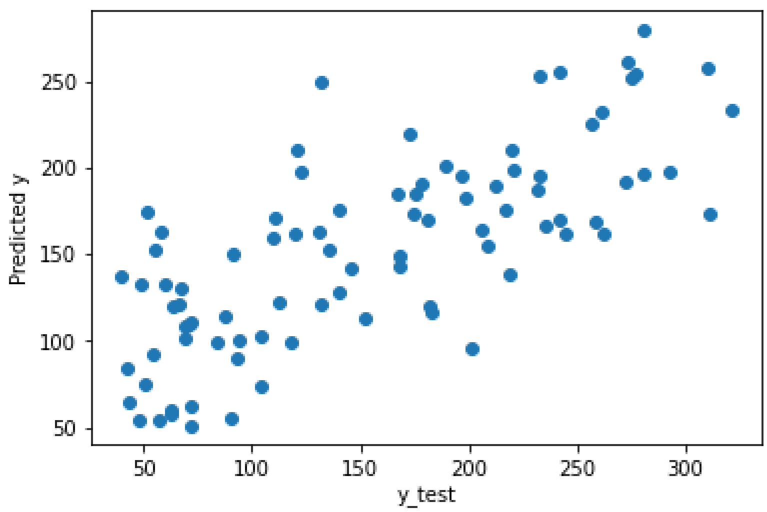
The gradient is: $\nabla C = -\frac{2}{N}\mathbf{X}^T\left(\mathbf{y} - \mathbf{X}\mathbf{w}\right)$

```
import numpy as np

class LinearRegression:
  def __init__(self,learning_rate=0.0001, n_iter=500, fit_intercept=True):
    self.learning_rate = learning_rate
    self.n_iter = n_iter
    self.fit_intercept = fit_intercept
    self.weights = []

  def fit(self,X,y):
    if self.fit_intercept:
      column_of_ones = np.ones((X.shape[0],1))
      X = np.append(column_of_ones,X,axis=1)

    n_samples, n_features = X.shape

    self.weights = np.zeros( (n_features,1), dtype=np.float32 )

    # Gradient descent
    for i in range(self.n_iter):

      y_pred = np.matmul(X, self.weights)

      grad_w = -(2/n_samples) * np.matmul( X.T, (y-y_pred)  )

      self.weights -= self.learning_rate * grad_w


  def predict(self,X):
    if self.fit_intercept:
      column_of_ones = np.ones((X.shape[0],1))
      X = np.append(column_of_ones,X,axis=1)

    return np.matmul(X, self.weights)
```

```
from sklearn import linear_model
from sklearn.metrics import mean_squared_error, r2_score
```

```
lr_skmodel = linear_model.LinearRegression()

lr_skmodel.fit(X,y)

print('Coefficients:',lr_skmodel.coef_)

y_pred = lr_skmodel.predict(Xt) # compare y_test

print('MSE:',mean_squared_error(y_pred,yt))
# R2 score = 1 - (MSE / (variance of data))
print('R2 score:',r2_score(y_pred,yt))


plt.scatter(yt,y_pred)
plt.xlabel('y_test')
plt.ylabel('Predicted y')
```

```
Coefficients: [[0.000000 -3.742844 -247.762264 520.395997 298.512841 -637.839376
  342.218503 35.404933 168.083838 678.601534 87.564379]]
MSE: 2910.2069332665283
R2 score: 0.0744687502046969
Text(0, 0.5, 'Predicted y')
```




```
import sklearn
sklearn.metrics.r2_score(yt,yhat)
```

```
0.5502492259658616
```

```
import numpy as np

class LinearRegression:
  def __init__(self,learning_rate=0.0001, n_iter=500, fit_intercept=True):
    self.learning_rate = learning_rate
    self.n_iter = n_iter
    self.fit_intercept = fit_intercept
    self.weights = []
```

```
  def fit(self,X,y):
    if self.fit_intercept:
      column_of_ones = np.ones((X.shape[0],1))
      X = np.append(column_of_ones,X,axis=1)

    n_samples, n_features = X.shape

    self.weights = np.zeros( (n_features,1), dtype=np.float32 )

    # Gradient descent
    for i in range(self.n_iter):

      y_pred = np.matmul(X, self.weights)

      grad_w = -(2/n_samples) * np.matmul( X.T, (y-y_pred)  )

      self.weights -= self.learning_rate * grad_w


  def predict(self,X):
    if self.fit_intercept:
      column_of_ones = np.ones((X.shape[0],1))
      X = np.append(column_of_ones,X,axis=1)

    return np.matmul(X, self.weights)
```

```
lrmodel = LinearRegression(learning_rate=0.0005,n_iter=100000,fit_intercept=True)
lrmodel.fit(X,y)
print(lrmodel.weights)
```

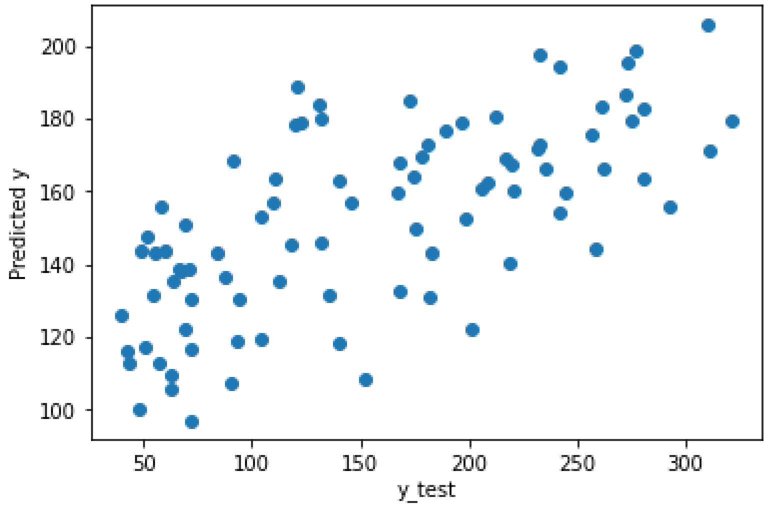
```
[[75.765945]
 [75.765945]
 [37.187241]
 [-12.957762]
 [154.003891]
 [110.085091]
 [41.531887]
 [29.075857]
 [-98.409035]
 [103.784019]
 [145.793289]
 [93.424103]]
```

```
y_pred1 = lrmodel.predict(Xt)
```

```
plt.scatter(yt,y_pred1)
```

```
plt.xlabel('y_test')
plt.ylabel('Predicted y')
```

```
Text(0, 0.5, 'Predicted y')
```




```
print('MSE:',mean_squared_error(y_pred1,yt))
# R2 score = 1 - (MSE / (variance of data))
print('R2 score:',r2_score(y_pred1,yt))
```

```
MSE: 4322.297674447385
R2 score: -5.461138132061364
```

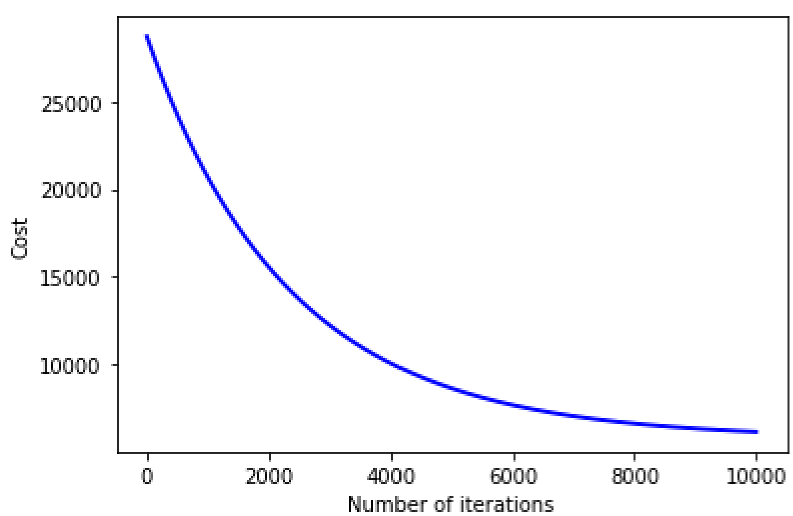
```
import sklearn
sklearn.metrics.r2_score(yt,y_pred1)
```

```
0.3320211348315707
```

```
import matplotlib.pyplot as plt
def linear_regression(x, y, m_current=0, b_current=0, epochs=10000, learning_rate=0.0001):
    cost_list = []
    N = float(len(y))
    for i in range(epochs):
        y_current = (m_current * x) + b_current
        cost = sum([data**2 for data in (y-y_current)]) / N
        cost_list.append(cost)
        m_gradient = -(2/N) * sum(x * (y - y_current))
        b_gradient = -(2/N) * sum(y - y_current)
        m_current = m_current - (learning_rate * m_gradient)
        b_current = b_current - (learning_rate * b_gradient)
    return m_current, b_current, cost, cost_list

m_current, b_current, cost, cost_list = linear_regression(X,y)
```

```
repeat = 10000
```

```
lrate = 0.01
theta = np.zeros((10+1))


def computeCost(X, y, theta):
  m = len(y) # number of training examples
  diff = np.matmul(X, theta)-y
  J = 1 / (2 * m) * np.matmul(diff, diff)
  return J


def gradientDescent(X, y, theta, alpha, num_iters):
  # Initialize some useful values
  m = len(y) # number of training examples
  J_history = []
  # repeat until convergance
  for i in range(num_iters):
    hc = np.matmul(X, theta) - y
    theta -= alpha / m . dot(np.matmul(X.transpose(), hc))
    # Save the cost J in every iteration
    J_history.append(computeCost(X, y, theta))
  return theta, J_history


costs=[]
for cost in cost_list:
  costs.append(cost.mean())
plt.plot(np.arange(10000), costs, '-b', LineWidth=2)
plt.xlabel('Number of iterations')
plt.ylabel('Cost')
plt.show()
```



Question 2

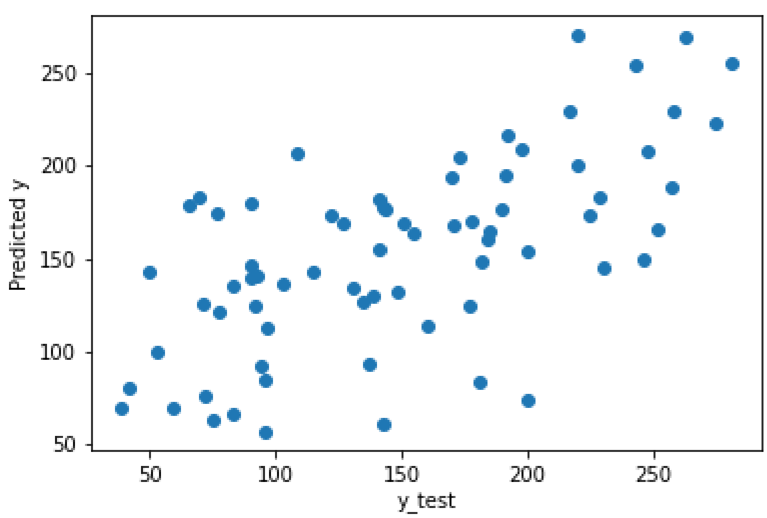
I am downloading sklearn.datasets.fetch_openml(name="house_prices") dataset and Split it into training (80%) and testing (20%) parts.

After that I used sklearn.linear_model module to get solution and plot the model prediction and actual values on test set and use sklearn.metrics.r2_score to calculate performance.

```
from sklearn.datasets import fetch_openml
housing = fetch_openml(name="house_prices")
# to do splitting
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X,y,test_size=0.2)
```

```
from sklearn import linear_model
from sklearn.metrics import mean_squared_error, r2_score

lr_skmodel = linear_model.LinearRegression()

lr_skmodel.fit(X_train,y_train)

print('Coefficients:',lr_skmodel.coef_)

y_pred = lr_skmodel.predict(X_test) # compare y_test

print('MSE:',mean_squared_error(y_pred,y_test))
# R2 score = 1 - (MSE / (variance of data))
print('R2 score:',r2_score(y_pred,y_test))


plt.scatter(y_test,y_pred)
plt.xlabel('y_test')
plt.ylabel('Predicted y')
```

```
Coefficients: [[0.000000 48.677257 -241.924793 522.394429 316.228880 -724.119525
  460.770283 7.366032 63.694368 737.943529 104.806340]]
MSE: 2595.7884990183557
R2 score: 0.027869589143429963
Text(0, 0.5, 'Predicted y')
```



```
import sklearn
sklearn.metrics.r2_score(y_test,y_pred)
```

```
0.38192037963462633
```

End of the Problem Set 1

Hessam Kaveh

22.10.2022