ECE580 FunWork#3

Hamidreza LotfAlizadeh

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For this homework I relied mostly on the matlab script found at

http://www.engr.colostate.edu/ echong/ece520/matlab_demos/optimgui.m

For finding alpha in 1-D linesearch fibonacci method is used. In initial bracketing, $\epsilon = 0.001$ is considered.

In steepest descent and Powell's conjugate gradient algorithms, $\alpha = 0$ is considered as stopping criterion.

In quasi-newton methods, $\Delta g^{(k)} = 0$ is considered as stopping criterion.

I'va had hard time trying to draw the lines with arrows. Eventually I gave up on the arrows and relied on the graphics found in the aforementioned source. Nevertheless, the figures perfectly show progression of optimization. In the following pages, tables indicating sequence points and other information per point and algoirhtm are found.

Method:Steepest Descent
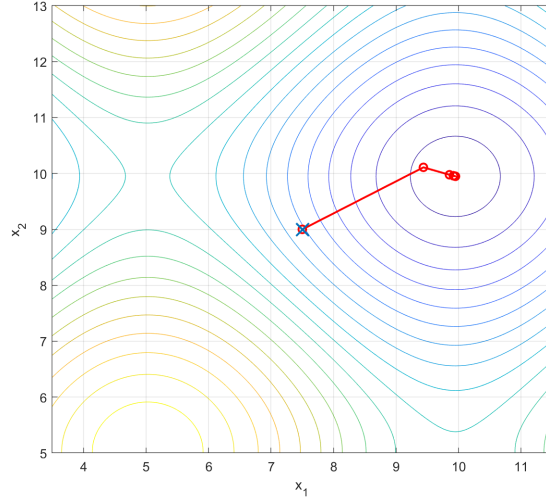
$$x^{(0)} = \begin{bmatrix} 7.5 \\ 9.5 \end{bmatrix}$$

Figure 1: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[9.437e+00 , 1.011e+01]'	2.5557041662e+00	[-1.9884205331e+00 , 6.3392546944e-01]'
2	[9.853e+00 , 9.977e+00]'	2.0096995093e+00	[-3.8087646741e-01 , 1.0733281447e-01]'
3	[9.917e+00 , 9.959e+00]'	1.9921782506e+00	[-1.2888551120e-01 , 3.6146735799e-02]'
4	[9.943e+00 , 9.951e+00]'	1.9900003210e+00	[-2.4589849011e-02 , 6.8797206870e-03]'
5	[9.947e+00 , 9.950e+00]'	1.9899286405e+00	[-8.7994725450e-03 , 2.4614199313e-03]'
6	[9.950e+00 , 9.949e+00]'	1.9899186426e+00	[1.9715005345e-03 , -5.5154166993e-04]'
7	[9.950e+00 , 9.949e+00]'	1.9899185976e+00	[1.8856744896e-03 , -5.2753150215e-04]'
8	[9.950e+00 , 9.949e+00]'	1.9899185546e+00	[1.7998484846e-03 , -5.0352131903e-04]'
9	[9.950e+00 , 9.949e+00]'	1.9899185136e+00	[1.7140225195e-03 , -4.7951112055e-04]'
10	[9.950e+00 , 9.949e+00]'	1.9899184746e+00	[1.6281965944e-03 , -4.5550090671e-04]'
11	[9.950e+00 , 9.949e+00]'	1.9899184376e+00	[1.5423707093e-03 , -4.3149067748e-04]'
12	[9.950e+00 , 9.949e+00]'	1.9899184026e+00	[1.4565448642e-03 , -4.0748043284e-04]'
13	[9.950e+00 , 9.949e+00]'	1.9899183696e+00	[1.3707190592e-03 , -3.8347017278e-04]'
14	[9.950e+00 , 9.949e+00]'	1.9899183386e+00	[1.2848932942e-03 , -3.5945989726e-04]'
15	[9.950e+00 , 9.950e+00]'	1.9899183096e+00	[1.1990675693e-03 , -3.3544960627e-04]'
16	[9.950e+00 , 9.950e+00]'	1.9899182827e+00	[1.1132418845e-03 , -3.1143929977e-04]'
17	[9.950e+00 , 9.950e+00]'	1.9899182577e+00	[1.0274162398e-03 , -2.8742897772e-04]'
18	[9.950e+00 , 9.950e+00]'	1.9899182347e+00	[9.4159063524e-04 , -2.6341864008e-04]'
19	[9.950e+00 , 9.950e+00]'	1.9899182137e+00	[8.5576507090e-04 , -2.3940828683e-04]'
20	[9.950e+00 , 9.950e+00]'	1.9899181948e+00	[7.6993954677e-04 , -2.1539791787e-04]'
21	[9.950e+00 , 9.950e+00]'	1.9899181778e+00	[6.8411406290e-04 , -1.9138753315e-04]'
22	[9.950e+00 , 9.950e+00]'	1.9899181628e+00	[5.9828861931e-04 , -1.6737713258e-04]'
23	[9.950e+00 , 9.950e+00]'	1.9899181499e+00	[5.1246321607e-04 , -1.4336671606e-04]'
24	[9.950e+00 , 9.950e+00]'	1.9899181389e+00	[4.2663785323e-04 , -1.1935628343e-04]'
25	[9.950e+00 , 9.950e+00]'	1.9899181300e+00	[3.4081253085e-04 , -9.5345834477e-05]'
26	[9.950e+00 , 9.950e+00]'	1.9899181230e+00	[2.5498724906e-04 , -7.1335368905e-05]'
27	[9.950e+00 , 9.950e+00]'	1.9899181181e+00	[1.6916200799e-04 , -4.7324886192e-05]'
28	[9.950e+00 , 9.950e+00]'	1.9899181151e+00	[8.3336807972e-05 , -2.3314385282e-05]'
29	[9.950e+00 , 9.950e+00]'	1.9899181142e+00	[-7.7184729910e-07 , 2.1592653202e-07]'

Method:Powell

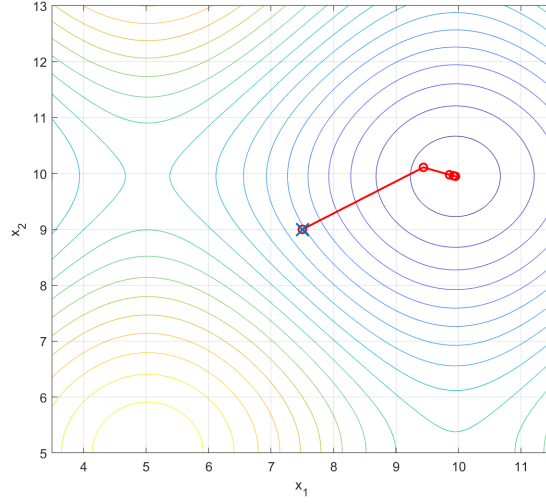$$x^{(0)} = \begin{bmatrix} 7.5 \\ 9.5 \end{bmatrix}$$

Figure 2: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[9.437e+00 , 1.011e+01]'	2.5557041662e+00	[-1.9884205331e+00 , 6.3392546944e-01]'
2	[9.853e+00 , 9.977e+00]'	2.0096995093e+00	[-3.8087646741e-01 , 1.0733281447e-01]'
3	[9.917e+00 , 9.959e+00]'	1.9921782506e+00	[-1.2888551120e-01 , 3.6146735799e-02]'
4	[9.943e+00 , 9.951e+00]'	1.9900003210e+00	[-2.4589849011e-02 , 6.8797206870e-03]'
5	[9.947e+00 , 9.950e+00]'	1.9899286405e+00	[-8.7994725450e-03 , 2.4614199313e-03]'
6	[9.950e+00 , 9.949e+00]'	1.9899186426e+00	[1.9715005345e-03 , -5.5154166993e-04]'
7	[9.950e+00 , 9.949e+00]'	1.9899185976e+00	[1.8856744896e-03 , -5.2753150215e-04]'
8	[9.950e+00 , 9.949e+00]'	1.9899185546e+00	[1.7998484846e-03 , -5.0352131903e-04]'
9	[9.950e+00 , 9.949e+00]'	1.9899185136e+00	[1.7140225195e-03 , -4.7951112055e-04]'
10	[9.950e+00 , 9.949e+00]'	1.9899184746e+00	[1.6281965944e-03 , -4.5550090671e-04]'
11	[9.950e+00 , 9.949e+00]'	1.9899184376e+00	[1.5423707093e-03 , -4.3149067748e-04]'
12	[9.950e+00 , 9.949e+00]'	1.9899184026e+00	[1.4565448642e-03 , -4.0748043284e-04]'
13	[9.950e+00 , 9.949e+00]'	1.9899183696e+00	[1.3707190592e-03 , -3.8347017278e-04]'
14	[9.950e+00 , 9.949e+00]'	1.9899183386e+00	[1.2848932942e-03 , -3.5945989726e-04]'
15	[9.950e+00 , 9.950e+00]'	1.9899183096e+00	[1.1990675693e-03 , -3.3544960627e-04]'
16	[9.950e+00 , 9.950e+00]'	1.9899182827e+00	[1.1132418845e-03 , -3.1143929977e-04]'
17	[9.950e+00 , 9.950e+00]'	1.9899182577e+00	[1.0274162398e-03 , -2.8742897772e-04]'
18	[9.950e+00 , 9.950e+00]'	1.9899182347e+00	[9.4159063524e-04 , -2.6341864008e-04]'
19	[9.950e+00 , 9.950e+00]'	1.9899182137e+00	[8.5576507090e-04 , -2.3940828683e-04]'
20	[9.950e+00 , 9.950e+00]'	1.9899181948e+00	[7.6993954677e-04 , -2.1539791787e-04]'
21	[9.950e+00 , 9.950e+00]'	1.9899181778e+00	[6.8411406290e-04 , -1.9138753315e-04]'
22	[9.950e+00 , 9.950e+00]'	1.9899181628e+00	[5.9828861931e-04 , -1.6737713258e-04]'
23	[9.950e+00 , 9.950e+00]'	1.9899181499e+00	[5.1246321607e-04 , -1.4336671606e-04]'
24	[9.950e+00 , 9.950e+00]'	1.9899181389e+00	[4.2663785323e-04 , -1.1935628343e-04]'
25	[9.950e+00 , 9.950e+00]'	1.9899181300e+00	[3.4081253085e-04 , -9.5345834477e-05]'
26	[9.950e+00 , 9.950e+00]'	1.9899181230e+00	[2.5498724906e-04 , -7.1335368905e-05]'
27	[9.950e+00 , 9.950e+00]'	1.9899181181e+00	[1.6916200799e-04 , -4.7324886192e-05]'
28	[9.950e+00 , 9.950e+00]'	1.9899181151e+00	[8.3336807972e-05 , -2.3314385282e-05]'
29	[9.950e+00 , 9.950e+00]'	1.9899181142e+00	[-7.7184729910e-07 , 2.1592653202e-07]'

3

Method: rank-1

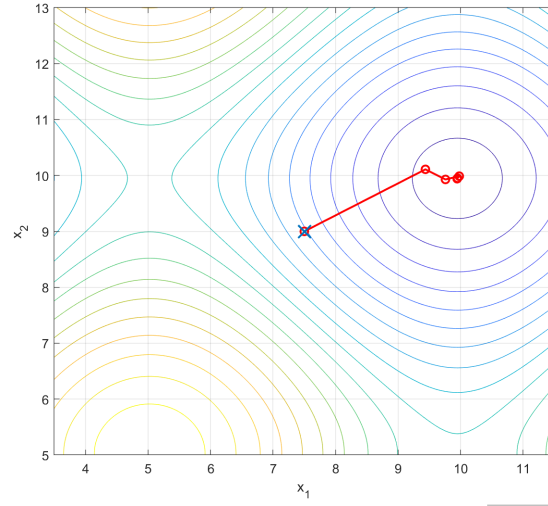$x^{(0)} = \begin{bmatrix} 7.5 \\ 9.5 \end{bmatrix}$

Figure 3: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[9.437e+00 , 1.011e+01]'	2.5557041662e+00	[-1.9884205331e+00 , 6.3392546944e-01]'
2	[9.758e+00 , 9.929e+00]'	2.0635962079e+00	[-7.5769880012e-01 , -8.2085975512e-02]'
3	[9.978e+00 , 9.988e+00]'	1.9944037621e+00	[1.1238923235e-01 , 1.5152423581e-01]'
4	[9.945e+00 , 9.941e+00]'	1.9900960455e+00	[-1.9174469482e-02 , -3.2303221803e-02]'
5	[9.951e+00 , 9.951e+00]'	1.9899245962e+00	[3.6334665692e-03 , 6.1816305090e-03]'
6	[9.950e+00 , 9.950e+00]'	1.9899193030e+00	[1.5562501502e-03 , 2.6472178417e-03]'
7	[9.950e+00 , 9.950e+00]'	1.9899181144e+00	[2.0611113280e-05 , 3.5052824279e-05]'

Method: DFP
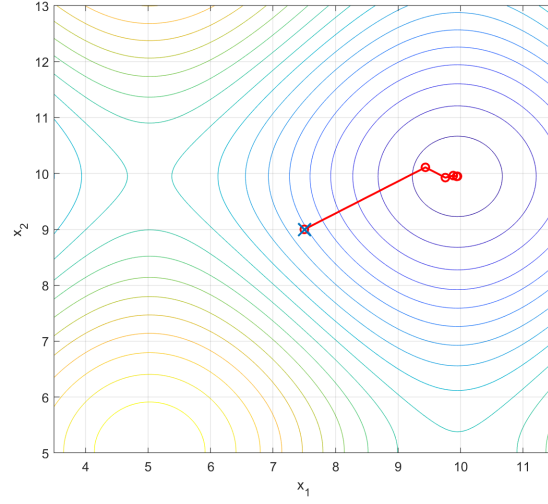
$$x^{(0)} = \begin{bmatrix} 7.5 \\ 9.5 \end{bmatrix}$$

Figure 4: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[9.437e+00 , 1.011e+01]'	2.5557041662e+00	[-1.9884205331e+00 , 6.3392546944e-01]'
2	[9.756e+00 , 9.926e+00]'	2.0653252217e+00	[-7.6509317980e-01 , -9.5206968517e-02]'
3	[9.883e+00 , 9.962e+00]'	1.9990578409e+00	[-2.6421794973e-01 , 5.0901736413e-02]'
4	[9.936e+00 , 9.949e+00]'	1.9902939794e+00	[-5.4515676832e-02 , -2.9504152753e-03]'
5	[9.944e+00 , 9.950e+00]'	1.9899782999e+00	[-2.1847723932e-02 , -1.2978362260e-04]'
6	[9.950e+00 , 9.950e+00]'	1.9899181963e+00	[7.8759872419e-04 , -1.7506854313e-04]'
7	[9.950e+00 , 9.950e+00]'	1.9899181791e+00	[7.0060123196e-04 , -1.5573070691e-04]'
8	[9.950e+00 , 9.950e+00]'	1.9899181640e+00	[6.1360386023e-04 , -1.3639249615e-04]'
9	[9.950e+00 , 9.950e+00]'	1.9899181509e+00	[5.2660623585e-04 , -1.1705558975e-04]'
10	[9.950e+00 , 9.950e+00]'	1.9899181398e+00	[4.3960964262e-04 , -9.7714212475e-05]'
11	[9.950e+00 , 9.950e+00]'	1.9899181306e+00	[3.5260954831e-04 , -7.8388757462e-05]'
12	[9.950e+00 , 9.950e+00]'	1.9899181235e+00	[2.6562258058e-04 , -5.9004470952e-05]'
13	[9.950e+00 , 9.950e+00]'	1.9899181184e+00	[1.7862264537e-04 , -3.9678635844e-05]'
14	[9.950e+00 , 9.950e+00]'	1.9899181153e+00	[9.1622780621e-05 , -2.0352651671e-05]'
15	[9.950e+00 , 9.950e+00]'	1.9899181142e+00	[4.6227478141e-06 , -1.0275923207e-06]'

Method: BFGS

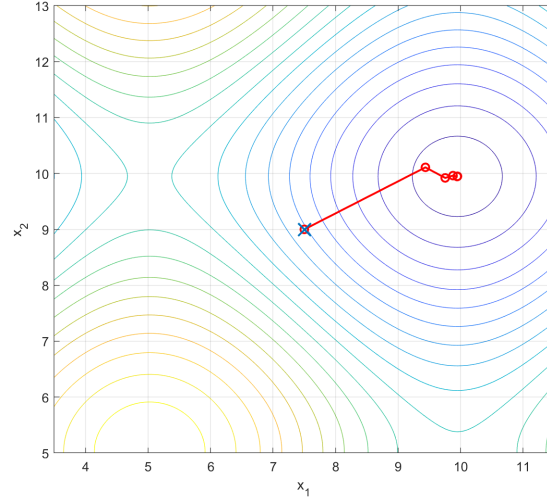$$x^{(0)} = \begin{bmatrix} 7.5 \\ 9.5 \end{bmatrix}$$

Figure 5: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[9.437e+00 , 1.011e+01]'	2.5557041662e+00	[-1.9884205331e+00 , 6.3392546944e-01]'
2	[9.753e+00 , 9.920e+00]'	2.0681825485e+00	[-7.7688125890e-01 , -1.1549744388e-01]'
3	[9.879e+00 , 9.962e+00]'	2.0002271259e+00	[-2.8161835172e-01 , 4.8432815651e-02]'
4	[9.945e+00 , 9.945e+00]'	1.9899916525e+00	[-1.7456176266e-02 , -1.6689301275e-02]'
5	[9.948e+00 , 9.948e+00]'	1.9899258371e+00	[-6.2290781099e-03 , -4.7385244936e-03]'
6	[9.949e+00 , 9.949e+00]'	1.9899193992e+00	[-2.5261892574e-03 , -1.9521006745e-03]'
7	[9.950e+00 , 9.950e+00]'	1.9899181175e+00	[-1.2855625949e-04 , -9.9338422583e-05]'
8	[9.950e+00 , 9.950e+00]'	1.9899181149e+00	[-5.8034086336e-05 , -4.4849191123e-05]'
9	[9.950e+00 , 9.950e+00]'	1.9899181142e+00	[1.1057262181e-05 , 8.5766690548e-06]'

Method:Steepest Descent

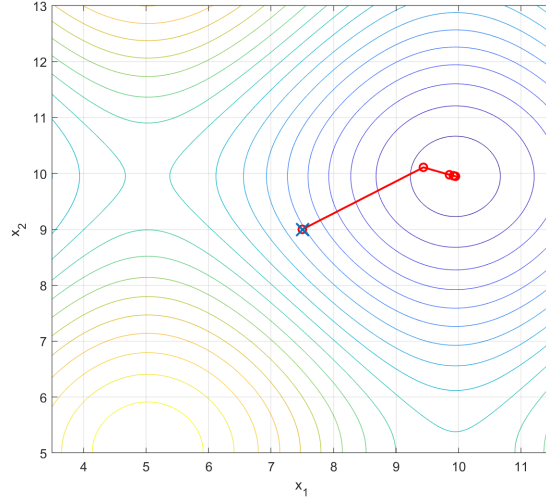$$x^{(0)} = \begin{bmatrix} -7.0 \\ -7.5 \end{bmatrix}$$

Figure 6: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[-8.459e+00 , -9.034e+00]'	7.6497459409e+00	[5.0067743173e+00 , 3.4040145716e+00]'
2	[-9.335e+00 , -9.629e+00]'	2.9303216036e+00	[2.3640856161e+00 , 1.2594178822e+00]'
3	[-9.827e+00 , -9.891e+00]'	2.0263492442e+00	[4.8443566071e-01 , 2.3129645280e-01]'
4	[-9.926e+00 , -9.938e+00]'	1.9912914977e+00	[9.4293005257e-02 , 4.4704770787e-02]'
5	[-9.941e+00 , -9.945e+00]'	1.9901077718e+00	[3.5046534556e-02 , 1.6609738831e-02]'
6	[-9.951e+00 , -9.950e+00]'	1.9899225806e+00	[-5.3782427958e-03 , -2.5497443728e-03]'
7	[-9.950e+00 , -9.950e+00]'	1.9899185468e+00	[-1.6738414836e-03 , -7.9355921758e-04]'
8	[-9.950e+00 , -9.950e+00]'	1.9899185062e+00	[-1.5933120692e-03 , -7.5538081945e-04]'
9	[-9.950e+00 , -9.950e+00]'	1.9899184676e+00	[-1.5127826900e-03 , -7.1720242185e-04]'
10	[-9.950e+00 , -9.950e+00]'	1.9899184309e+00	[-1.4322533462e-03 , -6.7902402478e-04]'
11	[-9.950e+00 , -9.950e+00]'	1.9899183963e+00	[-1.3517240378e-03 , -6.4084562822e-04]'
12	[-9.950e+00 , -9.950e+00]'	1.9899183637e+00	[-1.2711947647e-03 , -6.0266723216e-04]'
13	[-9.950e+00 , -9.950e+00]'	1.9899183331e+00	[-1.1906655270e-03 , -5.6448883659e-04]'
14	[-9.950e+00 , -9.950e+00]'	1.9899183045e+00	[-1.1101363247e-03 , -5.2631044150e-04]'
15	[-9.950e+00 , -9.950e+00]'	1.9899182779e+00	[-1.0296071579e-03 , -4.8813204686e-04]'
16	[-9.950e+00 , -9.950e+00]'	1.9899182533e+00	[-9.4907802654e-04 , -4.4995365265e-04]'
17	[-9.950e+00 , -9.950e+00]'	1.9899182307e+00	[-8.6854893066e-04 , -4.1177525887e-04]'
18	[-9.950e+00 , -9.950e+00]'	1.9899182101e+00	[-7.8801987028e-04 , -3.7359686547e-04]'
19	[-9.950e+00 , -9.950e+00]'	1.9899181915e+00	[-7.0749084544e-04 , -3.3541847244e-04]'
20	[-9.950e+00 , -9.950e+00]'	1.9899181749e+00	[-6.2696185618e-04 , -2.9724007972e-04]'
21	[-9.950e+00 , -9.950e+00]'	1.9899181603e+00	[-5.4643290252e-04 , -2.5906168727e-04]'
22	[-9.950e+00 , -9.950e+00]'	1.9899181477e+00	[-4.6590398451e-04 , -2.2088329502e-04]'
23	[-9.950e+00 , -9.950e+00]'	1.9899181371e+00	[-3.8537510221e-04 , -1.8270490290e-04]'
24	[-9.950e+00 , -9.950e+00]'	1.9899181285e+00	[-3.0484625569e-04 , -1.4452651079e-04]'
25	[-9.950e+00 , -9.950e+00]'	1.9899181220e+00	[-2.2431744503e-04 , -1.0634811849e-04]'
26	[-9.950e+00 , -9.950e+00]'	1.9899181174e+00	[-1.4378867042e-04 , -6.8169725695e-05]'
27	[-9.950e+00 , -9.950e+00]'	1.9899181148e+00	[-6.3259932174e-05 , -2.9991331744e-05]'
28	[-9.950e+00 , -9.950e+00]'	1.9899181142e+00	[1.5658194885e-05 , 7.4234978295e-06]'

Method:Powell

$$x^{(0)} = \begin{bmatrix} -7.0 \\ -7.5 \end{bmatrix}$$

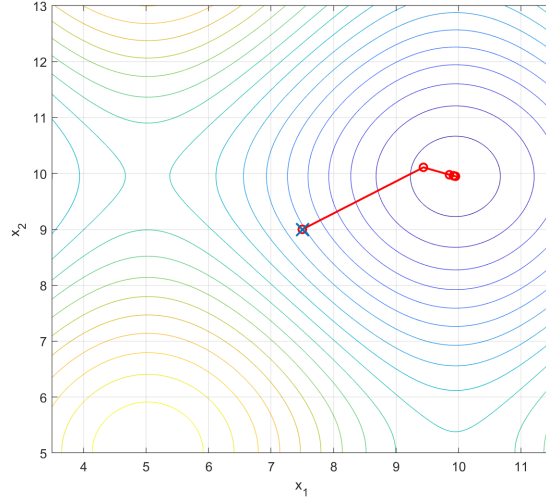Figure 7: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[-8.459e+00 , -9.034e+00]'	7.6497459409e+00	[5.0067743173e+00 , 3.4040145716e+00]'
2	[-9.335e+00 , -9.629e+00]'	2.9303216036e+00	[2.3640856161e+00 , 1.2594178822e+00]'
3	[-9.827e+00 , -9.891e+00]'	2.0263492442e+00	[4.8443566071e-01 , 2.3129645280e-01]'
4	[-9.926e+00 , -9.938e+00]'	1.9912914977e+00	[9.4293005257e-02 , 4.4704770787e-02]'
5	[-9.941e+00 , -9.945e+00]'	1.9901077718e+00	[3.5046534556e-02 , 1.6609738831e-02]'
6	[-9.951e+00 , -9.950e+00]'	1.9899225806e+00	[-5.3782427958e-03 , -2.5497443728e-03]'
7	[-9.950e+00 , -9.950e+00]'	1.9899185468e+00	[-1.6738414836e-03 , -7.9355921758e-04]'
8	[-9.950e+00 , -9.950e+00]'	1.9899185062e+00	[-1.5933120692e-03 , -7.5538081945e-04]'
9	[-9.950e+00 , -9.950e+00]'	1.9899184676e+00	[-1.5127826900e-03 , -7.1720242185e-04]'
10	[-9.950e+00 , -9.950e+00]'	1.9899184309e+00	[-1.4322533462e-03 , -6.7902402478e-04]'
11	[-9.950e+00 , -9.950e+00]'	1.9899183963e+00	[-1.3517240378e-03 , -6.4084562822e-04]'
12	[-9.950e+00 , -9.950e+00]'	1.9899183637e+00	[-1.2711947647e-03 , -6.0266723216e-04]'
13	[-9.950e+00 , -9.950e+00]'	1.9899183331e+00	[-1.1906655270e-03 , -5.6448883659e-04]'
14	[-9.950e+00 , -9.950e+00]'	1.9899183045e+00	[-1.1101363247e-03 , -5.2631044150e-04]'
15	[-9.950e+00 , -9.950e+00]'	1.9899182779e+00	[-1.0296071579e-03 , -4.8813204686e-04]'
16	[-9.950e+00 , -9.950e+00]'	1.9899182533e+00	[-9.4907802654e-04 , -4.4995365265e-04]'
17	[-9.950e+00 , -9.950e+00]'	1.9899182307e+00	[-8.6854893066e-04 , -4.1177525887e-04]'
18	[-9.950e+00 , -9.950e+00]'	1.9899182101e+00	[-7.8801987028e-04 , -3.7359686547e-04]'
19	[-9.950e+00 , -9.950e+00]'	1.9899181915e+00	[-7.0749084544e-04 , -3.3541847244e-04]'
20	[-9.950e+00 , -9.950e+00]'	1.9899181749e+00	[-6.2696185618e-04 , -2.9724007972e-04]'
21	[-9.950e+00 , -9.950e+00]'	1.9899181603e+00	[-5.4643290252e-04 , -2.5906168727e-04]'
22	[-9.950e+00 , -9.950e+00]'	1.9899181477e+00	[-4.6590398451e-04 , -2.2088329502e-04]'
23	[-9.950e+00 , -9.950e+00]'	1.9899181371e+00	[-3.8537510221e-04 , -1.8270490290e-04]'
24	[-9.950e+00 , -9.950e+00]'	1.9899181285e+00	[-3.0484625569e-04 , -1.4452651079e-04]'
25	[-9.950e+00 , -9.950e+00]'	1.9899181220e+00	[-2.2431744503e-04 , -1.0634811849e-04]'
26	[-9.950e+00 , -9.950e+00]'	1.9899181174e+00	[-1.4378867042e-04 , -6.8169725695e-05]'
27	[-9.950e+00 , -9.950e+00]'	1.9899181148e+00	[-6.3259932174e-05 , -2.9991331744e-05]'
28	[-9.950e+00 , -9.950e+00]'	1.9899181142e+00	[1.5658194885e-05 , 7.4234978295e-06]'

Method: rank-1

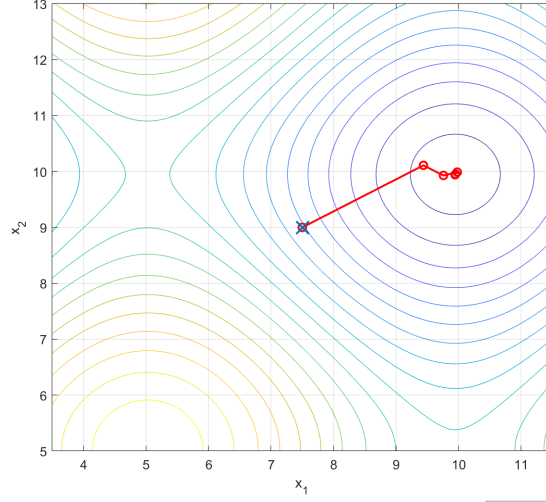$$x^{(0)} = \begin{bmatrix} -7.0 \\ -7.5 \end{bmatrix}$$

Figure 8: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[-8.459e+00 , -9.034e+00]'	7.6497459409e+00	[5.0067743173e+00 , 3.4040145716e+00]'
2	[-9.376e+00 , -9.562e+00]'	2.9272304801e+00	[2.2115207758e+00 , 1.5156419909e+00]'
3	[-9.858e+00 , -9.844e+00]'	2.0285537648e+00	[3.6270442321e-01 , 4.1736241770e-01]'
4	[-9.941e+00 , -9.954e+00]'	1.9901147534e+00	[3.5613668112e-02 , -1.7064869842e-02]'
5	[-9.951e+00 , -9.949e+00]'	1.9899216949e+00	[-4.2849304588e-03 , 3.1686890663e-03]'
6	[-9.949e+00 , -9.950e+00]'	1.9899185643e+00	[1.5182806057e-03 , -1.1247353953e-03]'
7	[-9.949e+00 , -9.950e+00]'	1.9899185228e+00	[1.4466696094e-03 , -1.0716856092e-03]'
8	[-9.949e+00 , -9.950e+00]'	1.9899184834e+00	[1.3750572247e-03 , -1.0186376765e-03]'
9	[-9.949e+00 , -9.950e+00]'	1.9899184459e+00	[1.3034490861e-03 , -9.6558399139e-04]'
10	[-9.949e+00 , -9.950e+00]'	1.9899184105e+00	[1.2318289745e-03 , -9.1254644995e-04]'
11	[-9.949e+00 , -9.950e+00]'	1.9899183770e+00	[1.1602300583e-03 , -8.5948027742e-04]'
12	[-9.949e+00 , -9.950e+00]'	1.9899183456e+00	[1.0886115497e-03 , -8.0644052965e-04]'
13	[-9.949e+00 , -9.950e+00]'	1.9899183161e+00	[1.0170004625e-03 , -7.5339074148e-04]'
14	[-9.949e+00 , -9.950e+00]'	1.9899182887e+00	[9.4538836289e-04 , -7.0034229930e-04]'
15	[-9.949e+00 , -9.950e+00]'	1.9899182633e+00	[8.7377943474e-04 , -6.4728955523e-04]'
16	[-9.949e+00 , -9.950e+00]'	1.9899182398e+00	[8.0216090236e-04 , -5.9424975659e-04]'
17	[-9.949e+00 , -9.950e+00]'	1.9899182184e+00	[7.3056271653e-04 , -5.4118247436e-04]'
18	[-9.949e+00 , -9.950e+00]'	1.9899181990e+00	[6.5894219149e-04 , -4.8814532508e-04]'
19	[-9.949e+00 , -9.950e+00]'	1.9899181815e+00	[5.8733109076e-04 , -4.3509543091e-04]'
20	[-9.949e+00 , -9.950e+00]'	1.9899181661e+00	[5.1571935239e-04 , -3.8204637685e-04]'
21	[-9.949e+00 , -9.950e+00]'	1.9899181527e+00	[4.4410972189e-04 , -3.2899445678e-04]'
22	[-9.949e+00 , -9.950e+00]'	1.9899181413e+00	[3.7249299125e-04 , -2.7595210131e-04]'
23	[-9.950e+00 , -9.950e+00]'	1.9899181319e+00	[3.0089557277e-04 , -2.2288366009e-04]'
24	[-9.950e+00 , -9.950e+00]'	1.9899181245e+00	[2.2927076265e-04 , -1.6985217396e-04]'
25	[-9.950e+00 , -9.950e+00]'	1.9899181190e+00	[1.5766071244e-04 , -1.1680073806e-04]'
26	[-9.950e+00 , -9.950e+00]'	1.9899181156e+00	[8.6050390384e-05 , -6.3749648548e-05]'
27	[-9.950e+00 , -9.950e+00]'	1.9899181142e+00	[1.4441494887e-05 , -1.0696612951e-05]'

Method: DFP
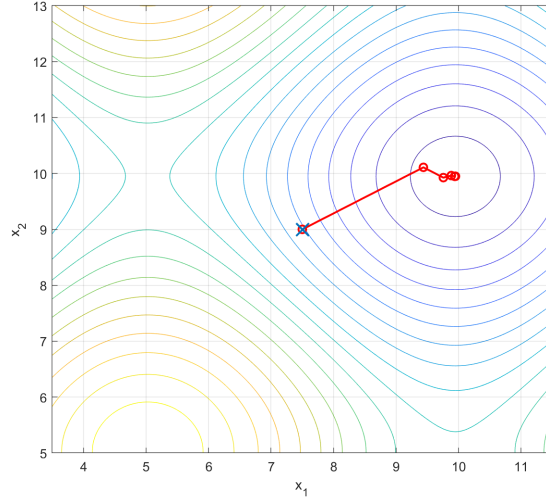
$$x^{(0)} = \begin{bmatrix} -7.0 \\ -7.5 \end{bmatrix}$$

Figure 9: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[-8.459e+00 , -9.034e+00]'	7.6497459409e+00	[5.0067743173e+00 , 3.4040145716e+00]'
2	[-9.459e+00 , -9.383e+00]'	3.0902271714e+00	[1.9075209519e+00 , 2.1874288848e+00]'
3	[-9.872e+00 , -1.012e+01]'	2.0578212915e+00	[3.0792949918e-01 , -6.6586022617e-01]'
4	[-9.982e+00 , -9.972e+00]'	1.9929213273e+00	[-1.2734668668e-01 , -8.7238497851e-02]'
5	[-9.945e+00 , -9.946e+00]'	1.9899946176e+00	[1.9964884877e-02 , 1.4428077655e-02]'
6	[-9.948e+00 , -9.948e+00]'	1.9899266652e+00	[6.6753896783e-03 , 4.8230899120e-03]'
7	[-9.949e+00 , -9.949e+00]'	1.9899183838e+00	[1.1854125282e-03 , 8.5645998134e-04]'
8	[-9.949e+00 , -9.949e+00]'	1.9899183520e+00	[1.1131718947e-03 , 8.0427120692e-04]'
9	[-9.949e+00 , -9.949e+00]'	1.9899183221e+00	[1.0409413568e-03 , 7.5206841093e-04]'
10	[-9.949e+00 , -9.949e+00]'	1.9899182943e+00	[9.6867870979e-04 , 6.9991002173e-04]'
11	[-9.949e+00 , -9.949e+00]'	1.9899182684e+00	[8.9651839083e-04 , 6.4761010461e-04]'
12	[-9.949e+00 , -9.949e+00]'	1.9899182445e+00	[8.2403101684e-04 , 5.9576438476e-04]'
13	[-9.949e+00 , -9.949e+00]'	1.9899182227e+00	[7.5180943282e-04 , 5.4354900238e-04]'
14	[-9.949e+00 , -9.949e+00]'	1.9899182028e+00	[6.7958770476e-04 , 4.9133376939e-04]'
15	[-9.949e+00 , -9.949e+00]'	1.9899181850e+00	[6.0736634827e-04 , 4.3911797267e-04]'
16	[-9.949e+00 , -9.949e+00]'	1.9899181692e+00	[5.3514365695e-04 , 3.8690397247e-04]'
17	[-9.949e+00 , -9.950e+00]'	1.9899181553e+00	[4.6292535615e-04 , 3.3468385001e-04]'
18	[-9.949e+00 , -9.950e+00]'	1.9899181435e+00	[3.9069190000e-04 , 2.8248464346e-04]'
19	[-9.950e+00 , -9.950e+00]'	1.9899181337e+00	[3.1847184090e-04 , 2.3026685326e-04]'
20	[-9.950e+00 , -9.950e+00]'	1.9899181258e+00	[2.4625169646e-04 , 1.7804913138e-04]'
21	[-9.950e+00 , -9.950e+00]'	1.9899181200e+00	[1.7403176594e-04 , 1.2583106390e-04]'
22	[-9.950e+00 , -9.950e+00]'	1.9899181162e+00	[1.0181081602e-04 , 7.3614356654e-05]'
23	[-9.950e+00 , -9.950e+00]'	1.9899181144e+00	[2.9594893026e-05 , 2.1390647709e-05]'

Method: BFGS

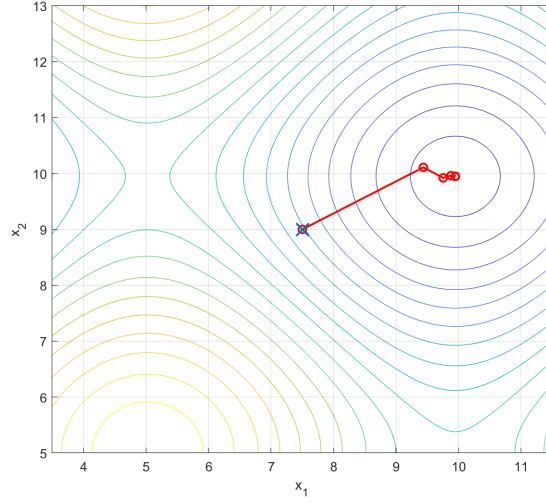$$x^{(0)} = \begin{bmatrix} -7.0 \\ -7.5 \end{bmatrix}$$

Figure 10: Optimization starts from the point at the center of the picture.

Iteration k	$x^{(k)}$	$f(x^{(k)})$	$g^{(k)}$
1	[-8.459e+00 , -9.034e+00]'	7.6497459409e+00	[5.0067743173e+00 , 3.4040145716e+00]'
2	[-9.480e+00 , -9.313e+00]'	3.2107612864e+00	[1.8262835250e+00 , 2.4406771576e+00]'
3	[-9.792e+00 , -1.017e+01]'	2.1361012837e+00	[6.2469708894e-01 , -8.7522670590e-01]'
4	[-9.952e+00 , -1.002e+01]'	2.0002501695e+00	[-8.5279510189e-03 , -2.8620695830e-01]'
5	[-9.946e+00 , -9.953e+00]'	1.9899732184e+00	[1.4815183973e-02 , -1.4750618342e-02]'
6	[-9.949e+00 , -9.950e+00]'	1.9899197889e+00	[2.2784774421e-03 , -2.8446619603e-03]'
7	[-9.950e+00 , -9.949e+00]'	1.9899182861e+00	[-7.3007139712e-04 , 9.1152127545e-04]'
8	[-9.950e+00 , -9.949e+00]'	1.9899182609e+00	[-6.7436076680e-04 , 8.4196004372e-04]'
9	[-9.950e+00 , -9.949e+00]'	1.9899182377e+00	[-6.1864085263e-04 , 7.7240623632e-04]'
10	[-9.950e+00 , -9.949e+00]'	1.9899182164e+00	[-5.6295092985e-04 , 7.0282840172e-04]'
11	[-9.950e+00 , -9.949e+00]'	1.9899181972e+00	[-5.0716326488e-04 , 6.3332889964e-04]'
12	[-9.950e+00 , -9.949e+00]'	1.9899181800e+00	[-4.5169771566e-04 , 5.6357204721e-04]'
13	[-9.950e+00 , -9.949e+00]'	1.9899181647e+00	[-3.9596129088e-04 , 4.9403142210e-04]'
14	[-9.950e+00 , -9.949e+00]'	1.9899181515e+00	[-3.4022561234e-04 , 4.2449018709e-04]'
15	[-9.950e+00 , -9.949e+00]'	1.9899181403e+00	[-2.8448732891e-04 , 3.5495102812e-04]'
16	[-9.950e+00 , -9.950e+00]'	1.9899181311e+00	[-2.2875825486e-04 , 2.8540447675e-04]'
17	[-9.950e+00 , -9.950e+00]'	1.9899181238e+00	[-1.7299518473e-04 , 2.1588516887e-04]'
18	[-9.950e+00 , -9.950e+00]'	1.9899181186e+00	[-1.1736546424e-04 , 1.4625909659e-04]'
19	[-9.950e+00 , -9.950e+00]'	1.9899181154e+00	[-6.1588582729e-05 , 7.6750845895e-05]'
20	[-9.950e+00 , -9.950e+00]'	1.9899181142e+00	[-5.8119741506e-06 , 7.2423645173e-06]'

Figure 11: Optimization starts from the point at the center of the picture.

Figure 12: Problem 4, Part (b)