ECE580 FunWork#5

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1 Solving TSP using GA

In this exercise I implemented a variation of the GA, as expected in the assignment description. First 20 random points were generated and the distance between each pair was calculated and stored in a proximity matrix for later reference in calculating the fitness value.

Order of visiting cities (visiting-order vector) are initially set randomly, with the exception of the first and the last city being the 20th city. Therefore the visiting-order includes 21 elements. The crossover was simply done by randomly selecting two different indices from the range [2,20] and swapping them on visiting-order vector. Ordinary mutation of an element of the visiting-order vector is not feasible in this genetic representation, since it will destroy the circle in the vising-order. Instead we could have implemented the mutation as a random number of crossovers, i.e. in case of mutation, there would be more than one crossover. However, implementing this kind of mutation did not yield that large of an improvement.

The fitness function is the length of the visiting-order, which was basically the sum of distances between consecutive cities in the visiting-order vector.

For the roulette-wheel selection algorithm, I used the function implemented by Rami AbouSleiman (https://www.mathworks.com/matl roulette-wheel-selection).

The population includes $N = 80$ members and ran the algorithm through 1000 generations.

I implemented this project in two variations. In the first variation, the new generation only comprised of the off springs. In the second variation though, the new generation included the best parent from the previous generation as well. Second variation produced better results.

For experimental results, I ran the algorithm in 8×20 iterations. On each of the 8 iterations, a new map was generated. Then on each new map, the GA was applied 20 times, with each time generating new set of visiting-orders for the zeroth population. In general, second variation produced better results by just keeping the best member of the previous generation in the new generation. The final result for the two variations are as follows.

	variation 1	variation 2
	119.5	95.9
	136.5	107.4
	132.1	92.8
	130.4	95.7
	138.7	100.3
	132.2	92.6
	140.0	98.0
	114.3	95.4
Average	130.46	97.26

As shown in the above table, keeping one best parent from the previous generation in the new one would drastically help the optimality of the new generation. In addition, according to the graphs, the average and worst fitness values had the same deviation between the two implementations.

In the following I have included a two graphs of the first and second implementations variations.

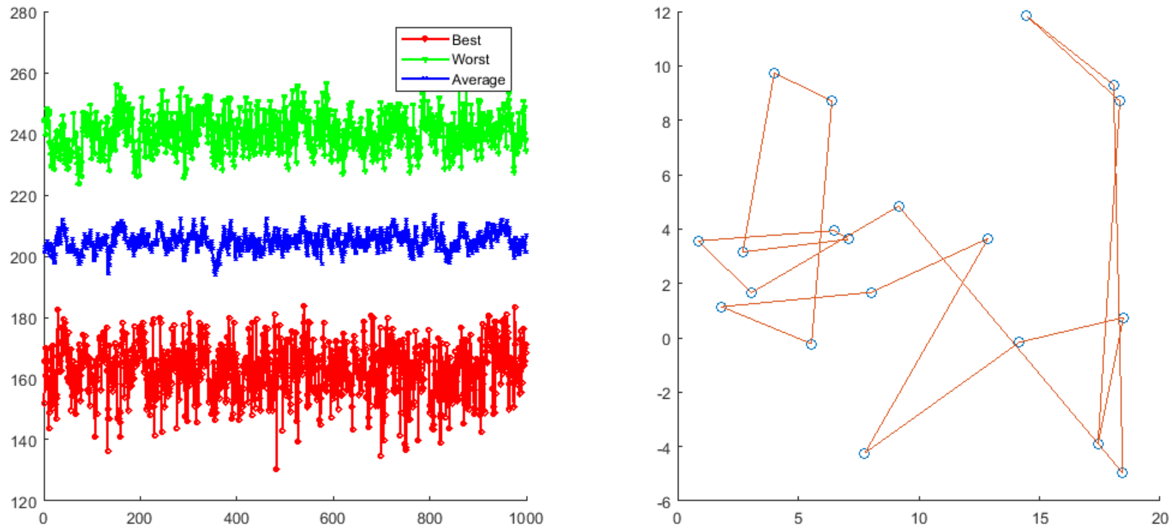

Figure 1: Solving TSP using GA. A sample map. In this variation, all member of the new generation are off springs.

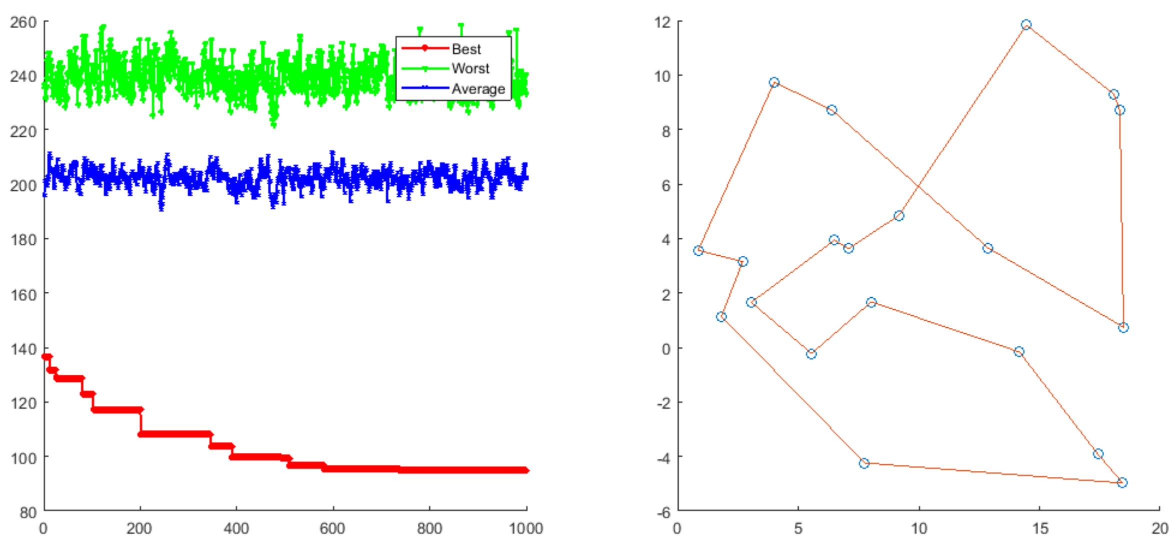

Figure 2: Solving TSP using GA. A sample map. In this variation, the best parent of the previous generation is also included in the new generation. Therefore we can see that the best member can remain the same through various generations.

2 Exercise 16.4

In standard form:

minimize $-2x_1 - x_2$
subject to
$x_1 + x_3 = 5$
$x_2 + x_4 = 7$
$x_1 + x_2 + x_5 = 9$
$x_1, x_2, ..., x_5 \geq 0$

The tableau would be as follows: $\begin{matrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 7 \\ 1 & 1 & 0 & 0 & 1 & 9 \\ -2 & -1 & 0 & 0 & 0 & 0 \end{matrix}$ Performing simplex procedure, pivoting around element (1,1):

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 7 \\ 0 & 1 & -1 & 0 & 1 & 4 \\ 0 & -1 & 2 & 0 & 0 & 10 \end{matrix}$$

Then pivoting around element (3,2):

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 1 & -1 & 3 \\ 0 & 1 & -1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 1 & 14 \end{matrix}$$

Now the coefficients are nonnegative. And solutions is $[5, 4, 0, 3, 0]^T$. And the cost is -14;

3 Exercise 16.12

In standard form:

minimize $5x_1 + 3x_2$
subject to
$5x_1 + x2 - x_3 = 11$
$2x_1 + x_2 - x_4 = 8$
$x_1 + 2x_2 - x_5 = 7$
$x_1, x_2, ..., x_5 \geq 0$

This problem can be solved by applying two phase method.
Phase 1:

Making the tableau with extra artificial variables v, w, z:

a_1	a_2	a_3	a_4	a_5	a_v	a_w	a_z	b
5	1	−1	0	0	1	0	0	11
2	1	0	−1	0	0	1	0	8
1	2	0	0	−1	0	0	1	7
0	0	0	0	0	1	1	1	0

The revised tableau would be as follows:

Variable	B^{-1}			y_0
v	1	0	0	11
w	0	1	0	8
z	0	0	1	7

$\lambda = [1, 1, 1]^T$
$r_D = [r_1, ..., r_5] = [-8, -4, 1, 1, 1]^T$

(1) The augmented revised tableau with

$$y_1 = B^{-1}a_1$$

would be as follows:

Variable	B^{-1}			y_0	y_1
v	1	0	0	11	5
w	0	1	0	8	2
z	0	0	1	7	1

Pivoting around $y_{1,1}$ (first component of y_1):

Variable	B^{-1}			y_0
x_1	1/5	0	0	11/5
w	-2/5	1	0	18/5
z	-1/5	0	1	24/5

$\lambda = [-3/5, 1, 1]^T$
$r_D = [r_2, ..., r_6] = [-12/5, -3/5, 1, 1, 8/5]^T$

(2) The augmented revised tableau with $y_2 = B^{-1}a_2$would be as follows:

Variable	B^{-1}			y_0	y_2
x_1	1/5	0	0	11/5	1/5
w	-2/5	1	0	18/5	3/5
z	-1/5	0	1	24/5	9/5

Pivoting around $y_{2,3}$ (third component of y_2):

Variable	B^{-1}			y_0
x_1	2/9	0	-1/9	5/3
w	-3/9	1	-3/9	6/3
x_2	-1/9	0	5/9	8/3

$\lambda = [-1/3, 1, -1/3]^T$
$r_D = [r_3, ..., r_7] = [-12/5, -3/5, 1, 1, 8/5]^T$

(3) The augmented revised tableau with $y_3 = B^{-1}a_3$ would be as follows:

Variable	B^{-1}			y_0	y_3
x_1	2/9	0	-1/9	5/3	-2/9
w	-3/9	1	-3/9	6/3	3/9
x_2	-1/9	0	5/9	8/3	1/9

Pivoting around $y_{3,2}$ (second component of y_3):

Variable	B^{-1}			y_0
x_1	0	2/3	-1/3	3
x_3	-1	9/3	-3/3	6
x_2	-0	-1/3	2/3	2

$\lambda = [0, 0, 0]^T$
$r_D = [r_4, ..., r_8] = [0, 0, 1, 1, 1]^T$

By the end of phase one the obtained basic feasible solution is $[3, 2, 6, 0, 0]^T$.

Phase 2:
Forming the tableau:

a_1	a_2	a_3	a_4	a_5	b
5	1	-1	0	0	11
2	1	0	-1	0	8
1	2	0	0	-1	7
4	3	0	0	0	0

Using the final tableau from the previous phase:

$\lambda = [0, 5/3, 2/3]^T$
$r_D = [r_4, r_5] = [5/3, 2/3]^T$

Hence, the optimal solution is $[3, 2]^T$

Now the problem in standard form would be as follows:

minimize $-6x_1 - 4x_2 - 7x_3 - 5x_4$
subject to
$x_1 + 2x_2 + x_3 + 2x_4 + x_5 = 20$
$6x_1 + 5x_2 + 3x_3 + 2x_4 + v = 100$
$3x_1 + 4x_2 + 9x_3 + 12x_4 + w = 75$
$x_1, x_2, ..., x_5, v, w \geq 0$

The tableau corresponding to the obtained feasible solution is

Variable	B^{-1}			y_0
x_5	1	0	0	20
v	0	1	0	100
w	0	0	1	75

$\lambda = [0, 0, 0]^T$
$r_D = [r_1, ..., r_4] = [-6, -4, -7, -5]^T$

(1) Including $y_3 = B^{-1}a_3$ in the basis:

Variable	B^{-1}			y_0	y_3
x_5	1	0	0	20	1
v	0	1	0	100	3
w	0	0	1	75	9

Pivoting around $y_{3,3}$

Variable	B^{-1}			y_0
x_5	1	0	-1/9	35/3
v	0	1	-3/9	225/3
x_3	0	0	1/9	25/3

$\lambda = [0, 0, -7/9]^T$
$r_D = [r_1, r_2, r_4, w] = [-33/9, -8/9, 39/9, 7/9]^T$

(2) Including $y_1 = B^{-1}a_1$ in the basis:

Variable	B^{-1}			y_0	y_3
x_5	1	0	-1/9	35/3	2/3
v	0	1	-3/9	225/3	5
x_3	0	0	1/9	25/3	1/3

Pivoting around $y_{1,2}$

Variable	B^{-1}			y_0
x_5	1	-2/15	-1/15	5/3
x_1	0	3/15	-1/15	45/3
x_3	0	-1/15	2/15	10/3

$\lambda = [0, -11/15, -8/15]^T$
$r_D = [r_2, r_4, v, w] = [27/15, 43/15, 11/15, 8/15]^T$

And finally the obtained optimal solution for the original problem is $[15, 0, 10/3, 0]^T$

4 Exercise 17.3

(part a)
In standard form:

minimize $-2x_1 - 3x_2$
subject to
$x_1 + 2x_2 + x_3 = 4$
$2x_1 + x_2 + x_4 = 5$
$x_1, x_2, x_3, x_4 \geq 0$

The initial tableau would be

1	2	1	0	4
2	1	0	1	5
-2	-3	0	0	0

Pivoting around element row 1 column 2:

1/2	1	1/2	0	2
3/2	0	-1/2	1	3
-1/2	0	3/2	0	6

Then pivoting around element row 2 column 1:

0	1	2/3	-1/3	1
1	0	-1/3	2/3	2
0	0	4/3	1/3	7

The solution to the standard form is $[2, 1, 0, 4]^T$ which means $x_1 = 2$ and $x_2 = 1$

(part b)
Dual of the linear program is

maximize $4\lambda_1 + 5\lambda_2$
subject to
$\lambda_1 + 2\lambda_2 \leq -2$
$2\lambda_1 + \lambda_2 \leq -3$
$\lambda_1, \lambda_2 \leq 0$

According to properties of duality, we have $\lambda = c_1 - r_1 = [-4/3, -1/3]^T$

5 Exercise 20.7

Part a. Using the Lagrange's theorem:

$2x_1 + 2\lambda x_1 = 0$
$6x_1 + 2\lambda x_2 = 0$
$1 + 2\lambda x_3 = 0$
$x_1^2 + x_2^2 + x_3^2 = 16$

We will end up with six satisfying conditions

x_1	x_2	x_3	λ
$\sqrt{63}/2$	0	1/2	-1
$-\sqrt{63}/2$	0	1/2	1
0	0	4	-1/8
0	0	-4	1/8
0	$\sqrt{575}/6$	1/6	-3
0	$-\sqrt{575}/6$	1/6	-3

Since all the above x are regular and $\lambda \in \mathbb{R}$ we can apply second order condition:

$F(x) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix}, H(x) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

And the tangent space $T(x^*) = \{y \in \mathbb{R}^3 : [2x_1, 2x_2, 2x_3]y = 0\}$

Therefore for the first point we have:

$$L(x^{(1)}, \lambda^{(1)}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$
$$T(x^{(1)}) = \{[a, b, -a\sqrt{63}]^T : a, b \in \mathbb{R}\}$$

Testing if $x^{(1)}$ satisfies SONC. Let $y = [a, b, -a\sqrt{63}]^T$. We have:

$$y^T L(x^{(1)}, \lambda^{(1)})y = 4b^2 - 2a^2$$

The above equation can have positive or negative values depending on a and b. Therefore, $x^{(1)}$ does not satisfy SONC.

Similarly $x^{(2)}$ will not satisfy SONC.

Now testing if $x^{(3)}$ satisfies SONC.

$$L(x^{(3)}, \lambda^{(3)}) = \begin{bmatrix} 7/4 & 0 & 0 \\ 0 & 23/4 & 0 \\ 0 & 0 & -1/4 \end{bmatrix}$$
$$T(x^{(3)}) = \{[a, b, 0]^T : a, b \in \mathbb{R}\}$$

Let $y = [a, b, 0]^T$ we have:

$$y^T L(x^{(3)}, \lambda^{(3)})y = 7/4 \times a^2 + 23/4 \times b^2$$

Because a and b are not zero simultaneously, the above equation is positive for all possible values of a and b. Therefore, $x^{(3)}$ satisfy SOSC and is a strict local minimizer.

Point $x^{(4)}$ has the same calculations as $x^{(3)}$ and is a local minimizer.

Now testing if $x^{(5)}$ satisfies SONC.

$$L(x^{(5)}, \lambda^{(5)}) = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$
$$T(x^{(5)}) = \{[a, b, -\sqrt{575}b]^T : a, b \in \mathbb{R}\}$$

Let $y = [a, b, -\sqrt{575}b]^T$ we have:

$$y^T L(x^{(5)}, \lambda^{(5)}) y = -4a^2 - 6 \times 575b^2 \leq 0$$

Because a and b are not zero simultaneously, the above equation is negative for all possible values of a and b. Therefore, $x^{(5)}$ satisfy SOSC and is a strict local maximizer. Same calculation goes for $x^{(6)}$, which is a strict local maximizer.

Part b. Using Lagrange's theorem:

$2x_1 + \lambda(6x_1 + 4x_2) = 0$
$2x_2 + \lambda(4x_1 + 12x_2) = 0$
$3x_2^2 + 4x_1x_2 + 6x_2^2 - 140 = 0$

We can write the first two equations as follows

$$\begin{bmatrix} 2 + 6\lambda & 4\lambda \\ 4\lambda & 2 + 12\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

We note due to the third equation (constraint) $x = [0, 0]^T$ cannot be an answer. Therefore for the above equation to hold, the left matrix must be singular. Therefore its determinant must be zero. Solving for λ we get two values -1/2 and -1/7. Inserting the two values of λ in the above equation, we will have the following four critical points:

x_1	x_2	λ
2	4	-1/7
-2	-4	-1/7
$-\sqrt{14}/7$	$\sqrt{14}$	-1/2
$\sqrt{14}/7$	$-\sqrt{14}$	-1/2

Again applying SOSC:

$$F(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, H(x) = \begin{bmatrix} 6 & 4 \\ 4 & 12 \end{bmatrix}$$

Testing for $x^{(1)}$

$$L(x^{(1)}, \lambda^{(1)}) = \begin{bmatrix} 8/7 & -4/7 \\ -4/7 & 2/7 \end{bmatrix}$$
$$T(x^{(1)}) = \{[2a, -a]^T : a \in \mathbb{R}\}$$

Let $y = [2a, -a]^T$

$y^T L(x^{(1)}, \lambda^{(1)}) y = 50a^2 > 0$

Hence, $x^{(1)}$ is a strict local minimizer. With similar calculations for other points, we can conclude that $x^{(2)}$ is a strict local minimizer, while $x^{(3)}$ and $x^{(4)}$ are strict local maximizers.