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Universiteit Leiden
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Opleiding Informatica

Simplicial Coalgebras

for Concurrent Regular Languages

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BACHELOR THESIS

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Abstract

This is where you write an abstract that concisely summarizes your thesis. Keep it short. No references here — exceptions do occur.

Contents

1 Introduction

In this section we give an introduction to the problem addressed in this thesis.

1.1 The situation

Sections may include subsections.

To make sure that this document renders correctly, execute these commands:

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pdflatex thesis
bibtex thesis
pdflatex thesis
pdflatex thesis
```

Here, the `pdflatex` command may need to be executed three times in order to generate the table of contents and so on. Note that a good thesis has figures and tables; examples can be found in Figure 1 and Table 1. And every thesis has references, like [DT19].

Figure 1: Every thesis should have figures. Source: `www.marxbrothers.org`.

Column A	Column B
Point 1	Good
Point 2	Bad

Table 1: Every thesis should have tables.

Final reminder: this template is just an example, if you want you can make adjustments; also discuss with your supervisor which layout he or she likes. But the front page should be as it is now.

TODO: quite a lot!

1.2 Thesis overview

It is recommended to end the introduction with an overview of the thesis. This chapter contains the introduction; Section **??** includes the definitions; Section 3 discusses related work; Section 4 describes the experiments and their outcome; Section 5 concludes. By the way, different section titles are certainly possible.

Also, produce a nice sentence with "bachelor thesis", LIACS and the names of the supervisors.

2 Notes

Connected components Let S be a simplicial set with an iPomset label set L and a labeling map $\ell : S \to L$. Say that $x, y \in S$ are *iPomset neighbors* iff there is a face map from one to the other, and $\ell(x)$,$\ell(y)$ are equal when disregarding event order.

This gives us path connected components of S where $x \sim y$ (are connected) iff there is a path of neighbors connecting x and y.

In this way (when denoting simplicial set elements by their labels)

$$\binom{a}{b} \sim \{a, b\} \sim \binom{b}{a}$$

in the figure below.

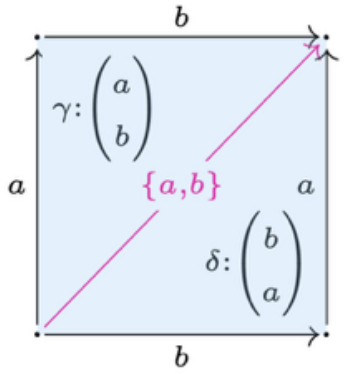

Figure 2: equivalence of iPomsets

We then take the quotient over this relation to get something workable for a transition coalgebra:

$$H := S/\sim$$

We can partition this into $H = H_1 \sqcup H_2 \sqcup H_3, \ldots$ Where $[x]_\sim \in H_k$ iff its shared iPomset event set contains k events. This also entails that if $[x] \in H_k$ then the maximum dimension of an element of $[x]$ is k.

We first intuitively look at what happens when we transition. From each state (class) in H we have maps What we can then do to transition is as follows: Let the current state of the system be $[x]_\sim \in H_k$, the equivalence class of some $x \in S$ under $\sim$, from here on written as $[x]$.

Define the transition maps $\downarrow_a^k \colon H_k \to H_{k-1}$ and $\uparrow_a^k \colon H_k \to H_{k+1}$ as follows:

- The *downward transition* map $\downarrow_a^k$ represents the transition when action a ends:

  $$\downarrow_a^k \colon [x] \mapsto [y]$$

  iff there are $x' \in [x]$, $y' \in [y]$ for which there exists a face map $d_i : x' \to y'$ in S such that

  $$P_{[y]} = P_{[x]} \setminus \{a\}.$$

  where $P_{[x]}$ denotes the event set of $\ell(x)$

- The *upward transition* map $\uparrow_a^k$ represents the transition when action a begins:

  $$\uparrow_a^k \colon [x] \mapsto [y]$$

  iff $\downarrow_a^{k+1} ([y]) = [x]$, which therefore adds a to the currently running events:

  $$P_{[y]} = P_{[x]} \cup \{a\}.$$

Now the directionality of a HDA can be easily constructed. It is simply a choice of allowed which will then define our coalgebra.

3 Related Work

4 Experiments

5 Conclusions and Further Research

References

[DT19] B. Dylan and D. Trump. How to write a good thesis in three months. *International Journal of Computer Science*, 42:123–456, 2019.