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Opleiding Informatica

Simplicial Coalgebras

for Concurrent Regular Languages

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BACHELOR THESIS

Abstract

This is where you write an abstract that concisely summarizes your thesis. Keep it short. No references here — exceptions do occur.

Contents

1 Introduction

In this section we give an introduction to the problem addressed in this thesis.

1.1 The situation

Sections may include subsections.

To make sure that this document renders correctly, execute these commands:

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pdflatex thesis
bibtex thesis
pdflatex thesis
pdflatex thesis
```

Here, the `pdflatex` command may need to be executed three times in order to generate the table of contents and so on. Note that a good thesis has figures and tables; examples can be found in Figure 1 and Table 1. And every thesis has references, like [DT19].

Figure 1: Every thesis should have figures. Source: `www.marxbrothers.org`.

Column A	Column B
Point 1	Good
Point 2	Bad

Table 1: Every thesis should have tables.

Final reminder: this template is just an example, if you want you can make adjustments; also discuss with your supervisor which layout he or she likes. But the front page should be as it is now.

TODO: quite a lot!

1.2 Thesis overview

It is recommended to end the introduction with an overview of the thesis. This chapter contains the introduction; Section ?? includes the definitions; Section 3 discusses related work; Section 4 describes the experiments and their outcome; Section 5 concludes. By the way, different section titles are certainly possible.

Also, produce a nice sentence with "bachelor thesis", LIACS and the names of the supervisors.

2 Notes

Definition 1. *Ranked term graph*

A ranked term graph *consists of:*

- *A directed acyclic graph (DAG) g,*

- *A* rank $(d_r, d_v) \in \mathbb{N}^2$. *Denoting the size of the root and target interfaces,*

- *Let $[n] := \{1, 2, \ldots, n\}$. Maps $r : [d_r] \to \mathcal{P}(g)$ and $v : [d_v] \to \mathcal{P}(g)$, where $r([d_r])$ consists of minimal vertices of g and $v([d_v])$ consists of maximal vertices called the* root *and* variable *maps.*

2.1 Monoidal Structure

Definition 2. *Let $G = (g^G, (d_r^G, d_v^G), r^G, v^G)$, $F = (g^F, (d_r^F, d_v^F), r^F, v^F)$ be ranked term graphs. We define a monoidal product functor as:*

$$G \oplus F = \left(g^G \sqcup g^F, (d_r^G + d_r^F, d_v^G + d_v^F), r', v'\right)$$

Where r', v' are the concatenations of r^G and r^F, v^G and v^F respectively:

$$r'(k) = \begin{cases} r^G(k) & k \leq d_r^G \\ r^F(k - d_r^G) & d_r^G < k \leq d_r^G + d_r^F \end{cases}$$

$$v'(k) = \begin{cases} v^G(k) & k \leq d_v^G \\ v^F(k - d_v^G) & d_v^G < k \leq d_v^G + d_v^F \end{cases}$$

2.2 Composition of Morphisms

Definition 3. *Let $G = (g^G, (d_r^G, d_v^G), r^G, v^G)$, $F = (g^F, (d_r^F, d_v^F), r^F, v^F)$ be ranked term graphs such that $d_v^G = d_r^F$, their composition is defined as follows:*

$$G \circ F = (H, (d_r^G, d_v^F), r^G, v^F)$$

Where $H = g^G \sqcup g^F \sqcup \{(x, x') \in g^G \times g^F | \exists i \in [d_v^G].x \in v^G(i) \wedge x' \in r^F(i)\}$ which is equal to the two union of the two graphs with added arrows going from roots to variables which are equally numbered.

A category with ranked term graphs as morphisms, $\mathbb{N}$ as the set of objects, the disjoint union as monoidal product, and the defined composition of morphisms gives a monoidal category.

2.3 Simplicial set

To get a simplicial set we need an extra piece of equipment: parallel composition with identification of the interfaces.

Definition 4. *Let G, F be ranked term graphs of equal rank (d_r, d_v). Then:*

$$G \otimes F = (g^G \sqcup g^F, (d_r, d_v), r^G \sqcup r^F, v^G \sqcup v^F)$$

Which allows for a identity element for our morphisms: $id_n = (\varnothing, (n, n), r, v)$ where $r(k) = \varnothing = v(k) \forall k \in [n]$

Let G be a ranked graph of rank (n, n) we get clearly: $id_n \otimes G = G \otimes id_n = G$

And lastly we are able to identify two consecutive interface points:

Definition 5. *Let G be a ranked term graph of rank (n, n), define p_m to be the identification of the $m, m + 1$-th interface points, for $m \in \{0\} \cup [n]$*

$$p_m(G) = (g, (n - 1, n - 1), r', v')$$

Where we have:

$$r'(k) = \begin{cases} r(k) & k < m \\ r(m) \cup r(m + 1) & k = m \\ r(k + 1) & m < k < n \end{cases}$$

$$v'(k) = \begin{cases} v(k) & k < m \\ v(m) \cup v(m + 1) & k = m \\ v(k + 1) & m < k \leq n - 1 \end{cases}$$

Definition 6. *Let G be a ranked graph, define a_m to be addition of an extra interface point at position m:*

$$a_m(G) = (g, (n + 1, n + 1), r', v')$$

Where we have:

$$r'(k) = \begin{cases} r(k) & k \leq m \\ \varnothing & k = m + 1 \\ r(k - 1) & m + 1 < k \leq n + 1 \end{cases}$$

$$v'(k) = \begin{cases} v(k) & k < m \\ v(m) \cup v(m + 1) & k = m \\ v(k + 1) & m < k < n \end{cases}$$

Clearly $p_m \circ a_m(G) = G$

Let $\mathcal{L}_n$ be the set of ranked term graphs of rank (n, n) and $\mathcal{L} = \bigcup_{n \in \mathbb{N}} \mathcal{L}_n$ we define a simplicial set $N\mathcal{L}$ as follows:

Definition 7. *Simplicial set of ranked term graphs*

- $(N\mathcal{L})_n = \mathcal{L}_n^n$

- $d_n^0(G_1, G_2, \ldots G_n) = (p_0(G_2), \ldots p_0(G_n))$

- $d_n^k(G_1, G_2, \ldots G_n) = (p_k(G_1), \ldots p_k(G_k \otimes G_{k+1}), \ldots p_k(G_n))$

- $s_n^k(G_1, \ldots, G_n) = (a_k(G_1), \ldots, a_k(G_k), id_{n+1}, a_k(G_{k+1}), \ldots a_k(G_n))$

3 Related Work

4 Experiments

5 Conclusions and Further Research

References

[DT19] B. Dylan and D. Trump. How to write a good thesis in three months. *International Journal of Computer Science*, 42:123–456, 2019.