Model Companion

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Contents

Foreword

When we started the development of this model, we wanted to have a simple code that would allow to understand the behavior of a reverberation chamber in pulsed regime. We did not imagine that a model that simple, would carry the physics of a reverberation chamber. The model we have developed is able to explore both the time domain and the frequency domain and can be helpful to understand the physics beyond a reverberation chamber.

Thank you for using the model we developed. This model based on the image theory can simulate high quality factor rectangular cavities. Even if this model is relatively straightforward, this document can be helpful to have an overview of its possibilities. This document may change in the future. Critics, comments and suggestions are welcomed.

The Authors.

Licensing

This work was supported by the French ministry of defence DGA (Délégation Générale de l'Armement).

Content of the Archive

Included in the Zip file are this document and four Matlab/Octave programs:

- `ImageCreator.m` is a program that generates the sources.
- `CIR.m` is a function that calculates the channel impulse response for a given position.
- `Example.m` is a sample program that uses the `CIR.m` function. It returns the channel impulse response, the frequency response and the response for a chosen signal at a given reception point in the cavity.
- `CIRFFTCONV.m` returns the same results than `Example.m`, it is a all-in-one sample program that does not use the `CIR.m` function, it may increase the performance if a large number of channel impulse responses are calculated.
- `lite.zip` contains a low memory usage version of the programs above (see chapter 5).

Release Notes

- v 1.0 - Image Creator & CIR function and program examples.

Quick Start

- Open Matlab and change the current directory to the directory of the model.
- run `ImageCreator.m` and wait 2 or 3 minutes.
- run `Example.m` or `CIRFFTCONV.m`

The first program creates and saves a matrix of sources for a cavity which dimensions match the dimensions of the reverberation chamber (RC) at the IETR. The second program loads the matrix and returns the channel impulse responses, the frequency response and the responses to an arbitrary signal calculated for a given reception point along the three rectangular components.

Chapter 1

Understanding the Model

1.1 Image Theory

Image theory is generally introduced with electric charges. Let a positive charge placed at a distance d of an infinite perfectly conducting plane (Fig. 1.1-(a)). This conductive plane is an anti-symmetrical plane, thus a negative charge is facing the negative charge. The resulting field of the positive charge and the plane is the field created by an electrostatic dipole with the two charges (Fig. 1.1-(b)). Image theory can be applied to moving charges. Fig. 1.2 sums up the different possible configurations with an electric current vector $\overrightarrow{i}$.

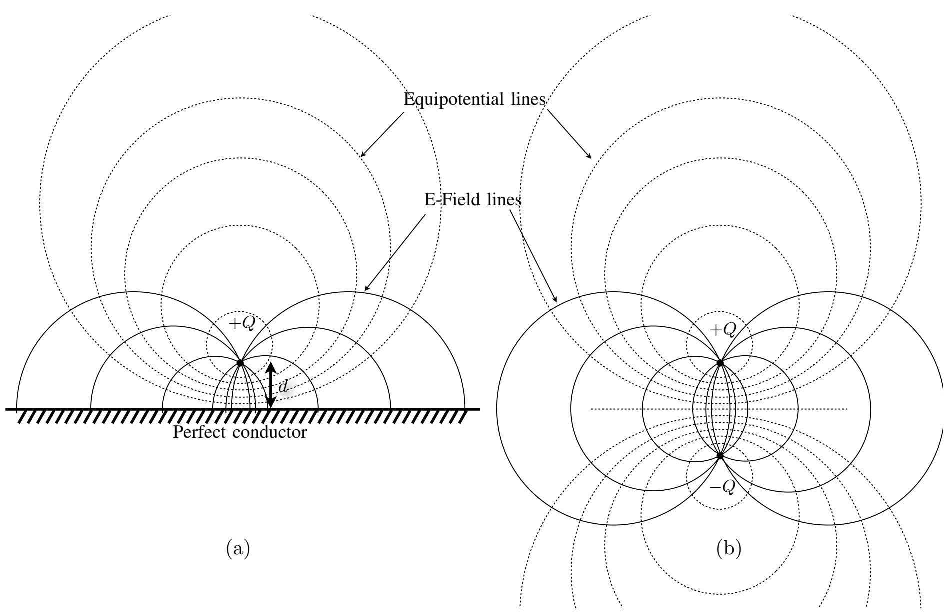


Figure 1.1: Image theory applied to an electric dipole.

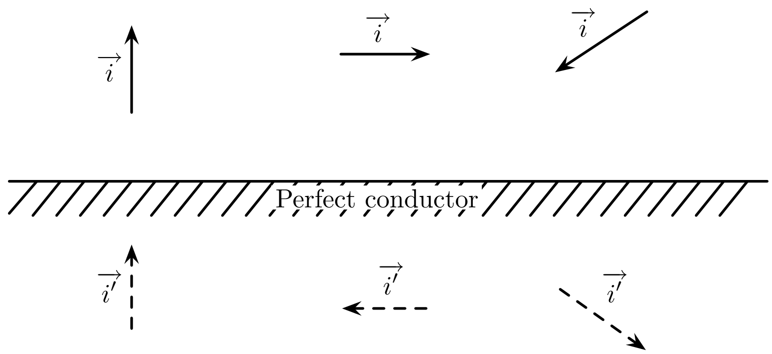


Figure 1.2: Image theory applied to electrical currents.

1.2 Image Theory Applied to Cavities

1.2.1 Images

Fig. 1.3 inspired by [1] presents a vertical and an horizontal view of the image currents created by applying the construction rules presented in Fig. 1.2 to an arbitrarily oriented current in a rectangular cavity. The real cavity (in bold line, in the middle) is surrounded by image cavities. Each image cavity contains an image current. We define the order of an image current, *i.e* the order of an image cavity, as the number of reflections involved in its creation. The number of cavities for a given order n is given by:

$$N_n = 4n^2 + 2, \tag{1.1}$$

and the total number of cavities till the order n is given by:

$$\begin{aligned} M_n &= 1 + \sum_{i=1}^{n}(4i^2 + 2) \\ &= 1 + 2n + \frac{2n(n+1)(2n+1)}{3}. \end{aligned} \tag{1.2}$$

The growth of M_n is therefore proportional to n^3.

1.2.2 Cavity Loss

Image theory is applied with perfectly conducting materials. To simulate a lossy rectangular cavity, we introduce three loss coefficients R_x, R_y, R_z corresponding to the three pairs of conducting walls of our cavity.[1] In this section we focus on an elementary current a inside a n^{th} order image-cavity. This elementary current is created by i reflections along the Ox axis, j reflections along the Oy axis and k reflections along the Oz axis. The attenuation associated

[1] In some situation it can be helpful to have three distinct coefficients to simulate an open door or a lossy wall

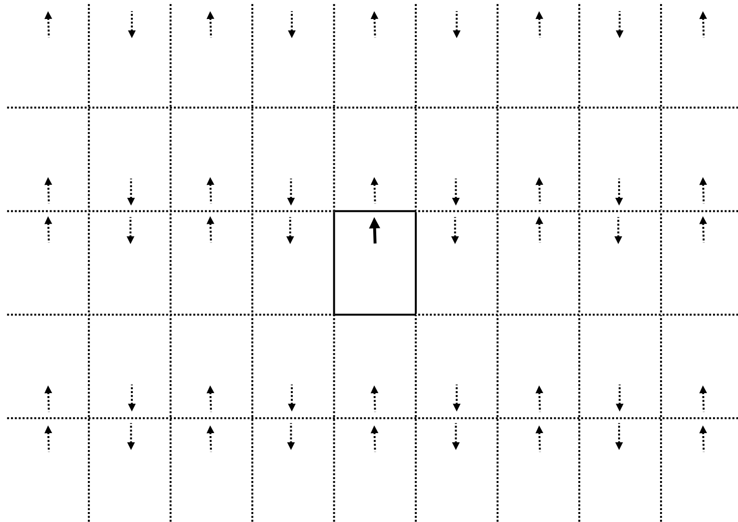


Figure 1.3: Image theory applied to a rectangular cavity.

to this elementary current a is:

$$R_a = R_x^{|i|} R_y^{|j|} R_z^{|k|}, \text{ with } |i| + |j| + |k| = n \tag{1.3}$$

The intensity of this current can be written:

$$I_a = I_0 \cdot R_a, \tag{1.4}$$

where I_0 is the intensity of every current in the system if the walls are perfectly conducting. Assuming $R = R_x = R_y = R_z$, the total amount of energy $\mathcal{E}_{tot}$ found in the system is proportional to:

$$\mathcal{E}_{tot} \propto I_0^2 + \sum_{i=1}^{\infty} (4i^2 + 2) I_0^2 \cdot R^{2i}. \tag{1.5}$$

As $R < 1$, the sum above converges.

1.2.3 Channel Impulse Response Calculation

The intensity of the particular elementary current a, $\overrightarrow{I_a}$ can be written:

$$\overrightarrow{I_a} = I_a \cdot \overrightarrow{w} = I_0 R_x^{|i|} R_y^{|j|} R_z^{|k|} \cdot \overrightarrow{w}, \tag{1.6}$$

where $\overrightarrow{w}$ is the normalized vector along the direction of the considered elementary current. The current in the real cavity and all the image currents emit simultaneously an elementary impulse $f(t)$. The intensity of the elementary current a can be written:

$$I_a(t) = I_0 R_a \cdot f(t), \text{ with } f(t) = \begin{cases} 1 & \text{if } t = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1.7}$$

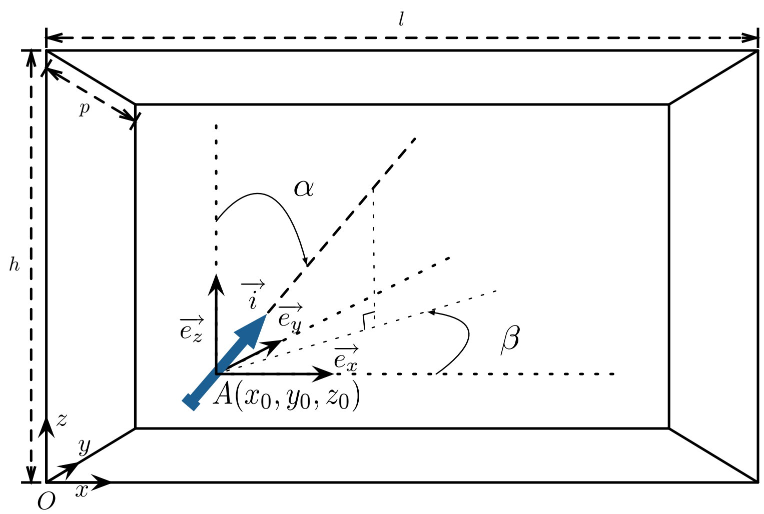


Figure 1.4: Angles and coordinates of the elementary dipole in the cavity.

The orientation of the current at this position is given by a tilt angle α, defined by the angle between $\overrightarrow{w}$ and $\overrightarrow{e_z}$ and an azimuthal angle β defined by the angle between $\overrightarrow{w} - (\overrightarrow{w} \cdot \overrightarrow{e_z})\overrightarrow{w}$ and $\overrightarrow{e_x}$ (Fig. 1.4). The electrical field created by the elementary current a and received at a reception point P within the real cavity can be written[2]:

$$\overrightarrow{E_a}(t) = -\omega\mu \frac{dhI_0R_af(t-t_a)}{4\pi d_a} \sin\theta_a \begin{cases} \cos\theta_a \cos\phi_a \cdot \overrightarrow{u} \\ \cos\theta_a \sin\phi_a \cdot \overrightarrow{v} \\ -\sin\theta_a \cdot \overrightarrow{w} \end{cases} \tag{1.8}$$

with:

$$\begin{cases} \overrightarrow{u} = \mathcal{R}_{\alpha,\beta} \cdot \overrightarrow{e_x} \\ \overrightarrow{v} = \mathcal{R}_{\alpha,\beta} \cdot \overrightarrow{e_y} \\ \overrightarrow{w} = \mathcal{R}_{\alpha,\beta} \cdot \overrightarrow{e_z} \end{cases} \tag{1.9}$$

where dh is the length of the elementary dipole, d_a is the distance between the position of the elementary current a and the reception point P, t_a is the time of arrival at the reception point, θ_a and ϕ_a are angular coordinates of the point P in the local spherical coordinate system attached to the elementary current a. $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ define the local rectangular basis attached to the elementary current. $\mathcal{R}_{\alpha,\beta}$ is the rotation matrix[3] that changes the rectangular basis $(\overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z})$ into the local basis $(\overrightarrow{u}, \overrightarrow{v}, \overrightarrow{w})$ and c is the celerity.

From the E-field expression (1.8) in the local rectangular coordinate system attached to the elementary current, we can deduce the expression in the rectangular coordinate system attached

[2] (1.8) is valid for a dipole radiation pattern ($-\sin\theta$). One should note that any 3-D radiation pattern can be employed.

[3] $\mathcal{R}_{\alpha,\beta}$ represents a rotation of an angle α around a unitary vector $\overrightarrow{e}_\beta = -\cos\beta\overrightarrow{e_x} + \sin\beta\overrightarrow{e_y}$,

$$\mathcal{R}_{\alpha,\beta} = \begin{pmatrix} \cos^2\beta + (1-\cos^2\beta)\cos\alpha & -\cos\beta\sin\beta(1-\cos\alpha) & \sin\beta\sin\alpha \\ -\cos\beta\sin\beta(1-\cos\alpha) & \sin^2\beta + (1-\sin^2\beta)\cos\alpha & \cos\beta\sin\alpha \\ -\sin\beta\sin\alpha & -\cos\beta\sin\alpha & \cos\alpha \end{pmatrix} \tag{1.10}$$

to the simulated cavity:

$$\begin{aligned}\overrightarrow{E}_{a_{(\overrightarrow{e_x},\overrightarrow{e_y},\overrightarrow{e_z})}} &= \mathcal{R}_{\alpha,\beta}^{-1}\cdot\overrightarrow{E}_{a_{(\overrightarrow{u},\overrightarrow{v},\overrightarrow{w})}} \\ &= \mathcal{R}_{-\alpha,\beta}\cdot\overrightarrow{E}_{a_{(\overrightarrow{u},\overrightarrow{v},\overrightarrow{w})}}. \end{aligned} \tag{1.11}$$

The channel impulse response is given by adding the contribution of every current in our system. If M is the total number of currents in our system, from (1.8) we can deduce three channel impulse responses corresponding to the three rectangular components:

$$s_{x,y,z}(t) = \sum_{i=0}^{M} \overrightarrow{E_i}(t)\cdot\overrightarrow{e_{x,y,z}} \tag{1.12}$$

The channel impulse response can be convoluted with a chosen signal to simulate the waveform obtained at the position P in the shielded cavity. By applying a Fourier transform on the channel impulse response, the frequency domain of the cavity can be studied.

One should note that the quantity $C = -\omega\mu\frac{dhI_0}{4\pi}$ equals 1 V in our model. As our model cannot pretend to be deterministic, obtaining absolute values is not necessary. The E-fields are expressed in V.m^{-1} but the values are arbitrary.

1.2.4 Assumptions

Using image theory to model a shielded cavity means that we omit the energy diffracted by the wall edges in the cavity. We only consider the energy reflected by the different walls. This optical approach is validated if the dimensions the cavity involved are substantially bigger than the wavelength[4]. In these conditions the geometrical laws of optics can be applied to the image currents' emissions. This model uses far-field radiation only, near-field radiations are neglected. The radiating current does not have any physical dimension, but this limitation can be easily bypassed by juxtaposing emitting currents to simulate a radiating line or an antenna in the cavity.

[4] However, results have shown a good adequacy between measurements and simulations even at low frequency.

Chapter 2

Image Creator

2.1 Usage

Image creator is a program (`ImageCreator.m`) that returns a matrix of images for a given cavity and a given position and orientation of one or more elementary currents within the rectangular cavity. The input parameters are the dimensions of the cavity, the desired time-window of the simulation, and the attributes of every emitting elementary current (position, angular orientation, amplitude, phase). This program allows to create array antennas by tweaking the positions, the phase and the amplitude of multiple elementary currents.

The matrix `POS` (for position) created by this program contains M elementary currents, each line describes a current (Tab. 2.1).

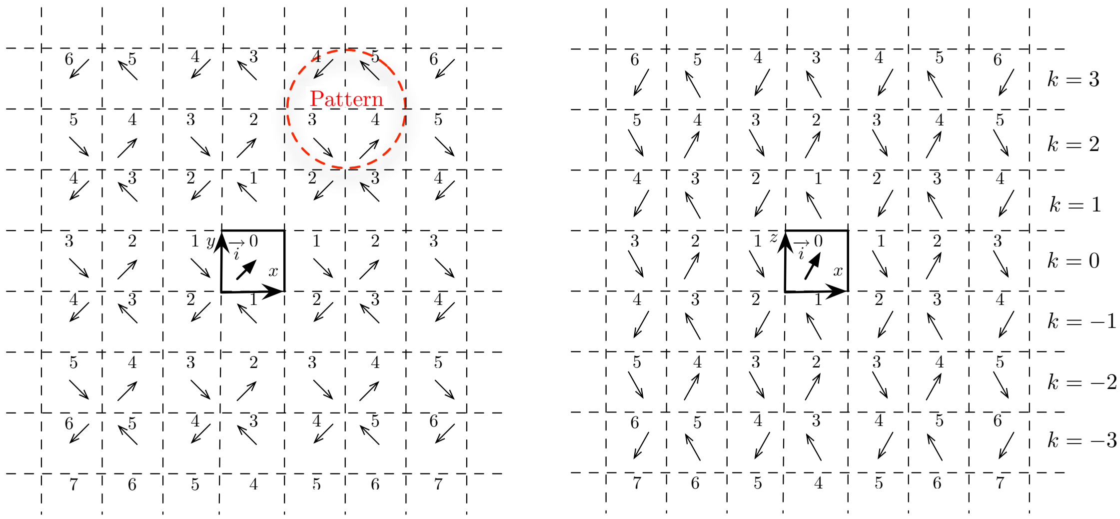
x-position	y-position	z-position	$\|i\|$	$\|j\|$	$\|k\|$	Phase	Amplitude	Tilt	Azimuth

Table 2.1: Description of a line of the matrix `POS`, corresponding to a n^{th} order current with i reflections along the axis Ox, j reflections along the axis Oy and k reflections along the axis Oz.

2.2 How to Construct Images When Time is Money

Let a rectangular cavity of length l, width p and height h. A corner of this cavity is the origin of the rectangular coordinates and the three main directions Ox, Oy and Oz are defined by edges of the cavity.

Let an elementary current be placed within this cavity at the point $A(x_0, y_0, z_0)$, its angular orientation in the cavity is defined by a tilt angle α and an azimuthal angle β as presented in Fig. 1.4. Generating the elementary image current means that we have to determine the



(a) (b)

Figure 2.1: Image cavities and image currents in an horizontal plane ($k = 0$) (a) and a vertical plane ($j = 0$) (b). The order of each cavity is indicated.

position and the angular orientation of every current created by the reflections with the walls and save the number of reflections involved for each direction. A one-by-one image creation process can take days to generate millions of sources. The first step is to identify patterns in Fig. 2.1 to speed-up the image creation process. An elementary current can be identified by the reflections involved in its creation. If we consider a n^{th} order current created by i reflections along the axis Ox, j reflections along the axis Oy, k reflections along the axis Oz, we have $n = |i| + |j| + |k|$[1].

If we examine the horizontal plane ($k = 0$) represented in Fig. 2.1(a), we can identify a pattern of four juxtaposed elementary currents (Fig. 2.1(a)). This pattern is duplicated in the whole horizontal plane to create the plane $k = 0$ of elementary currents. This process is subdivided in 4 loops, because the order of each current must be correctly incremented, we have four situations:

- $i > 0$ and $j > 0$,
- $i \leq 0$ and $j > 0$,
- $i \leq 0$ and $j \leq 0$,
- $i > 0$ and $j \leq 0$.

If we study the orientations of the elementary currents in a vertical plane (Fig. 2.1(b)) we can discern that the parity of k dictates the orientation of the current in the corresponding

[1]The numbers i, j and k can be negative if the reflections are in the decreasing direction along the respective axis.

horizontal plane. There are only two different horizontal planes. We already have the plane $k = 0$ and thus all the even horizontal planes. We can easily derive the odd planes from an even plane, the positions of the currents along the axis Ox and Oy are the same and the vertical positions of the elementary currents differ. The height of all the currents in the $k = 0$ horizontal plane is z_0 and the height of all the currents in the horizontal plane $k = 1$ is $2l - z_0$. The tilt angles are conserved but the azimuthal angles of the currents are reversed. We add π radians to the azimuthal angle of the currents of the plane $k = 0$ to obtain the azimuthal angles of the currents contained in an odd plane. These two horizontal plane configurations are then duplicated along the Oz direction, the vertical positions of the generated sources and the number of reflections along the axis Oz are adjusted accordingly.

Creating a full horizontal plane and duplicate it along the vertical direction speeds up considerably the image creation process. This process can certainly be improved, generating the first horizontal can be improved by duplicating odd and even values of lines... Every suggestion is welcomed.

Chapter 3

CIR Calculation and More

This chapter is dedicated to the program `Example.m`. It is a sample program that shows how to get a CIR (through the `CIR` function) a frequency response and an arbitrary signal response for a given cavity and a given position within the cavity. One should use this program or the all-in-one program (`CIRFFTCONV.m`) as a bases and add loops, functions...

3.1 CIR Computation

The CIR computation is the core of the program. It is assured by the function (`CIR`). The parameters are the three coordinates of the reception point. The loss coefficients (`Rx`, `Ry`, `Rz`),the position matrix (`POS`), the length of the time-window (`Lt`) become global variables. The function computes a CIR. After loading the position matrix `POS` in the memory, for every image current near enough to appear in the given time-window, the program calculate the position of the reception point in the local coordinate system attached to the image current by using a coordinates transformation matrix. The E-field is then computed in this system of coordinates through an antenna factor named `Antth`. Basically it is the radiation pattern associated to this image current, any 3D radiation pattern as a function of the local angles θ_a and ϕ_a can be used. By using the inverse coordinates transformation matrix the E-field is expressed in the usual rectangular coordinates. The contributions of all the image currents are summed and form three CIRs (`Sx`, `Sy`, `Sz`) presented in Fig. 3.1. The CIR computation is not vectorized. We are welcoming any suggestion to vectorize this part of the code. It would speed up dramatically the calculations of the model.

3.2 Frequency Response and Response to an Arbitrary Waveform

From the CIRs, we can easily derived a channel frequency response by using a fast Fourier transform. We obtain three frequency responses (`FFTx`, `FFTy`, `FFTz`) presented in Fig. 3.2.

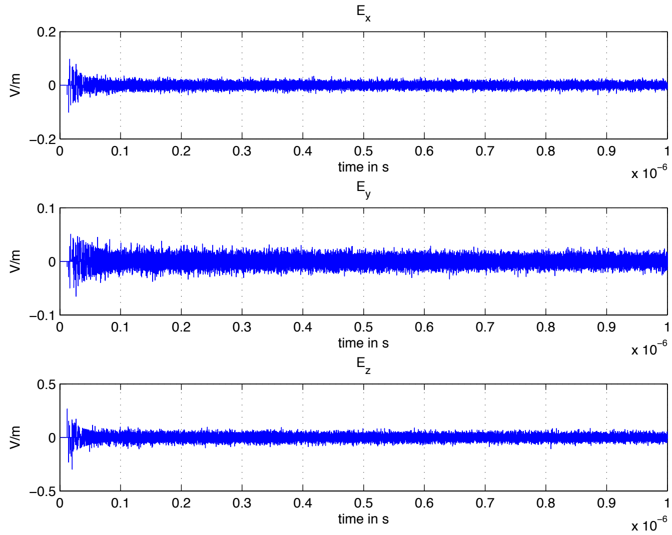


Figure 3.1: Channel impulse responses returned by the program.

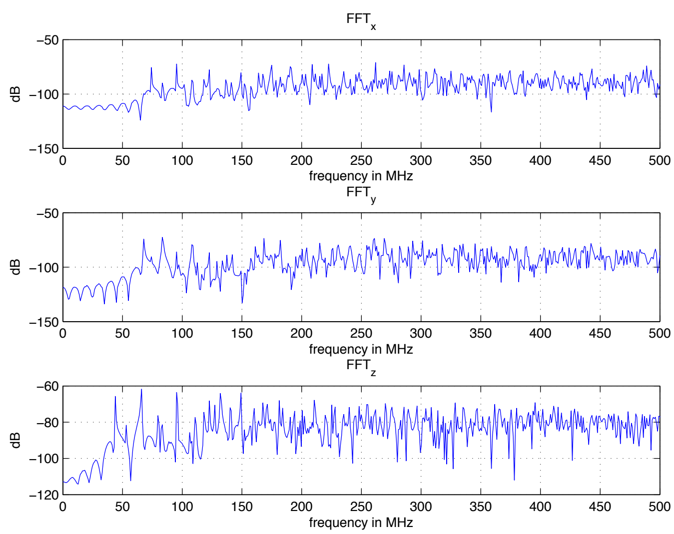


Figure 3.2: Fast Fourier transforms of the CIRs.

From the CIRs, we can derive the response of the cavity for an arbitrary signal by convoluting the CIRs with a given signal `s`. We obtain three responses (`Signalfinalx`, `Signalfinaly`, `Signalfinalz`) presented in Fig. 3.3.

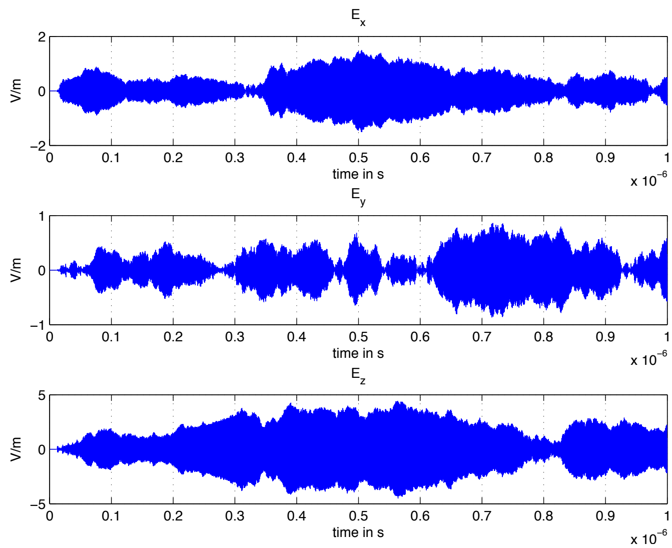


Figure 3.3: Reponses to a 300 ns pulse at 1GHz.

Chapter 4

Stirring Process

4.1 Reception Stirring

In our model, there is no mechanical stirring process. However Source stirring can be easily achieved by selecting independent positions of the receiver. The code below shows how N indepedent positions can be chosen in a cavity. We use the criterion that states that positions are independent if the distance between them is superior to $\lambda/2$ [2]. The routine below generates randomly $N = 100$ independent positions (according to the criterion... their true independency must be tested) for a frequency f_0 and a $l \times p \times h$ meters cavity:

```
N=100

lambda=c/f0;

X=[(l-lambda)*rand+lambda/2];
Y=[(p-lambda)*rand+lambda/2];
Z=[(h-lambda)*rand+lambda/2];

while (length(X)<N)

    X_1=(l-lambda)*rand+lambda/2;
    Y_1=(p-lambda)*rand+lambda/2;
    Z_1=(h-lambda)*rand+lambda/2;

    D=sqrt((X_1-X).^2+(Y_1-Y).^2+(Z_1-Z).^2);
    if min(D)>lambda/2
        X=[X;X_1];
        Y=[Y;Y_1];
        Z=[Z;Z_1];
    end
end
```

4.2 Paddle Simulation

We are currently trying to add a stirring method to our model in order to add a degree of freedom to our simulations. Our first approach would be to simulate an amount of M situations, in which the position of the emitting elementary dipole is different. By applying the superposition theorem, a stirring step could be a combination of N simulations among M. Another approach could be to change the dipole radiation pattern by a more directive pattern and to simulate a paddle with five or six of these directive sources rotating around a chosen axis.

Chapter 5

Memory Usage

If the model is relatively straightforward, the memory usage of the algorithm can be a problem. Image-currents are generated numerically and their positions and various attributes[1] are stored in the `POS` matrix. The number of sources is given by (1.2). In reality the main parameter of the simulation is not the maximum order but the duration of the simulated time-window L_T. It means that we only need the image-currents within a radius $c \cdot L_T$. This filtering can save a lot of memory if the RC is not cubic.

Tab. 5.1 sums up the memory usage for a given time window. On a 64 bits platform with 32 GB of memory, we manage to reach a time-window of 3 μs.

L_T	Number of currents M	Memory usage
500 ns	1.1×10^6	90 MB
1 μs	8.8×10^6	700 MB
3 μs	238×10^6	19 GB
10 μs	8.8×10^9	700 GB
100 μs	8.8×10^{12}	700 TB

Table 5.1: Length of the simulated time-windows and memory usage for the RC at the IETR

The sources are contained in a cylinder, their number M can be roughly obtained by calculating:

$$M \sim \frac{8\pi(cL_t)^3}{lph} \tag{5.1}$$

and the memory usage in bytes can be estimated by $M \times 80$. One should note that if $R_x = R_y = R_z$, then the fourth, fifth and sixth columns of the matrix can be replaced by one column containing the order of each source. If there is only one source, the seventh and eighth columns are not necessary. The size of the matrix `POS` can be easily reduced by 40%[2].

[1] For each current the attributes are: the rectangular coordinates, the number of reflections along each axis, tilt and azimuth angles, the amplitude and the phase of the current for array antenna.

[2] A lite version of the programs is provided in the `lite.zip` archive, the memory usage is reduced but the loss coefficients are exactly the same along each direction, and the amplitude and the phase of each element cannot be set independently

Increasing gradually the time-window in the image creator program can be helpful to determine the maximum time-window allowed by your system. Because 32 bits systems can only address 4 GB of memory, 64 bits systems are strongly recommended to run this program.

Measurements in our RC at the IETR have shown that 99% of the energy of an impulse response is contained in 20 μs when the cavity is empty. It means that the model we propose can only simulate the first moments of our empty cavity. Hopefully, we can note that 60% of the energy is contained in the first 3 μs (Fig. 5.1) when the RC is empty. Moreover this restriction is less problematic in pulse regime and/or with a loaded cavity. If the length of the pulse is smaller than the time-window, we can reasonably expect that the maximum levels would appear within the time-window. If we simulate a loaded cavity, the channel impulse response is rapidly reduced and most (if not the total amount) of the energy involved in the system would be simulated.

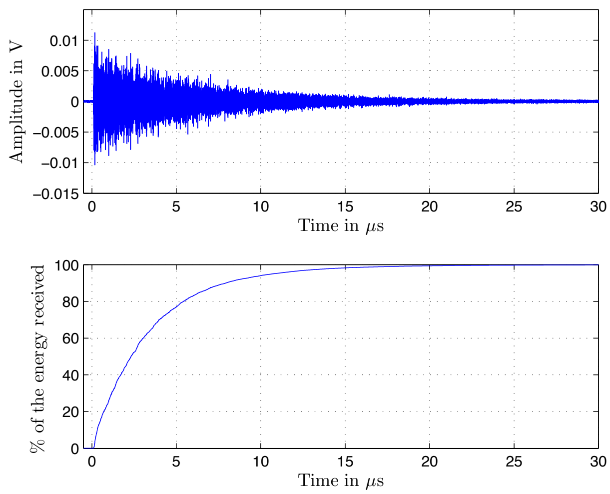


Figure 5.1: Measured channel impulse response in the empty IETR RC and Percentage of the total energy of the impulse response vs. time

Chapter 6

Loss Coefficient Estimation

The loss coefficient used in our model is empirically determined by using a measurement of the channel impulse response with a given loading. We used the following methodology. First, we measure a channel impulse response. We use an arbitrary waveform generator (Tektronix® AWG 7052) to create the shortest impulse possible (200 ps, 2.5 GHz of bandwith). The antennas used are generally a pair of wide band horn antennas or discone antennas. The signal is averaged to increase the SNR and recorded on a digital storage oscilloscope (Tektronix® TDS6124C at 40 GS/s). This signal is then transformed using Matlab®. We compute the square of the signal and we make several simulations with different values of R. In order to compare the simulations with the measurement, both the simulations and the measurement are normalized so that the cumulated power after 3 μs equals 1. These simulations are confronted with measurements and we identify graphically the correct value of R. Fig. 6.1 shows how the value of R is chosen, the idea is to find the value of R (among 501 values between 0.95 and 1) that fits the most the measurement. In this case the value $R = 0.9924$ fits the measurement made in our RC with one absorber.

Practically R values higher than 0.995 are used to simulate an empty cavity. Values under 0.995 are used to simulate a loaded cavity.

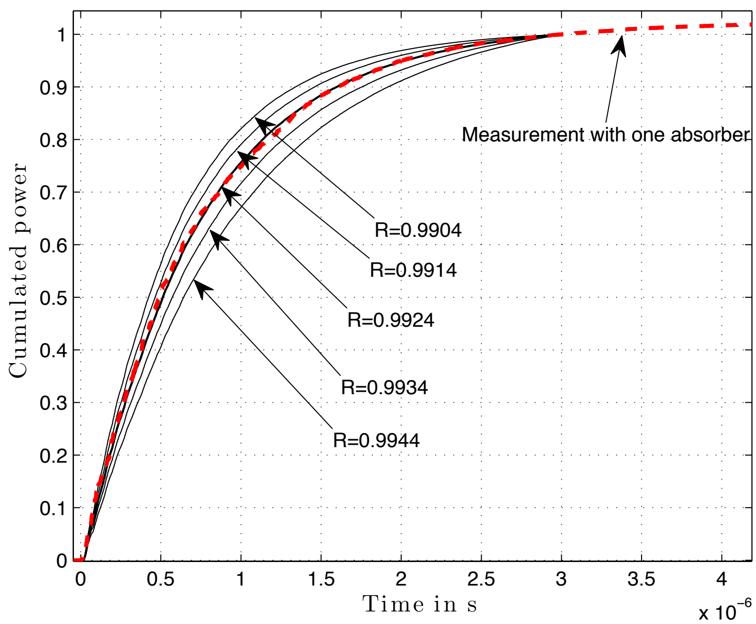


Figure 6.1: Measured cumulated power with one absorber vs. time and simulations with different values of R. In this case the value $R = 0.9924$ fits the measurement.

Chapter 7

Applications

Here is a non exhaustive list of applications of this model:

- Waveform simulation,
- Transients studies,
- Lowest usable frequency determination,
- Frequency domain behavior of a given cavity,
- Loading effect on the frequency-domain and the time-domain behavior,
- Time-reversing simulations in a RC or a canonic indoor environment,
- ...

Bibliography

[1] R. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill Book Company, 1961.

[2] D. Hill and J. Ladbury, "Spatial-correlation functions of fields and energy density in a reverberation chamber," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 44, no. 1, pp. 95–101, Feb 2002.