MAST20006 Probability for Statistics /MAST90057 Elements of Probability

Assignment 2, Semester 1 2020

Due date: 4pm, Friday April 24.

- Your assignment should show all working and reasoning. Marks will be given for method as well as for correct answers.
- Each assignment counts for 4% of your final assessment (with a total of five assignments). Tutors will not help you directly with assignment questions. However, they may give some appropriate guidance.
- This assignment must be **submitted online** on Canvas, following the guidelines for online assignment submission in the Modules Section (please read these guidelines carefully).

  In particular, we highlight that each assignment must be submitted as a **single pdf file**, with pages in the correct order (other file types cannot be uploaded). A scan of your handwritten work is acceptable. If you take pictures of/scan your solutions, please make sure all files are merged into a single pdf.
- In order to avoid any technical issue with submission, we highly recommend you to submit your assignment the day before the deadline.

  A late penalty of 4% of the total mark available will apply for every hour or part thereof up to 25 hours. Assignments cannot be submitted more than 25 hours late.

  If there are extenuating circumstances, you need to contact your Tutorial Coordinator, Dr Rob Maillardet (rjmail@unimelb.edu.au), preferably prior to the submission deadline. Medical certificates are usually required.
- Label your assignment with the following information: Your name and student number, your tutor's name, your tutorial group (day and time).
- Only **two questions** (the same 2 for all students, and decided by random selection) will be marked by your tutor. The other questions are mandatory with solutions to be given later. Two marks will be automatically deducted if you do not provide evidence on your submitted work that you have attempted all the questions. The total marks for this assignment is 20.

1. Among the 16 applicants for a job, 10 have university degrees. A sample of 3 applicants are to be randomly chosen for interviews. Let X be the number of applicants in the sample who have university degrees.

   (a) Give the name to the distribution of X if it has a name. Also specify the values of all parameters involved in this distribution.

   (b) Find the probability that exactly 1 applicant in the sample has a university degree.

   (c) Find the probability that at most 1 applicant in the sample has a university degree.

2. A bag contains 5 coins, one of which has a head on both sides while the other 4 coins are normal. A coin is chosen at random from the bag and tossed 2 times. The number of heads obtained is a random variable, say X.

   (a) What are the possible values of X? Also tabulate the pmf of X.
   (*Hint*: The coin chosen is either normal or with head on both sides. Find the conditional probability of X for each of these two situations. Then use the law of total probability and multiplication rule to find the probability of X.)

   (b) Calculate $E(X)$ and $\text{Var}(X)$.

3. A moment-generating function of X is given by $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{5t}$.

   (a) Find the pmf of X.

   (b) Find the values of μ and σ^2 for X.

   (c) Calculate $P(X \geq 2)$.

   (d) Calculate $E(2^X)$.

   (e) Find the mgf of $X^2 - 1$.

4. A moment-generating function of X is given by $M(t) = (0.25 + 0.75e^t)^4$.

   (a) Give the name of the distribution of X (if it has a name).

   (b) Find the values of μ and σ^2 for X. (*Note:* No need to derive μ and σ^2 if you know the name of the distribution.)

   (c) Calculate $P(1 \leq X \leq 2)$.

5. A moment-generating function of X is given by $M(t) = \frac{0.25e^t}{1-0.75e^t}, \quad t < -\ln(0.75)$.

   (a) Give the name of the distribution of X (if it has a name).

   (b) Find the values of μ and σ^2 for X. (*Note:* No need to derive μ and σ^2 if you know the name of the distribution.)

   (c) Calculate $P(X \geq 2)$.

   (d) Calculate $E[(1 - 0.75e^{-1})e^{-X+1} + 0.75]$.

6. An expert sharpshooter misses a target 10 percent of the time.

(a) What is the probability that she misses the target for the first time in her second shot?

(b) What is the probability that she misses the target for the first time in her xth shot?

(c) What is the probability that the first miss comes after the 3rd shot?

(d) How many shots does she expect to fire to suffer the first miss?

(e) What is the probability that she will suffer 2 misses from 10 shots?

(f) What is the probability that she will suffer at least 1 miss from the 10 shots?

(g) What is the probability that she misses the target for the second time in her 10th shot?