

Assignment 2

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1) (a) - Hypergeometric Distribution

- where $N_1 = 10, N_2 = 6, N = 16, 0 \leq x \leq N_1, 0 \leq n-x \leq N_2, n = 3$.

$$(b) P(X=1) = \frac{\binom{N_1}{x} \binom{N_2}{n-x}}{\binom{N}{n}} = \frac{\binom{10}{1} \binom{6}{3-1}}{\binom{16}{3}} = 0.2679$$

$$(c) P(X \leq 1) = P(X=0) + P(X=1) \\ = \frac{\binom{10}{0} \binom{6}{3}}{\binom{16}{3}} + 0.2679 \\ = 0.0357 + 0.2679 \\ = 0.3036$$

2) $x = \{0, 1, 2\}$

x	0	1	2
$P(X=x)$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

$P(X=0) = \frac{4}{5} \times \frac{1}{2}$
 $P(X=1) = \frac{4}{5} \times \frac{1}{5}$
 $P(X=2) = \frac{1}{5}$

$$(b) E(X) = (0 \times \frac{2}{5}) + (1 \times \frac{2}{5}) + (2 \times \frac{1}{5}) \\ = \frac{2}{5} + \frac{2}{5} \\ = \frac{4}{5}$$

$$Var(X) = E(X^2) - [E(X)]^2 \\ = (0 + \frac{2}{5} + \frac{4}{5}) - (\frac{4}{5})^2 \\ = 0.56$$

(3) (a) p.m.f of X , $M(t) = E(e^{tx})$

$$= \sum_{x=0}^{\infty} e^{tx} f(x) \\ = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{5t}$$

(a)

x	1	2	3	5
p.m.f $f(x)$	0.3	0.4	0.2	0.1

$$\begin{aligned} (b) \mu &= m'(0) \\ &= E(x) \\ &= 1(0.3) + 2(0.4) + 3(0.2) + 5(0.1) \\ &= 2.2 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= E(x^2) - (E(x))^2 \\ &= (0.3(1)^2 + 0.4(2)^2 + 0.2(3)^2 + 0.1(5)^2) - (2.2)^2 \\ &= 1.36 \end{aligned}$$

$$\begin{aligned} (c) P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - P(X=1) \\ &= 1 - 0.3 \\ &= 0.7 \end{aligned} \quad \text{Great}$$

$$\begin{aligned} (d) E(x^2) &= 0.3(2^1) + 0.4(2^2) + 0.2(2^3) + 0.1(2^5) \\ &= 7 \end{aligned}$$

$$\begin{aligned} (e) E(e^{t(x^2-1)}) &= 0.3e^{t(2^1-1)} + 0.4e^{t(2^2-1)} + 0.2e^{t(3^2-1)} + 0.1e^{t(5^2-1)} \\ &= 0.3 + 0.4e^{3t} + 0.2e^{8t} + 0.1e^{24t} \end{aligned}$$

4(a) - Binomial Distribution

$$\begin{aligned} (b) \mu &= E(x) \\ &= np \\ &= 4(0.75) \\ &= 3 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= E(x^2) - (E(x))^2 \\ &= np(1-p) \\ &= 4(0.75)(1-0.75) \\ &= 0.75 \end{aligned}$$

$$\begin{aligned} (c) P(1 \leq x \leq 2) &= P(X=1) + P(X=2) \\ &= \binom{4}{1}(0.75)^1(0.25)^3 + \binom{4}{2}(0.75)^2(0.25)^2 \\ &= 0.046875 + 0.2109 \\ &= 0.258 \end{aligned}$$

5(a) - Geometric Distribution

$$\begin{aligned} (b) \mu &= \frac{1}{p} \\ &= \frac{1}{0.25} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \frac{q}{p^2} \\ &= \frac{0.75}{(0.25)^2} \\ &= 12 \end{aligned}$$

$$\begin{aligned} (c) P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - P(X=1) \\ &= 1 - 0.25(0.75)^{1-1} \\ &= 0.75 \end{aligned}$$

$$\begin{aligned}
 (d) \quad E[(1-0.75)e^{-\frac{1}{10}}e^{-x+1} + 0.75] \\
 = E[(0.25e^{-\frac{1}{10}}e^{-x+1} + 0.75] \\
 = (0.25e^{-\frac{1}{10}}e^{-1+1} + 0.75) + (0.25e^{-\frac{1}{10}}e^{-2+1} + 0.75) + (0.25e^{-\frac{1}{10}}e^{-3+1} + 0.75) \\
 + (0.25e^{-\frac{1}{10}}e^{-4+1} + 0.75) \\
 = 3.143
 \end{aligned}$$

$$\begin{aligned}
 6) (a) P(X=2) &= q \cdot p \\
 &= 0.9(0.1) \\
 &= 0.09
 \end{aligned}$$

$$(b) P(X=x) = 0.9^{x-1} (0.1)$$

$$\begin{aligned}
 (c) P(X > 3) &= q^k \\
 &= 0.9^3 \\
 &= 0.729
 \end{aligned}$$

$$(d) P(X=x) = 1 \quad -0.5$$

$$0.9^{x-1} (0.1) = 1$$

$$0.9^{x-1} = 10$$

$$(x-1) \log 0.9 = \log 10$$

$$x-1 = \frac{\log 10}{\log 0.9}$$

$$x-1 = \log\left(\frac{10}{0.9}\right)$$

$$x-1 = 1.045$$

$$x = 2.045$$

$$x = 2$$

$$E(X) = 1/p = 10.$$

∴ 2 shots to suffer the first miss

-0.5

$$\begin{aligned}
 (e) \text{ Probability 2 misses in 10 shots} &= {}^9C_2 (0.1)^2 (0.9)^8 \\
 &= 0.155 \quad 0.1937
 \end{aligned}$$

$$\begin{aligned}
 (f) \quad n=10, p=0.1, q=0.9 \\
 P(X \geq 1) &= 1 - P(X=0) \\
 &= 1 - {}^{10}C_0 (0.1)^0 (0.9)^{10} \\
 &= 0.651
 \end{aligned}$$

$$\begin{aligned}
 (g) \text{ Probability misses 2nd time in 10 shot} &= (0.9^8 \times 0.1) {}^9C_1 \times 0.1 \\
 &= 0.03874
 \end{aligned}$$

