MAST20006 Probability for Statistics /MAST90057 Elements of Probability

Assignment 1, Semester 1 2020

Due date: 4pm, Friday March 20.

- Your assignment should show all working and reasoning. Marks will be given for method as well as for correct answers.
- Each assignment counts for 4% of your final assessment (with a total of five assignments). Tutors will not help you directly with assignment questions. However, they may give some appropriate guidance.
- This assignment must be **submitted online** on Canvas, following the guidelines for online assignment submission in the Modules Section (please read these guidelines carefully).

  In particular, we highlight that each assignment must be submitted as a **single pdf file**, with pages in the correct order (other file types cannot be uploaded). A scan of your handwritten work is acceptable. If you take pictures of/scan your solutions, please make sure all files are merged into a single pdf.
- In order to avoid any technical issue with submission, we highly recommend you to submit your assignment the day before the deadline.

  A late penalty of 4% of the total mark available will apply for every hour or part thereof up to 25 hours. Assignments cannot be submitted more than 25 hours late.

  If there are extenuating circumstances, you need to contact your Tutorial Coordinator, Dr Rob Maillardet (rjmail@unimelb.edu.au), preferably prior to the submission deadline. Medical certificates are usually required.
- Label your assignment with the following information: Your name and student number, your tutor's name, your tutorial group (day and time).

Only ***two questions*** *(the same 2 for all students, and decided by random selection) will be marked by your tutor. The other questions are mandatory with solutions to be given later. Two marks will be automatically deducted if you do not provide evidence on your submitted work that you have attempted all the questions. The total marks for this assignment is 20.*

1. Let $P(A) = P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{2}$. Find the following:
   (a) $P(B')$.
   (b) $P(A' \cap B')$.
   (c) $P(B \cap A')$.
   (d) $P(A \cup B')$.

2. A hockey team plays two games on a certain day. The probability that it wins the first game is 0.7; the probability that it wins the second game is 0.6. Also whether or not the team wins one game is **independent** of whether or not it wins the other game. Find the probability that
   (a) the team wins at least one game.
   (b) the team wins exactly one game.
   (c) the team wins neither games.

3. *Birthday paradox.* Consider a group of 3 students. Each student has a birthday that can be any one of the days numbered 1, 2, 3, ..., 365.
   (a) What is the probability that **none of them** have the same birthday with each other?
   (b) What is the probability that **some of them** have the same birthday with each other?
   (c) What is the probability that **all of them** have the same birthday with each other?

4. Binary digits 0 and 1 are transmitted over a communication channel. If a 1 is sent, it will be received as a 1 with probability 0.9 and as a 0 with probability 0.1; if a 0 is sent, it will be received as a 0 with probability 0.7 and as a 1 with probability 0.3. If the probability that a 0 is sent is the same as the probability that a 1 is sent, what is
   (a) the conditional probability that a 1 was sent given that a 1 was received?
   (b) the conditional probability that a 0 was sent given that a 0 was received?

5. It is known from experience that in a certain industry 60 percent of all labor-management disputes are over wages, 15 percent are over working conditions, and 25 percent are over fringe issues. Also, 45 percent of the disputes over wages are resolved without strikes, 70 percent of the disputes over working conditions are resolved without strikes, and 40 percent of the disputes over fringe issues are resolved without strikes. What is the probability that a labor-management dispute in this industry will be resolved without a strike? (*Hint:* Apply the law of total probability.)

6. In a certain community, 8 percent of all adults over 50 years old have diabetes. If a health service in this community correctly diagnoses 95 percent of all persons with diabetes as having the disease and incorrectly diagnoses 2 percent of all persons without diabetes as having the disease, find the probabilities that

   (a) the community health service will diagnose an adult over 50 as having diabetes;

   (b) a person over 50 diagnosed by the health service as having diabetes actually has the disease.