Lab 6 Solutions

1.
```
>  with(Statistics):
>  X := RandomVariable(Uniform(0, 8));
>  Y: = min(X,8-X);
>  CDF(Y,y);
```
Y has a Uniform(0,4) distribution.

2. (a)
```
> with(Statistics):
> f:=exp(-x)/(1+exp(-x))^2;
> f:=unapply(f,x);
> X:=RandomVariable(Distribution(PDF=f));
> CDF(X,x);
or
> int(PDF(X,x),x);
or
> int(f(x),x);
```

(b)
```
> Mean(X);
> Variance(X);
> Probability({X>3, X<5});
Or
> int(PDF(X,x),x=3..5);
or
> int(f(x),x=3..5);
```

(c)
```
> Percentile(X,85);
Or
> solve(CDF(X,x)=0.85,x);
```

(d)
```
> Y:=1/(1+exp(-X));
> CDF(Y,y);
```
Y has a Uniform(0,1) distribution.

3.
```
> with(Statistics):
> X:=RandomVariable(Exponential(theta));
> assume(theta>0, u>0, v>0);
> Probability(X>u);
> Probability(X>u+v);
> Probability(X>u+v)/Probability(X>v);
> simplify(%);
```
One can conclude that $P(X > u + v|X > v) = P(X > u)$. This is true because of the 'no-memory' property of the exponential distribution.

4. (a)
```
> with(Statistics):
> X:=RandomVariable(Gamma(2,8));
> assume(t<1/2);
```

```
> MGF(X,t); or  > int(exp(t*x)*PDF(X,x),x=0..infinity);
> Probability(X>3); or > int(PDF(X,x),x=3..infinity);
```

(b)
```
> Y:=RandomVariable(Poisson(3/2));
> Probability(Y<=7); or > sum((3/2)^k*exp(-3/2)/k!,k=0..7);
```

Same as $P(X > 3)$. The event that 'waiting time' for the 8th change is greater than 3 time units is the same as the event that there are at most 7 changes during a period of 3 units of time.

(c)
```
> Z:=RandomVariable(ChiSquare(16));
> MGF(Z,t);
> Probability(Z>3); or > int(PDF(Z,z),z=3..infinity);
```

Z and X have the same mgf and distribution and probability. ChiSquare distribution with df=r is also a Gamma distribution with $\theta = 2$ and $\alpha = r/2$.

5.
```
> 1-pgamma(3,shape=8,rate=1/2)
[1] 0.9998304
> ppois(7,lambda=3/2)
[1] 0.9998304
> 1-pchisq(3,df=16)
[1] 0.9998304
```