Fastest-Path Algorithm for Electrical Vehicles

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ABSTRACT

Keywords

Shortest-path algorithm, Electrical Vehicles, Road-network

1. INTRODUCTION

As the adoption rate of electric vehicles increases, the need for a better and more complex route planing solution appears. Electrical vehicles generally have a shorter range as well as a significantly slower recharge time, compared to traditional gasoline cars. As the recharge time have become a significant factor in the total travel time, the time spend recharging should be taken into account by a route planning system, to accommodate for an otherwise large margin of error. Time spend recharging batteries is inherently variable, as the charge rate is affected by both the charge station's charge rate as well as the battery's current charge. When planing a fastest-path route for an electric vehicle, it is therefore paramount to be able to incorporate charge station's charge rates and electric vehicle's battery.

Another important aspect is the electric vehicle's energy consumption. With the now significant time spend recharging, a high energy consumption rate while driving, can in turn result in a slower route. The energy consumption of electrical vehicles generally increases polynomially with speed so this should also be taken into account by the route planning system.

It should be clear that no traditional shortest path algorithm is able to accommodate for the variables and relationships between the travel speed, the charge rate and the current charge of the electrical vehicle. In this article we present two different approaches to solving the problem of optimising a route plan for electrical vehicles. One is a linear program..... The other is naive solution which uses heuristics..... Both approaches are analysed on their running time and reliability, based on a real world road network as input.

2. NOTATION

In this section we will briefly explain the mathematical notation and model used in the paper. The graph representation consists of edges which represents roads and vertices which represents either charge stations or road intersections. We define a **road network** as an ordered pair $G = (V, E)$ comprising of a set V of vertices together with a set E of edges. Where V is a finite set and E is a binary relation on V. We further define the following functions:

$$E_d(u, v) \to d$$

$$E_v(u, v) \to v$$

These are partial functions defined by the edge (u, v) and respectively returns a distance or a speed limit.

Each vertex is then defined as a *charge station*, described by the following partial function:

$$V_c(v) \to c$$

A road intersection is simply a vertex with charge rate $c = 0$, while a charge station is a vertex where $c > 0$. An electrical vehicle is specified by two parameters: It's battery capacity (kWh), consumption rate (kWh/km). The consumption rate is given by the following function:

$$C_R(v) = av^2 + bv + c$$

where v is the speed of the vehicle.

2.1 Shorter Not Equal Faster

A shorter path does not necessarily mean a faster path for electrical vehicles. This is partly due to the fact that an electrical vehicles uses exponentially more energy as it's speed increases but also due to the fact that charge times on charge stations varies a lot. Driving a longer path with a fast charge station on can therefore turn out to be a faster choice than driving a shorter path with a slow charge station on. This illustrated in **??**. In the example we assume our car has a battery capacity of 100 kWh and a energy-usages of 0,4 kWh/km. Path 1 consist of two edges with distance = 250 km and speed limit = 100 km/t and a charge station with

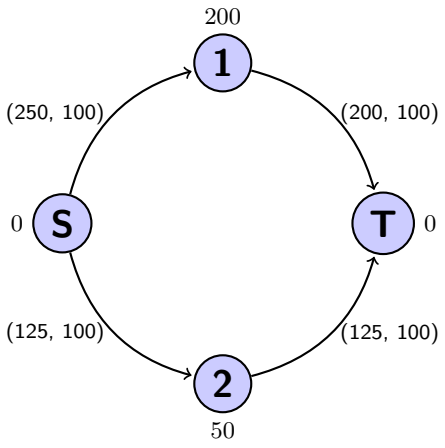

Figure 1: A simple road-network with starting point s and end point t

a charge speed of 200 kWh/t. Path 2 consist of two edges with distance 200 km and speed limit 100 km/t and a charge station with a charge speed of 30 kWh/t. The total time of each path:

Path 1: route time = ((250km + 250km) / 100km/h) + (100kWh / 200kWh/h) = 5,5h

Path 2: route time = ((200 km + 200 km) / 100 km/h) + (60 kWh / 30 kWh/h) = 6 h

2.2 Optimizing with a linear program

To solve the route planning problem with a linear program solver, we are going to get approximate solutions. Linear program solvers work on linear equations, and we ultimately want to use non-linear expressions for the energy consumption as a function of speed, as well as charge rate as a function of battery charge. These functions will each be represented by by a set of linear equations. The precision of these peicewise linearized expressions of course depend on the amount of linear expressions used. In section (experiment), you can see the effect of this varying degree of approximation on a route plan.

minimizing the sum of time spend driving and the sum of time spend charging.

$$\text{minimize} \quad \sum_{i=1}^{n} \frac{Charge\ rate[i]}{Velocity[i]} + \sum_{i=1}^{n} Charge\ station[i] \tag{1}$$

subject to:

$$Battery\ capacity \tag{2}$$

$$Velocity[1 \dots n, 1 \dots m] \tag{3}$$

$$Charge\ time[1 \dots n] \tag{4}$$

$$Charge\ rate[1 \dots n] \tag{5}$$

$$Edge\ distance[1 \dots n] \tag{6}$$

$$Points1[1 \dots n, 1 \dots m] \tag{7}$$

$$Points2[1 \dots n, 1 \dots m] \tag{8}$$

$$LinesA[1 \dots n, 1 \dots m] \tag{9}$$

$$LinesB[1 \dots n, 1 \dots m] \tag{10}$$

$$Selected\ lines[1 \dots n, 1 \dots m] \tag{11}$$

$$\forall_{i \in 1 \dots n} \ : \ 0 \leq \sum_{j=1}^{m} Selected\ lines[i,j] \leq 1 \tag{12}$$

$$\forall_{i \in 1 \dots n, j \in 1 \dots m} \ : \ Selected\ lines[i,j] = 1 \tag{13}$$

$$\forall_{i \in 1 \dots n, j \in 1 \dots m} \ : \ Velocity[i,j] \leq Selected\ points[i,j] * Points1[i,j] \tag{14}$$

$$\forall_{i \in 1 \dots n, j \in 1 \dots m} \ : \ Velocity[i,j] \geq Selected\ points[i,j] * Points2[i,j] \tag{15}$$

$$\forall_{k \in 1 \dots n} \ : \ 0 \leq \sum_{i=1}^{k} Charge\ rate[i] * Charge\ time - \sum_{i=1}^{k} Edge\ distance[i] (\sum_{j=1}^{m} LinesA[i,j] * Velocity[i,j] + \sum_{j=1}^{m} Selected\ edges[i,j] * LinesB[i,j]) \leq Battery\ capacity \tag{16}$$

3. EXPERIMENTS

In this section we will describe the experiments being conducted.

3.1 Setup

To facilitate the experiments, we've had to find a real world road network graph dataset which has road distances, speed limits and charging stations of varying power output. Such dataset does not openly exist to our knowledge, instead we have used OpenStreetMaps, henceforth *OSM*. One can read more about it at `http://www.openstreetmap.org/about`. OSM has a concept of *Ways* and *Nodes*. Ways represent geographical planar objects, e.g. roads. A node is a geographical point (latitude,longitude). A way is then constituted of several nodes, and some tags which describe meta-information such as the name of the way, and what type of construct it is. As such, it follows that if way A and B intersect in node x, they will share the node x, known as an intersection in the road network.

These objects can easily be converted into a road network structure, by filtering ways of type *road* and use these ways as edges in the road network. To get speed limits we can derive general speed limits from the type of the roads, OSM carries such information as motorway,residential,tertiary for roads. Since we are not interested in the way a road curves, we ignore intermediate points on a way, which do not correspond to an intersection.

3.2 Citations

4. CONCLUSIONS

4.1 References