About the QED of particles on Monopole fields and Gauge Symmetry

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How should Maxwell theory be modified when monopole exists in nature? Part of the answer has been provided by Dirac, where he gave the description of a vector potential with magnetic flux. However, there are unsatisfactory points in Dirac's description: 1. The motion of monopole cannot be described as a solution of the equation of motion. 2. One cannot describe how the monopole moves under the Maxwell fields. 3. It cannot explain the action and reaction between the field and the monopole. This situation is in sharp contrast to the electron, where its motion can be described within the Maxwell theory. In this study, we look for a new description of Electro- magnetic theory with particles having electric and/or magnetic charges. We propose a theory U(1) gauge group for identical particle system, SU(2) gauge groups for Dyon-electron interacting mode with two vector fields A_μ and B_μ. In this model, it can explain catchball-reaction breaking on B_μ fields.

I. INTRODUCTION

The former Maxwell theory of monopole fields can be explained by Mathematical expansion of Maxwell field from 4-electric currents J_e^μ:

$$\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r},t)=\varrho_e\ ,\qquad \boldsymbol{\nabla}\cdot\mathbf{B}(\mathbf{r},t)=\varrho_g\ , \tag{1}$$

$$\boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r},t)=-\left(\mathbf{J}_g(\mathbf{r},t)+\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t)\right)\ , \tag{2}$$

$$\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r},t)=\mathbf{J}_e(\mathbf{r},t)+\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t). \tag{3}$$

But here's some unsatisfactory points:

- The electrons, magnetic monopoles, Dyons are not be separated.
- It can not explain the catchball-reaction breaking on B_μ fields; Electrons only possible for responding to field B_μ. It can not release the field B_μ.
- No donsidering for electric charge-monopole charge interaction.

So that here we required the new gauge model which satisfies this demands. To construct new Maxwell theory on monopole fields, here we propose each particle, electron, magnetic monopole, dyon are fermion with spin $s = 1/2$.

I.1. Notations and Convention

In this paper, we use Natural Units:

$$c=\mu_0=\epsilon_0=\hbar=k_B=1.$$

Also, the sign convention of Mincowski metric tensor as

$$\eta_{\mu\nu}:=\mathrm{diag}(+,-,-,\cdots,-). \tag{4}$$

Else, other notations defined by:

Symbol	Physical quantity
$\Lambda^\mu{}_\nu$	Lorentz transformation
$g=\det g_{\mu\nu}$	determinant of metric tensor $g_{\mu\nu}$
$\hat{\mathcal{F}}_{\rho\sigma}$	generator of Lorentz group
σ^i $(i=1,2,3)$	Pauli matrices
γ^μ	Dirac matrices
ψ_e	Spinor of electron
$\psi_{\nu_{eg}}$	Spinor of dyon
A_μ/B_μ	4-vectors from electron/monopole

TABLE I: Nations and Convention

II. THE IDENTICAL PARTICLES

In this section, we construct the correction model of Maxwell theory on identical particle[1] system. The interaction of identical particles has U(1) gauge groups:

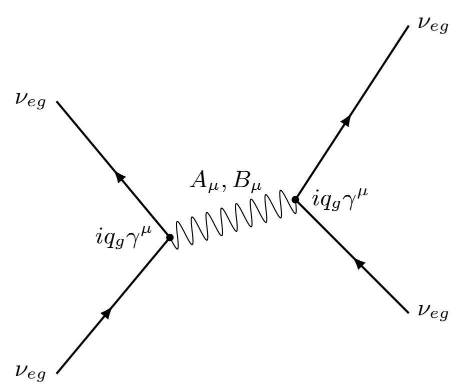

FIG. 1: The Dyon-Dyon U(1) interaction

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II.1. The Gauge transformation

Hereby we construct the gauge transformation of U(1) gauge field for identical particle system, which satisfies Feynman diagramme FIG. 1:

$$A_\mu \;\mapsto\; A'_\mu = A_\mu + \partial_\mu \Lambda, \tag{5}$$

$$B_\mu \;\mapsto\; B'_\mu = B_\mu + \partial_\mu \Gamma, \tag{6}$$

$$\psi(x) \;\mapsto\; \psi'(x) = e^{-iq_e\Lambda - iq_g\Gamma}\psi(x). \tag{7}$$

We denote ψ as spinor of dyon with electric charge q_e and monopole charge q_g, Λ and Γ as scalar quantity[2]. Transfromation of spinor (7) is suggested by superposition principle of dyons shown on Feynman diagramme.

II.2. The Dirac Field

From the condition of gauge invariance of Dirac field, here we define the covariant derivative D_μ by:

$$\mathcal{L}_{\text{Dirac}}: \quad \bar{\psi}(i\not{D} - m)\psi \quad \leftrightarrow \quad \bar{\psi}'(i\not{D} - m)\psi', \tag{8}$$

$$D_\mu := \partial_\mu + iq_e A_\mu + iq_g B_\mu. \tag{9}$$

Lagrangian density of Dirac field of Feynman diagramme FIG. 1 is represented by

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\not{\partial} - q_e\not{A} - q_g\not{B} - m)\psi. \tag{10}$$

Hence, we can confirm that Dirac field (10) goes back to monopoleless U(1) fields on limit $q_g, B_\mu \to 0$.

II.3. Curvature tensors of Gauge Field

The electromagnetic tensor $F_{\mu\nu}$ of U(1) gauge fields without monopoles is also derived by considering the curvature tensor of gauge fields:

$$F_{\mu\nu} = \frac{1}{ie}[D_\mu, D_\nu]_{q_g=0} = \partial_{[\mu}A_{\nu]}. \tag{11}$$

Here we put $D_\mu|_{q_g=0} = \partial_\mu + ieA_\mu$. So that we have new electromagnetic tensor $G_{\mu\nu}$ on monopole-interacting identical particles mode as simplic expansion of curvature tensor by

$$G_{\mu\nu} := -\frac{i}{\hat{q}}[D_\mu, D_\nu] = \frac{1}{\hat{q}}(q_e F_{\mu\nu} + q_g E_{\mu\nu}), \tag{12}$$

$$\hat{q} := \sqrt{q_e^2 + q_g^2}. \tag{13}$$

where we denote $E_{\mu\nu} = \partial_{[\mu}B_{\nu]}$.

II.4. Maxwell equation of Identical particles

Hence, we have following action integral from each calculation of $G_{\mu\nu}$ and Dirac field (10):

$$S_{\mathrm{U}(1)} = \int \left(-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi \right) \sqrt{-g}d^4x \tag{14}$$

hereby we recall Hamilton's principles to derive new Maxwell equations of identical particles. From the equation of variation $\delta_{A_\mu}S_{\mathrm{U}(1)} = \delta_{B_\mu}S_{\mathrm{U}(1)} = 0$, we have

$$q_e\partial_\mu(\sqrt{-g}G^{\mu\nu}) = \hat{q}^2\sqrt{-g}J^\nu, \tag{15}$$

$$q_g\partial_\mu(\sqrt{-g}G^{\mu\nu}) = \hat{q}^2\sqrt{-g}K^\nu. \tag{16}$$

Equations (15) and (16) shows classical electrodynamics of identical particles on curved spacetime. Here we denote each 4-currents from electron/monopole J^ν, K^ν as:

$$J^\nu := q_e\bar{\psi}\gamma^\nu\psi, \quad K^\nu := q_g\bar{\psi}\gamma^\nu\psi. \tag{17}$$

Now we can confirm that quantity (17) has 4-vector Lorentz transformation.

$$\bar{\psi}'\gamma^\nu\psi' = \bar{\psi}S^{-1}(\Lambda)\gamma^\nu S(\Lambda)\psi = \Lambda^\nu{}_\sigma\bar{\psi}\gamma^\sigma\psi. \tag{18}$$

Each quantity q_e, q_g shows elementary charges of each particles. It is determined by the nature of particles:

Elementary charges	Particles	spin
$q_e = e, q_g = 0$	electron	Dirac particles
$q_e = 0, q_g = g$	monopole	Dirac particles
$q_e = e, q_g = g$	dyon	Dirac particles

TABLE II: Nature of charges q_e, q_g

Else, here we determine each electric field $\mathbf{E}(\mathbf{r}, t)$ and magnetic field $\mathbf{B}(\mathbf{r}, t)$ from Gauss' law. Covariant index $\nu = 0$ takes Gauss' law:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q_e^2}{\hat{q}^2}\left(-\boldsymbol{\nabla}\varphi_e - \dot{\mathbf{A}}_e\right) + \frac{q_e q_g}{\hat{q}^2}\left(-\boldsymbol{\nabla}\varphi_g - \dot{\mathbf{A}}_g\right) + \alpha_e(\boldsymbol{\nabla} \times \mathbf{V}_e) \tag{19}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q_g^2}{\hat{q}^2}\left(-\boldsymbol{\nabla}\varphi_g - \dot{\mathbf{A}}_g\right) + \frac{q_e q_g}{\hat{q}^2}\left(-\boldsymbol{\nabla}\varphi_e - \dot{\mathbf{A}}_e\right) + \alpha_g(\boldsymbol{\nabla} \times \mathbf{V}_g) \tag{20}$$

Each third term on (19) and (20) can be supported by identity $\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\mathbf{V}) = 0$. From the symmerty of Feynman diagramme FIG. 1, we have

$$\alpha_e(\boldsymbol{\nabla} \times \mathbf{V}_e) = -\frac{q_g^2}{\hat{q}^2}(\boldsymbol{\nabla} \times \mathbf{A}_g) - \frac{q_e q_g}{\hat{q}^2}(\boldsymbol{\nabla} \times \mathbf{A}_e), \tag{21}$$

$$\alpha_g(\nabla \times \mathbf{V}_g) = \frac{q_e^2}{\hat{q}^2}(\nabla \times \mathbf{A}_e) + \frac{q_e q_g}{\hat{q}^2}(\nabla \times \mathbf{A}_g). \quad (22)$$

The new Maxwell equations of identical particles on (15) and (16) gives

$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \varrho_e, \qquad \nabla \cdot \mathbf{B}(\mathbf{r}, t) = \varrho_g, \quad (23)$$

$$\nabla \times \mathbf{B}(\mathbf{r}, t) = \mathbf{J}_e(\mathbf{r}, t) + \frac{\partial}{\partial t}\mathbf{E}(\mathbf{r}, t), \quad (24)$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\left(\mathbf{J}_g(\mathbf{r}, t) + \frac{\partial}{\partial t}\mathbf{B}(\mathbf{r}, t)\right). \quad (25)$$

Else, notation (19) and (20) also takes

$$G_{\mu\nu} = \begin{pmatrix} 0 & -\varepsilon_x & -\varepsilon_y & -\varepsilon_z \\ \varepsilon_x & 0 & \pi_z & -\pi_y \\ \varepsilon_y & -\pi_z & 0 & \pi_x \\ \varepsilon_z & \pi_y & -\pi_x & 0 \end{pmatrix} \quad (26)$$

where we denote each ε_i, π_i by

$$\varepsilon_i = \frac{1}{\hat{q}}(q_e E_i + q_g B_i), \quad \pi_i = \frac{1}{\hat{q}}(q_e B_i - q_g E_i). \quad (27)$$

Hereby we confirm that the former Maxwell equation on (1) to (3) gives only explanation of motion of identical particles on monopole fields.

II.5. Energy-momentum tensor

Noether's theorem gives the energy-momentum tensor of Maxwell fields (14) by

$$\mathcal{L}_f = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu}, \quad (28)$$

$$T^\mu_\nu = \frac{\partial \mathcal{L}_f}{\partial(\partial_\mu A_\sigma)}\partial_\nu A_\sigma + \frac{\partial \mathcal{L}_f}{\partial(\partial_\mu B_\sigma)}\partial_\nu B_\sigma - \delta^\mu_\nu \mathcal{L}_f, \quad (29)$$

which takes $\partial_\mu T^\mu_\nu = 0$. The Noether currents T^μ_ν is supported on variation with parametre λ:

$$\begin{aligned} \delta A_\mu(\lambda) &= \frac{\partial A_\mu(\lambda)}{\partial \lambda}\delta\lambda, \\ \delta B_\mu(\lambda) &= \frac{\partial B_\mu(\lambda)}{\partial \lambda}\delta\lambda. \end{aligned} \quad (30)$$

Now that we have separated-two energy momentum tensors. Hereby we have energy-momentum tensor[3] in vaccum

$$T^{\lambda\mu} = -\frac{1}{\hat{q}}G^{\lambda\sigma}G^\mu{}_\sigma + \frac{1}{4}g^{\lambda\mu}G_{\rho\sigma}G^{\rho\sigma} \quad (31)$$

for the contravariant component. The representation on (19) and (20) gives each energy and Poynting vector in vaccum by conservation law:

$$\mathcal{E} = \frac{1}{2}(\boldsymbol{\varepsilon}^2 + \boldsymbol{\pi}^2) = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \quad (32)$$

$$\mathbf{S} = \boldsymbol{\varepsilon} \times \boldsymbol{\pi} = \mathbf{E} \times \mathbf{B}. \quad (33)$$

which satisfies conservation law in vaccum:

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \mathbf{S} = 0. \quad (34)$$

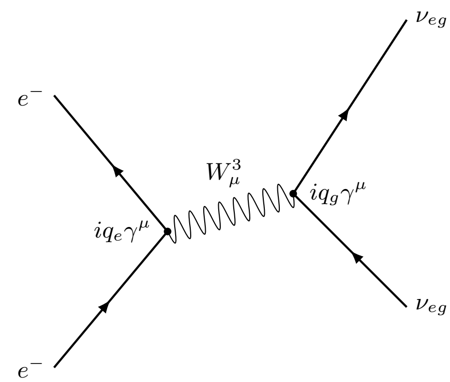
Else, on the fields with $q_e, q_g \neq 0$, the conservation law (34) goes to

$$\frac{\partial \mathcal{E}}{\partial t} + \mathbf{J}_e \cdot \mathbf{E} + \mathbf{J}_g \cdot \mathbf{B} + \nabla \cdot \mathbf{S} = 0 \quad (35)$$

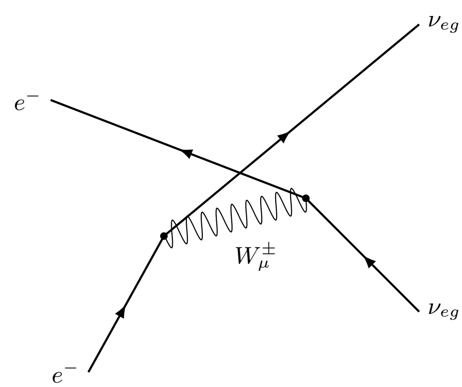
what shows expansion of Poynting theorem on monopole fields. Hereby we confirm that Poynting theorem (35) goes back to monopoleless U(1) field on limit $q_g \to 0$.

III. ELECTRON-DYON INTERACTIONS

The U(1) gauge group model on section II can not gives satisfactory explanation of symmetry breaking eletron-dyon interaction. Hereby we propose Yang-Mills SU(2) gauge theory:



(a)



(b)

FIG. 2: The Yang-Mills interaction

Now we propose probability of double decay of electron and dyon shown on Feynman diagramme FIG. 2. This double decay gives symmetry to system even the symmetry breaking occurred on electron by field B_μ.

III.1. The Gauge transformation

Hereby we construct the gauge transformation of SU(2) gauge field for identical particle system, which satisfies Feynman diagramme FIG. 2:

$$\mathbf{W}_\mu \cdot \boldsymbol{\sigma} \quad \mapsto \quad \mathbf{W}_\mu \cdot \boldsymbol{\sigma} + \epsilon_{ijk}\alpha^i W^j_\mu \sigma^k + \frac{1}{g}\partial_\mu \boldsymbol{\alpha}\cdot\boldsymbol{\sigma}, \quad (36)$$

$$\Psi \quad \mapsto \quad \exp\left(-\frac{i}{2}\boldsymbol{\alpha}(x^\mu)\cdot\boldsymbol{\sigma}\right)\Psi \quad (37)$$

where we denote spinor Ψ with doublet term of $\psi_e, \psi_{\nu_{eg}}$ from the probability of double decay, and the non abelian gauge field as

$$\Psi := \begin{pmatrix}\psi_e \\ \psi_{\nu_{eg}}\end{pmatrix}, \quad \mathbf{W}_\mu = (W^1_\mu, W^2_\mu, W^3_\mu). \quad (38)$$

Also, we notate each parametre $\boldsymbol{\alpha} = (\alpha^1, \alpha^2, \alpha^3)$ and Pauli matrices $\boldsymbol{\sigma} = (\sigma^i, \sigma^2, \sigma^3)$.

III.2. Determination of Gauge Fields

Here we remind that A_μ is possible for acting on both electron and dyon, but B_μ is only possible for generating from dyon:

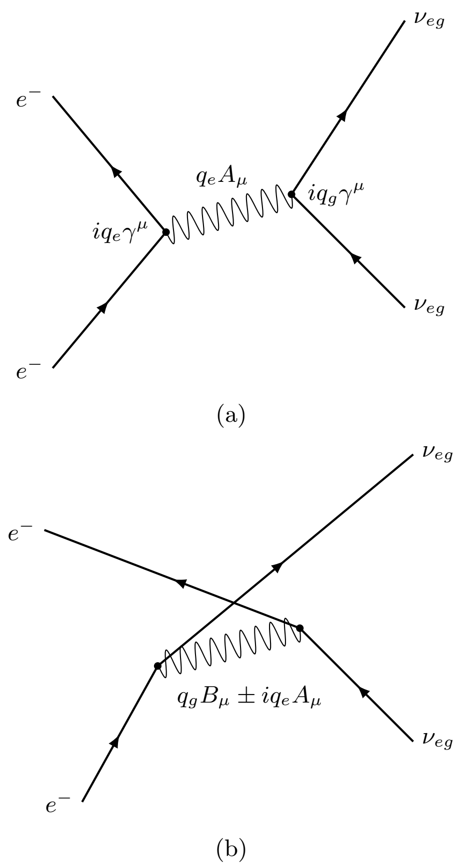


FIG. 3: The Electron-Dyon interaction

Now that the first-order Feynman diagramme of electromagnetic interaction can be constructed by FIG. 3. Each gauge fields have

$$W^1_\mu = q_g B_\mu, \quad W^2_\mu = q_e A_\mu, \quad W^3_\mu = q_e A_\mu. \quad (39)$$

The gauge field $\mathbf{W}_\mu$ on (39) gives gauge model which has satisfactory explanation of interaction of electrons on monopole fields.

III.3. The Dirac Field

From the gauge transformation of doublet term of spinor (37) and determination of gauge field (39) have covariant derivative

$$D_\mu := \partial_\mu - \frac{g}{2}\mathbf{W}_\mu\cdot\boldsymbol{\sigma}. \quad (40)$$

Lagrangian density of Dirac field of Feynman diagramme FIG. 3 is represented by

$$\begin{aligned}\mathcal{L}_{\text{Dirac}} = \bar{\psi}_e(i\partial\!\!\!/ - \frac{g}{2}q_e A\!\!\!/ - m)\psi_e - \frac{g}{2}\bar{\psi}_e Q\!\!\!/^- \psi_{\nu_{eg}} \\ + \bar{\psi}_{\nu_{eg}}(i\partial\!\!\!/ + \frac{g}{2}q_e A\!\!\!/ - m)\psi_{\nu_{eg}} - \frac{g}{2}\bar{\psi}_{\nu_{eg}} Q\!\!\!/^+ \psi_e\end{aligned} \quad (41)$$

where we put $Q^\pm_\mu = q_g B_\mu \pm i q_e A_\mu$. From the Feynman diagramme FIG. 3, we put the coupling coefficient $g = 2$.

III.4. The Monopoleless limit

Now we consider limit $q_g, B_\mu \to 0$ on Dirac field (41). Lagrangian density of SU(2) Dirac field goes to

$$\begin{aligned}\mathcal{L}_{\text{Dirac}} \quad \mapsto \quad \bar{\psi}_e(i\partial\!\!\!/ - q_e A\!\!\!/ - m)\psi_e + i\bar{\psi}_e A\!\!\!/ \psi_{\nu_{eg}} \\ + \bar{\psi}_{\nu_{eg}}(i\partial\!\!\!/ + q_e A\!\!\!/ - m)\psi_{\nu_{eg}} - i\bar{\psi}_{\nu_{eg}} A\!\!\!/ \psi_e\end{aligned} \quad (42)$$

for the monopoleless limit. Here we consider the charge-conjugation[4] of spinor ψ_e as:

$$\psi_e \quad \mapsto \quad \psi^c_e = \underbrace{(i\gamma^2\gamma^0)}_{C}\bar{\psi}^T_e. \quad (43)$$

Hence, we have following terms belonging to terms of S-matrices by

$$i(\bar{\psi}_e A\!\!\!/ \psi^c_e - \bar{\psi}^c_e A\!\!\!/ \psi_e) = 0 \quad (44)$$

where we expect the each term of S-matrices $\bar{\psi}_e \gamma^\mu A_\mu \psi^c_e$ be real:

$$\begin{aligned}(\bar{\psi}_e A\!\!\!/ \psi^c_e)^* &= -iA_\mu(\psi^\dagger_e \gamma^0\gamma^\mu\gamma^2\psi^*_e)^\dagger \\ &= A_\mu \psi^T_e \gamma^0 \underbrace{(-i\gamma^2\gamma^0)}_{C^\dagger}\gamma^0\gamma^\mu\psi_e = \bar{\psi}^c_e A\!\!\!/ \psi_e \in \mathbf{R}.\end{aligned} \quad (45)$$

We obtain $\psi_{\nu_{eg}} = \psi^c_e$ as one of soulution of Lagrangian density (42). The spinor of dyon $\psi_{\nu_{eg}}$ becomes positron on the monopoleless limit $q_g, B_\mu \to 0$:

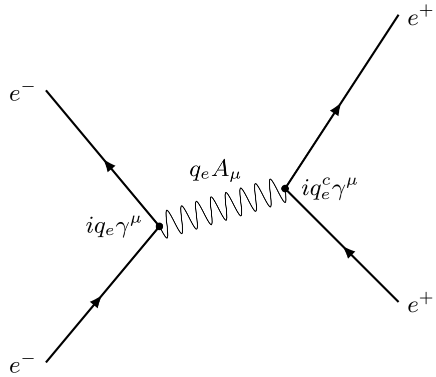


FIG. 4: Monopoleless limit on SU(2) gauge field

Hereby the SU(2) gauge model of electron-dyon interaction shows electron-positron scattering on monopoleless limit. The decay of electron and positron not allowed.

III.5. Curvature tensors of Gauge Field

The SU(2) gauge field $\mathbf{W}_\mu$ shown on Feynman diagramme FIG. 3 also gives curvature tensor $H_{\mu\nu}$ by

$$H^k_{\mu\nu} := \frac{1}{\hat{q}}(\partial_{[\mu}W^k_{\nu]} - 2\epsilon_{ijk}W^i_\mu W^j_\mu). \tag{46}$$

In each components, tensor (46) goes on

$$H^{k=1}_{\mu\nu} = \frac{q_g}{\hat{q}}E_{\mu\nu}, \tag{47}$$

$$H^{k=2}_{\mu\nu} = \frac{q_e}{\hat{q}}F_{\mu\nu} - 2\frac{q_e q_g}{\hat{q}^2}A_{[\mu}B_{\nu]}, \tag{48}$$

$$H^{k=3}_{\mu\nu} = \frac{q_e}{\hat{q}}F_{\mu\nu} + 2\frac{q_e q_g}{\hat{q}^2}A_{[\mu}B_{\nu]}, \tag{49}$$

The curvature tensors (47) to (49) gives action integral of gauge field by

$$S_f = \int \mathcal{L}_f \sqrt{-g}d^4x = \int -\frac{1}{8}\mathrm{tr}(H_{\mu\nu}H^{\mu\nu}) \sqrt{-g}d^4x. \tag{50}$$

Now that gauge field $H_{\mu\nu}$ also gives Lagrangian density of SU(2) gauge model:

$$\begin{aligned}\mathcal{L}_f = -\frac{q_g^2}{8\hat{q}^2}E_{\mu\nu}E^{\mu\nu} - \frac{q_e^2}{4\hat{q}^2}F_{\mu\nu}F^{\mu\nu} \\ + \frac{q_e^2 q_g^2}{2\hat{q}^4}f_{\mu\nu\rho\sigma}A^\mu B^\nu A^\rho B^\sigma\end{aligned} \tag{51}$$

where we put the $f_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\lambda}\epsilon_{\rho\sigma\lambda}$. Hence, the gauge boson coupling term appeared.

[1] Electron-electron, dyon-dyon, monopole-monopole, etc.
[2] Lorentz Scalar.
[3] Here we add Landau term $\frac{q_e}{\hat{q}} G^{\mu}{}_{\sigma} \partial^{\sigma} A^{\lambda}$ where it satisfies bianchi identity $\partial_{\mu}(G^{\mu}{}_{\sigma}\partial^{\sigma} A^{\lambda}) = \partial_{\mu}\partial_{\sigma}(G^{\mu\sigma} A^{\lambda}) = 0$ on energy-momentum tensor (29).
[4] The operator of charge-conjugation gives

$$C = i\gamma^2\gamma^0 = i\begin{pmatrix} 0 & \sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix}.$$

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