Simple Knowledge based Agent

An Example: The Wumpus World!

Performance measure:

gold +1000, death -1000,

-1 per step, -10 for using the arrow

Environment:

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

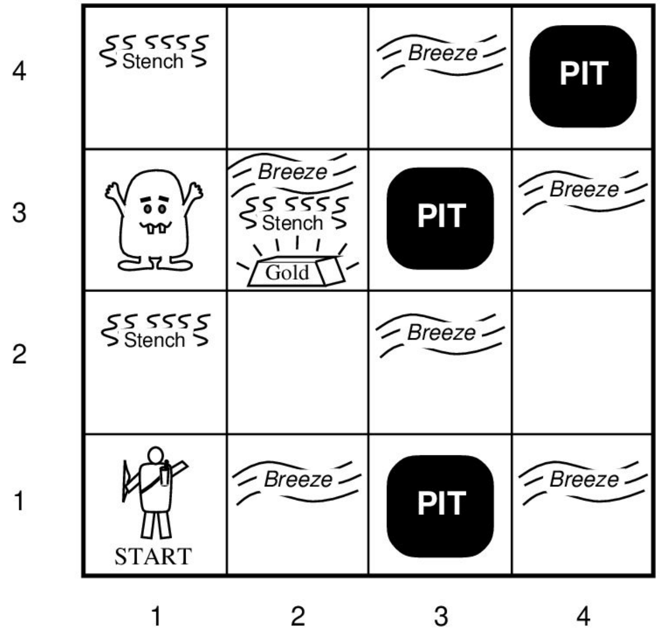
Shooting uses up the only arrow

Grabbing picks up gold if in same square

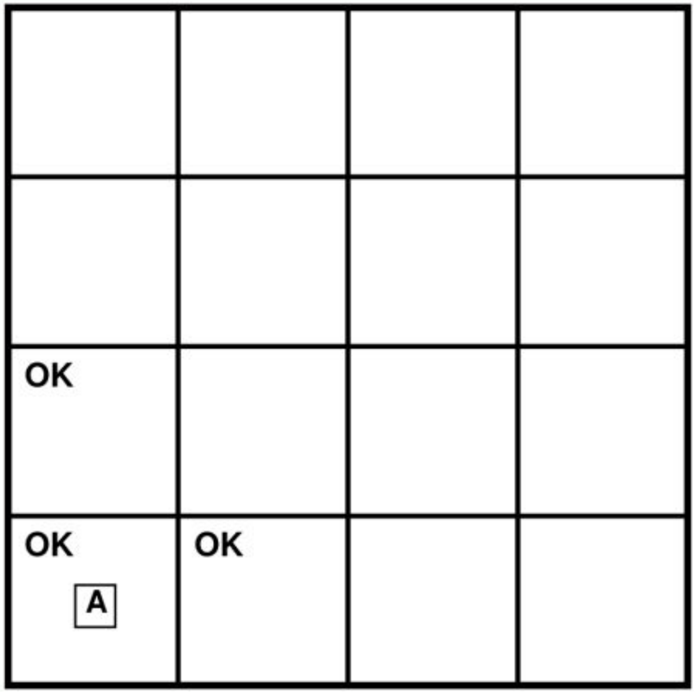
Releasing drops the gold in same square

Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Sensors: Breeze, Glitter, Smell



Exploring the Wumpus World



A: Agent; B: Breeze; G: Glitter; S: Stench
OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$
$B_{[1,1]} \leftrightarrow (P_{[1,2]} \vee P_{[2,1]})$
$S_{[1,1]} \leftrightarrow (W_{[1,2]} \vee W_{[2,1]})$
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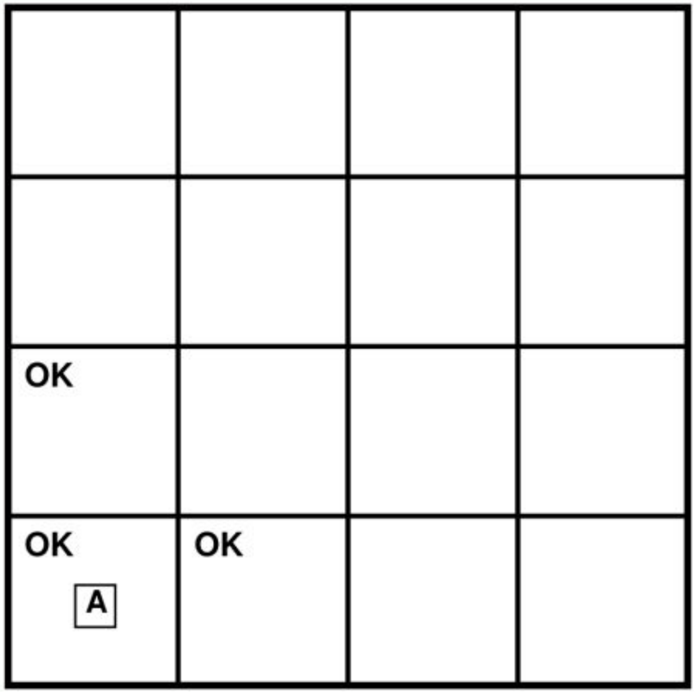

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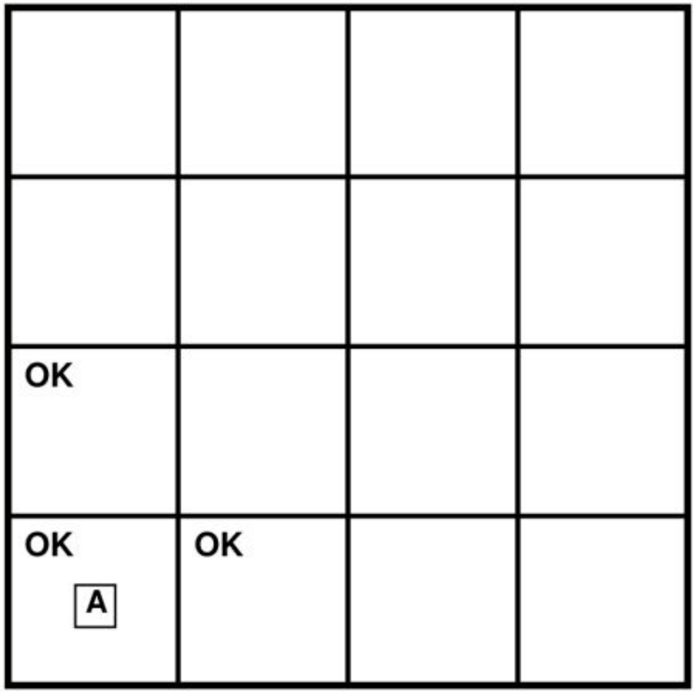
  ...
- Agent is initially in 1,1
- Percepts (no stench, no breeze):
  $\neg S_{[1,1]}, \neg B_{[1,1]}$



A: Agent; B: Breeze; G: Glitter; S: Stench
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Resolution

$$
\begin{aligned}
KB &= \{\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}),\ \neg B_{1,1}\ \} \\
\alpha &= \neg P_{1,2}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{CNF} &= \{\ \neg P_{1,2} \vee B_{1,1},\ \neg B_{1,1} \vee P_{1,2} \vee P_{2,1},\ \neg P_{1,2} \vee B_{1,1}, \\
&\qquad \neg B_{1,1},\ P_{1,2}\}
\end{aligned}
$$

$\boxed{\neg P_{2,1} \vee B_{1,1}}$ $\boxed{\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}}$ $\boxed{\neg P_{1,2} \vee B_{1,1}}$ $\boxed{\neg B_{1,1}}$ $\boxed{P_{1,2}}$

Resolution

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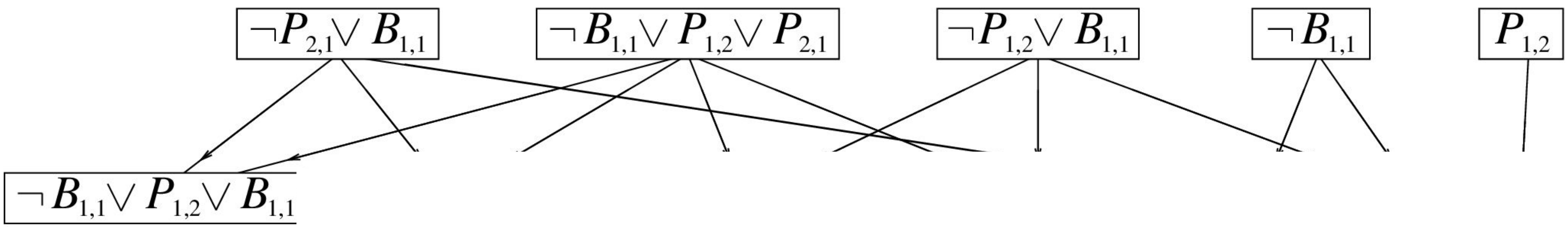
$$\begin{aligned} \mathsf{CNF} = \{\ & \underline{\neg P_{1,2} \vee B_{1,1}},\ \underline{\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}},\ \neg P_{1,2} \vee B_{1,1}, \\ & \neg B_{1,1},\ P_{1,2}\} \end{aligned}$$



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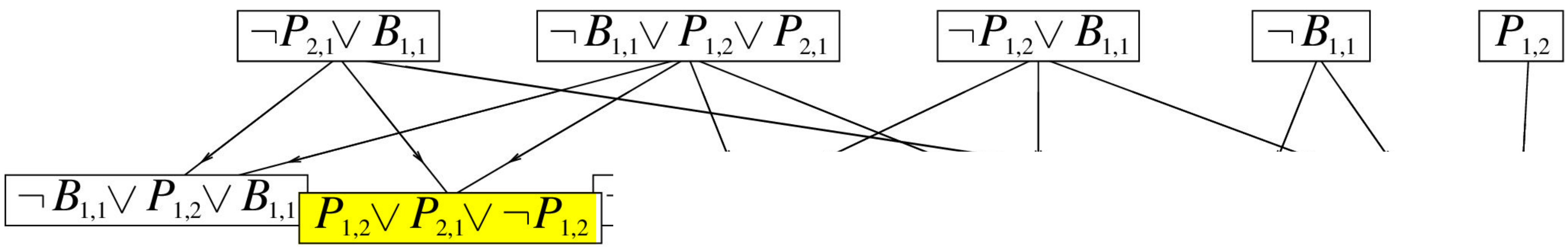

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$$KB = \{\ B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1}),\ \neg B_{1,1}\ \}$$
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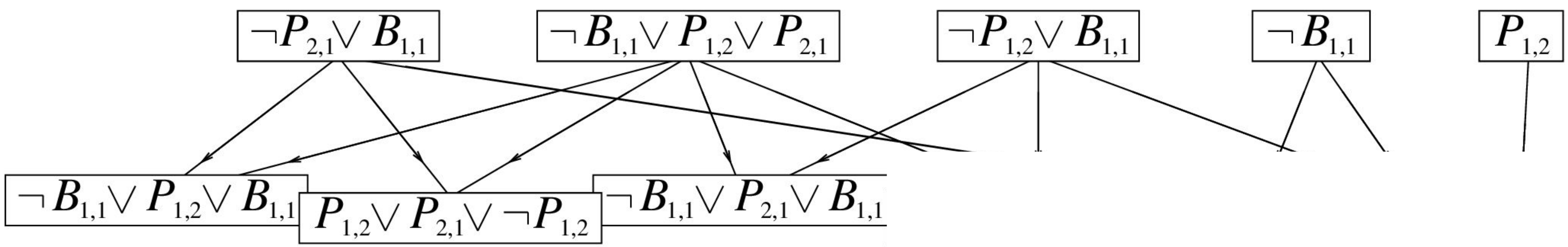

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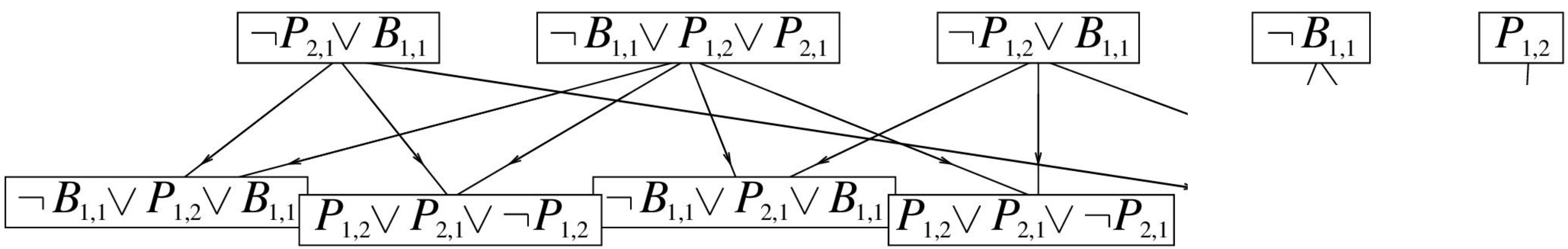

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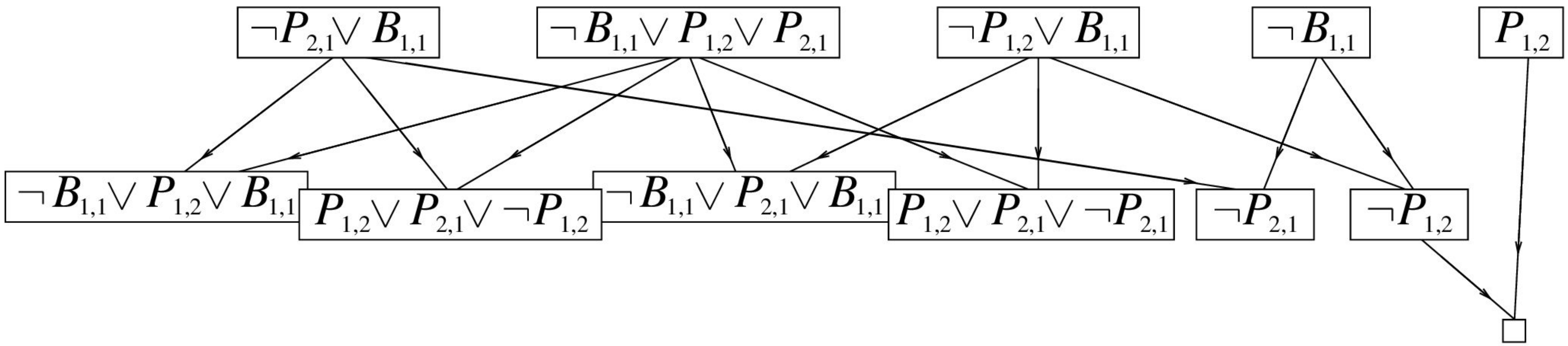

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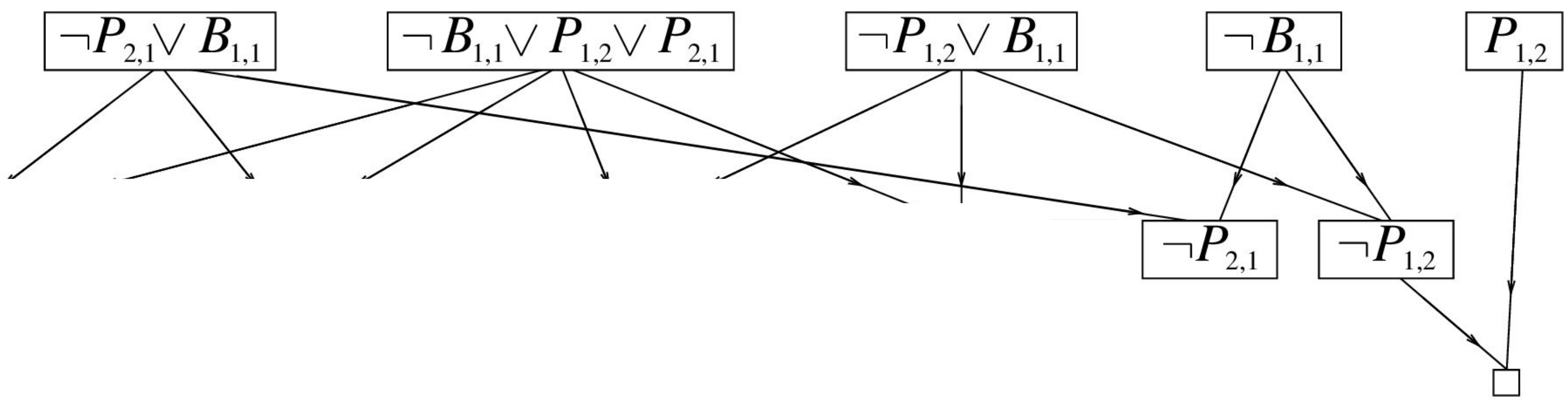

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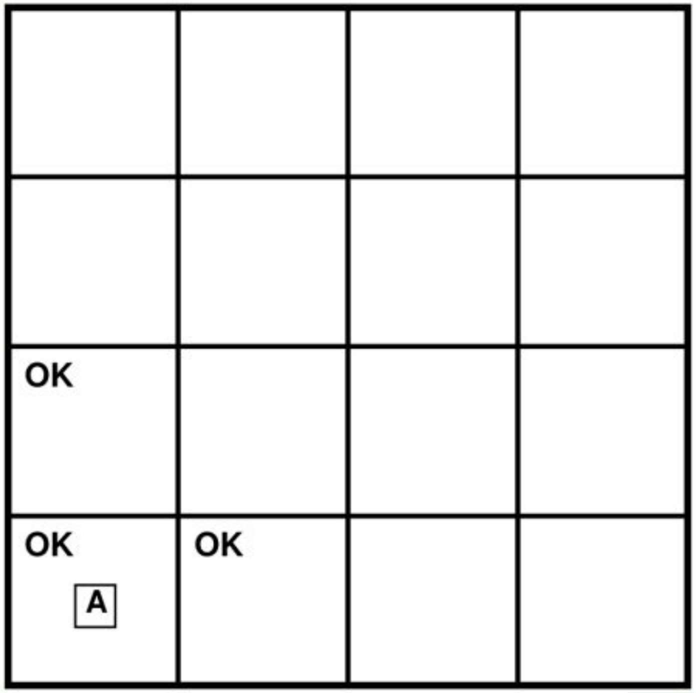
  ...
- Agent is initially in 1,1
- Percepts (no stench, no breeze):
  $\neg S_{[1,1]}, \neg B_{[1,1]}$

$\Longrightarrow$ $\neg P_{[1,2]}$



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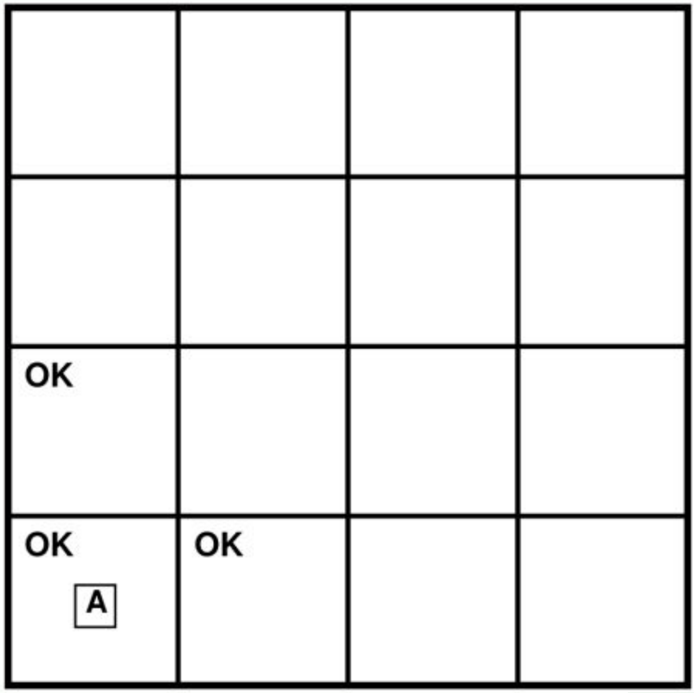

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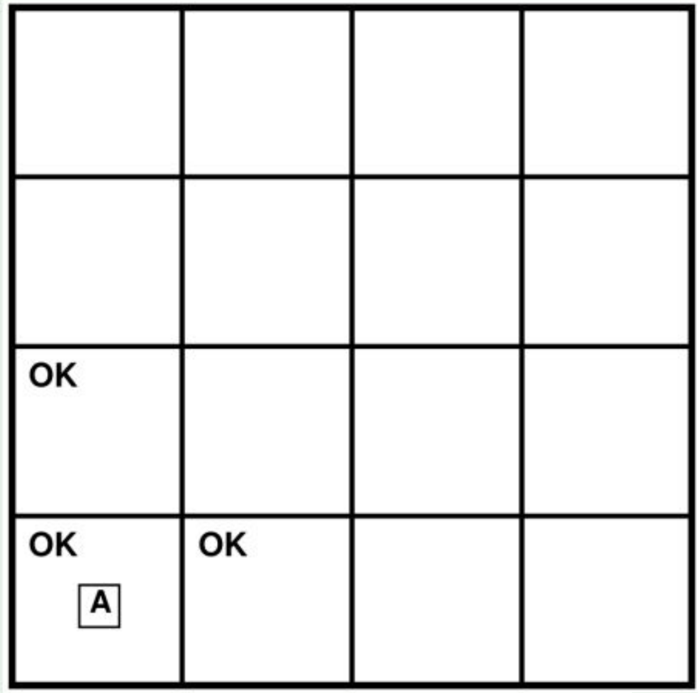
- Agent is initially in 1,1
- Percepts (no stench, no breeze):
  $\neg S_{[1,1]}, \neg B_{[1,1]}$

$\Longrightarrow$ $\neg W_{[1,2]}, \neg W_{[2,1]}, \neg P_{[1,2]}, \neg P_{[2,1]}$

$\Longrightarrow$ $OK_{[1,2]}, OK_{[2,1]}$ ([1,2]&[2,1] OK)

- Add all them to KB



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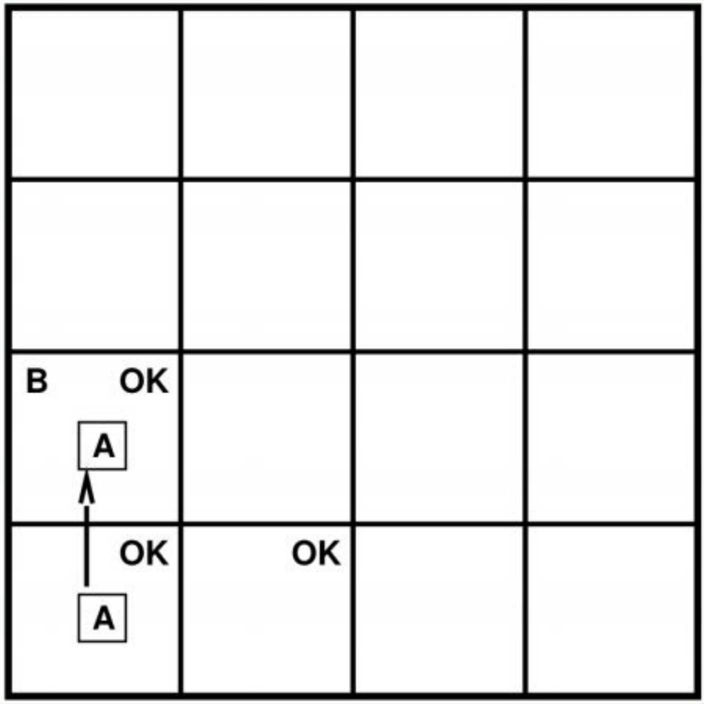

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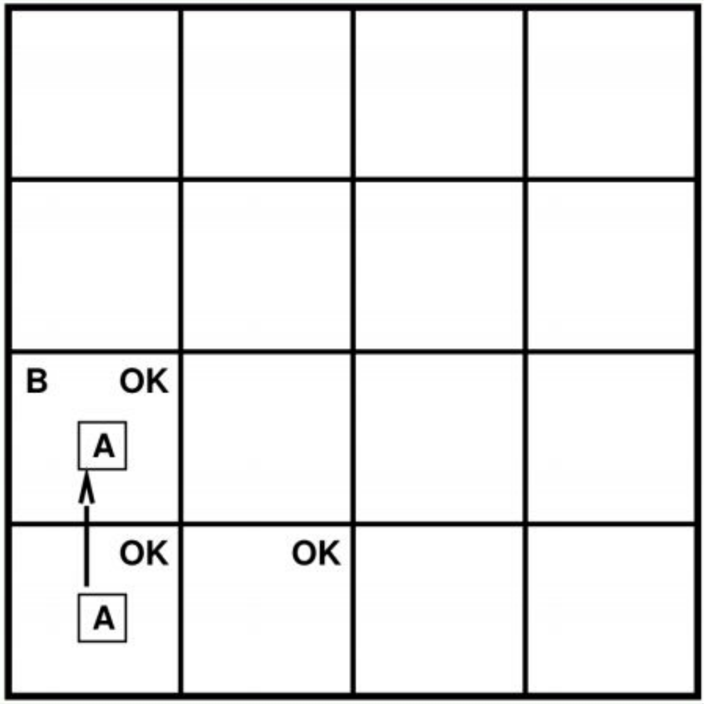
  $S_{[2,1]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[3,1]})$
- Agent moves to [2,1]
- perceives a breeze: $B_{[2,1]}$



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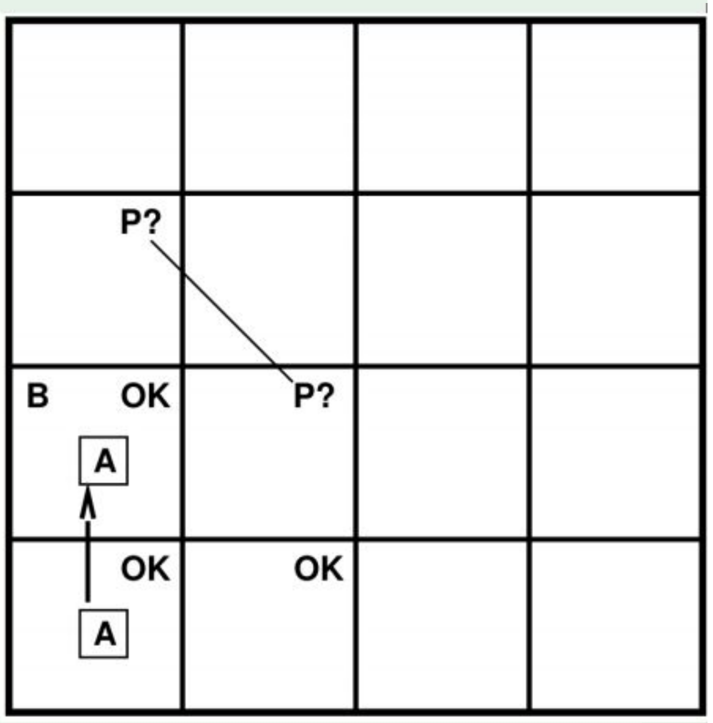
- Agent moves to [2,1]
- perceives a breeze: $B_{[2,1]}$

$\Longrightarrow$ $(P_{[3,1]} \vee P_{[2,2]})$ (pit in [3,1] or [2,2])



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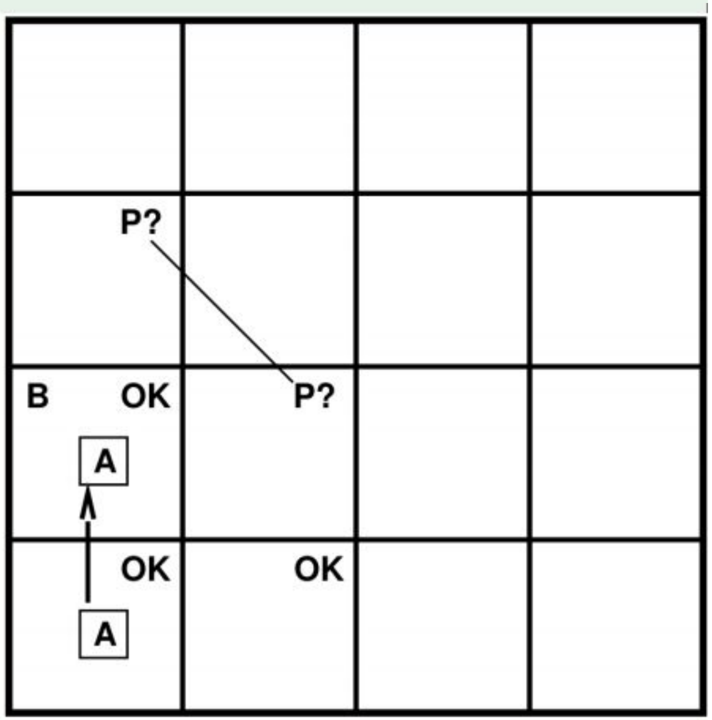
  $S_{[2,1]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[3,1]})$
- Agent moves to [2,1]
- perceives a breeze: $B_{[2,1]}$

$\Longrightarrow$ $(P_{[3,1]} \vee P_{[2,2]})$ (pit in [3,1] or [2,2])

- perceives no stench $\neg S_{[2,1]}$



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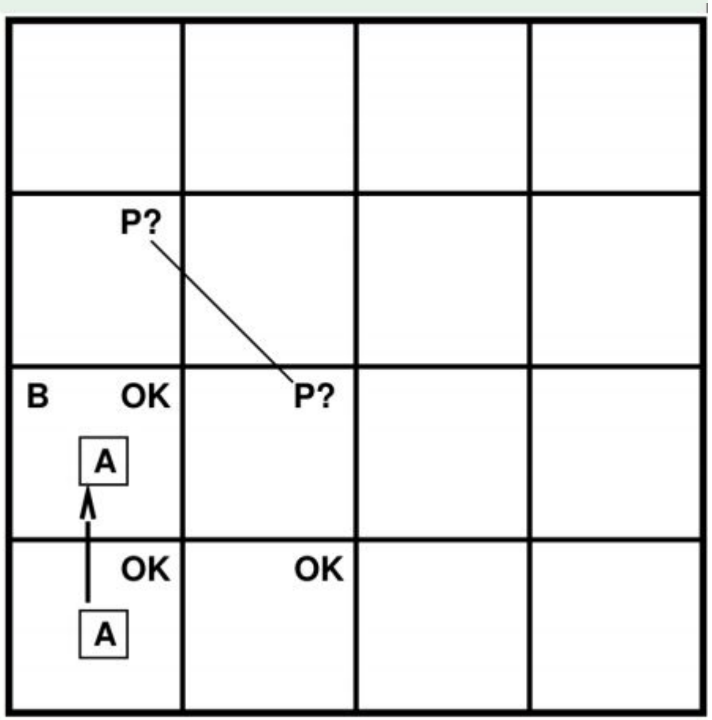
- Agent moves to [2,1]
- perceives a breeze: $B_{[2,1]}$

$\Longrightarrow$ $(P_{[3,1]} \vee P_{[2,2]})$ (pit in [3,1] or [2,2])

- perceives no stench $\neg S_{[2,1]}$

$\Longrightarrow$ $\neg W_{[3,1]}, \neg W_{[2,2]}$
(no Wumpus in [3,1], [2,2])

- Add all them to KB



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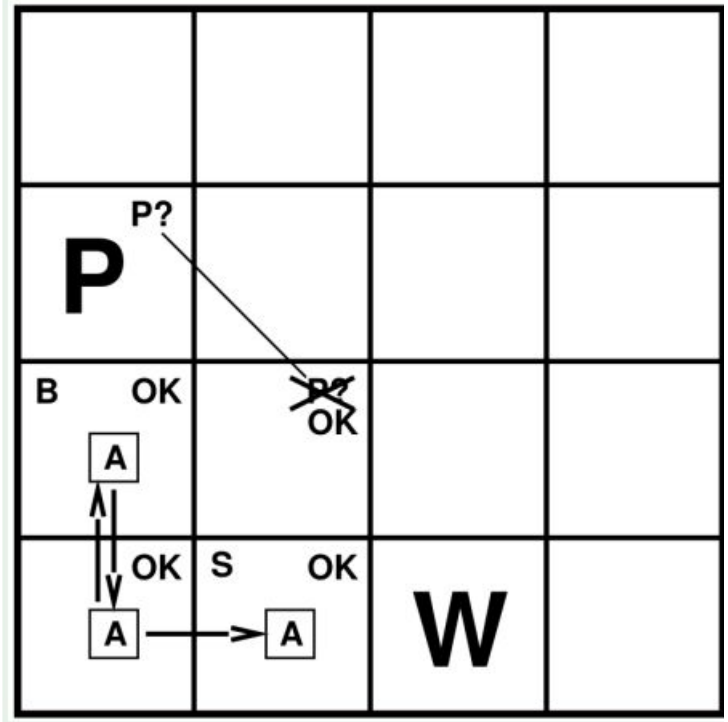

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- KB initially contains:
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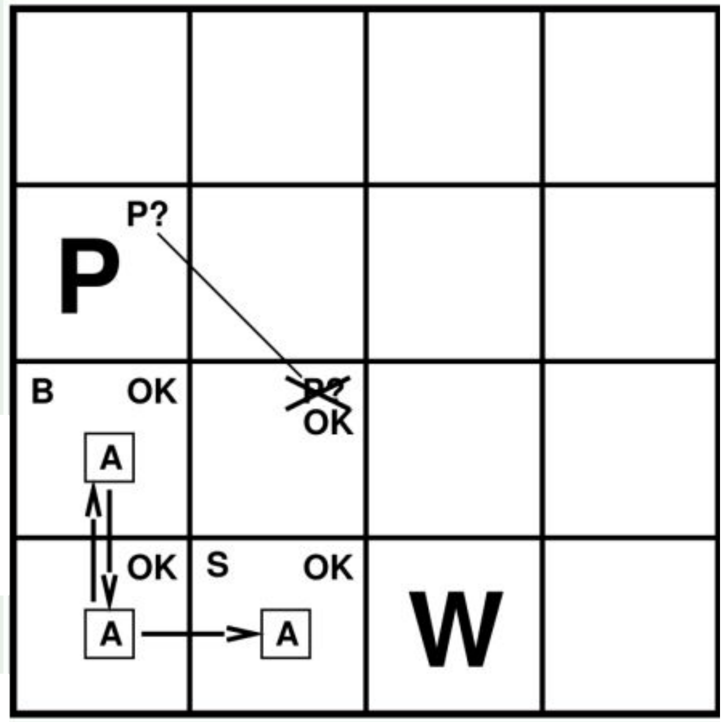
  $(P_{[3,1]} \vee P_{[2,2]}), \neg W_{[3,1]}, \neg W_{[2,2]}$
  $B_{[1,2]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[1,3]})$
  $S_{[1,2]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[1,3]})$
  $OK_{[2,2]} \leftrightarrow (\neg W_{[2,2]} \wedge \neg P_{[2,2]})$
- Agent moves to [1,1]-[1,2]
- perceives no breeze: $\neg B_{[1,2]}$



- perceives a stench: $S_{[1,2]}$

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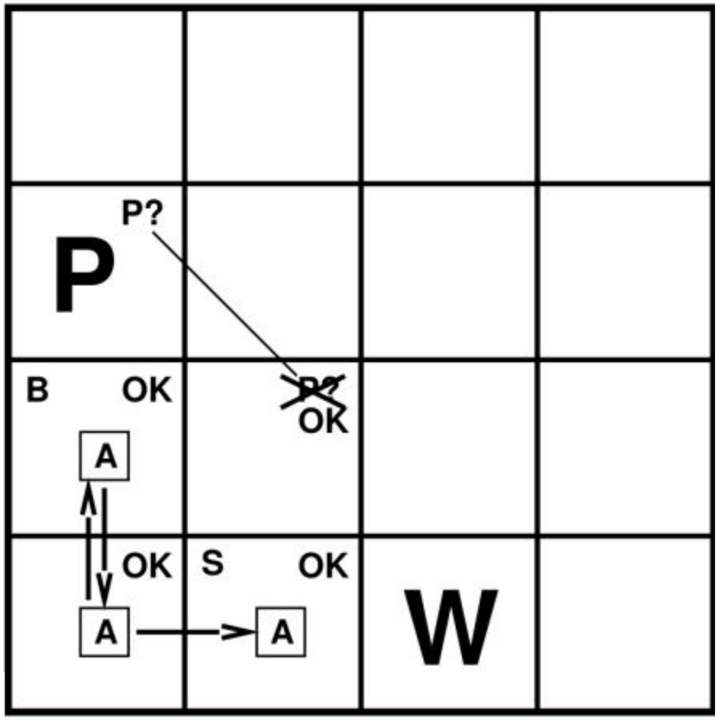
  $B_{[1,2]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[1,3]})$
  $S_{[1,2]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[1,3]})$
  $OK_{[2,2]} \leftrightarrow (\neg W_{[2,2]} \wedge \neg P_{[2,2]})$
- Agent moves to [1,1]-[1,2]
- perceives no breeze: $\neg B_{[1,2]}$

$\Longrightarrow$ $\neg P_{[2,2]}, \neg P_{[1,3]}$ (no pit in [2,2], [1,3])

$\Longrightarrow$ $P_{[3,1]}$ (pit in [3,1])

- perceives a stench: $S_{[1,2]}$



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  $(P_{[3,1]} \vee P_{[2,2]}), \neg W_{[3,1]}, \neg W_{[2,2]}$
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  $OK_{[2,2]} \leftrightarrow (\neg W_{[2,2]} \wedge \neg P_{[2,2]})$
- Agent moves to [1,1]-[1,2]
- perceives no breeze: $\neg B_{[1,2]}$

$\Longrightarrow$ $\neg P_{[2,2]}, \neg P_{[1,3]}$ (no pit in [2,2], [1,3])

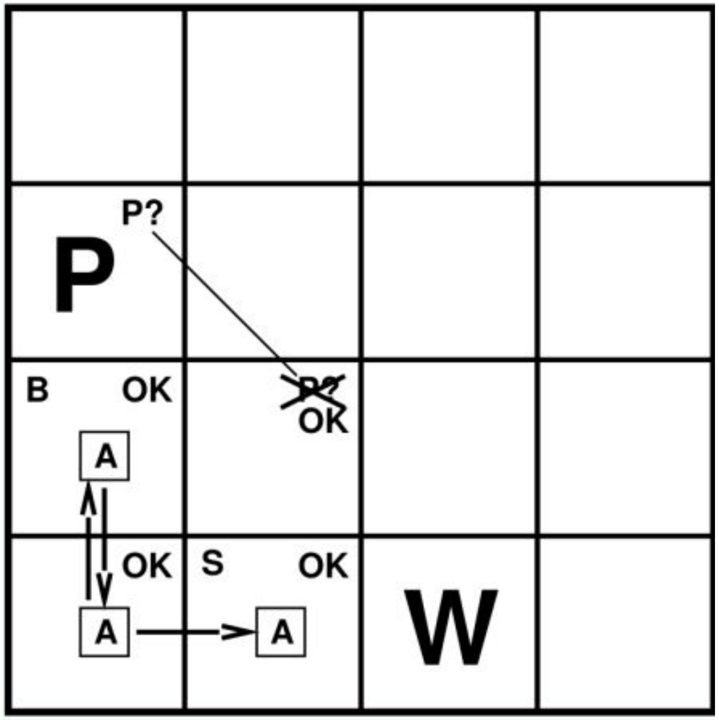
$\Longrightarrow$ $P_{[3,1]}$ (pit in [3,1])

- perceives a stench: $S_{[1,2]}$

$\Longrightarrow$ $W_{[1,3]}$ (Wumpus in [1,3]!)

$\Longrightarrow$ $OK_{[2,2]}$ ([2,2] OK)

- Add all them to KB



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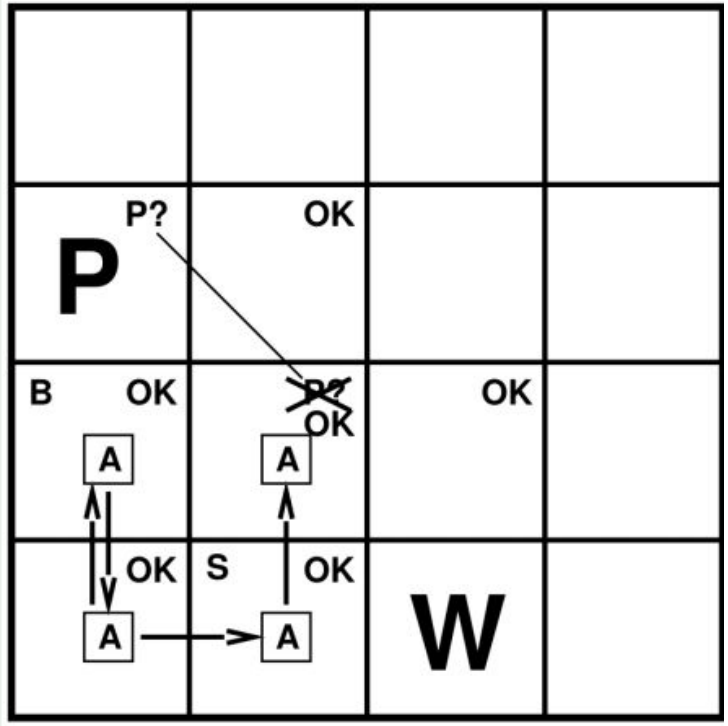
- Agent moves to [2,2]
- perceives no breeze: $\neg B_{[2,2]}$
- perceives no stench: $\neg S_{[2,2]}$



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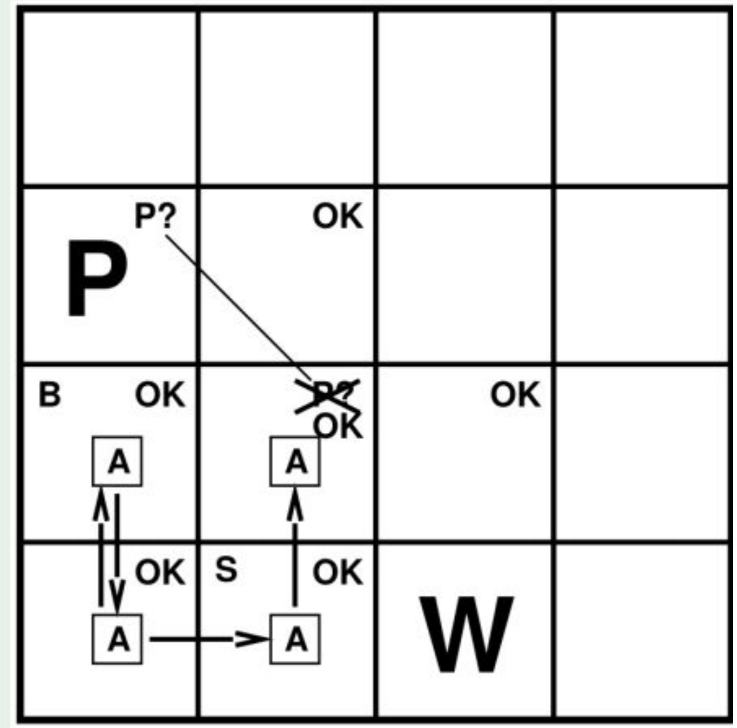
- Agent moves to [2,2]
- perceives no breeze: $\neg B_{[2,2]}$

$\Longrightarrow$ $\neg P_{[3,2]}, \neg P_{[3,2]}$ (no pit in [3,2], [2,3])

- perceives no stench: $\neg S_{[2,2]}$

$\Longrightarrow$ $\neg W_{[3,2]}, \neg W_{[3,2]}$ (no Wumpus in [3,2], [2,3])



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- Agent moves to [2,2]
- perceives no breeze: $\neg B_{[2,2]}$

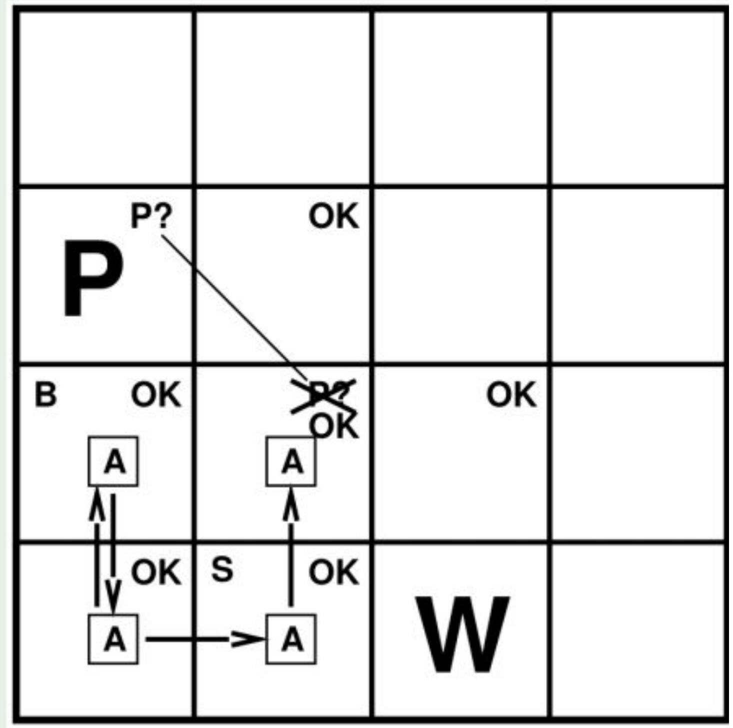
$\Longrightarrow$ $\neg P_{[3,2]}, \neg P_{[3,2]}$ (no pit in [3,2], [2,3])

- perceives no stench: $\neg S_{[2,2]}$

$\Longrightarrow$ $\neg W_{[3,2]}, \neg W_{[3,2]}$ (no Wumpus in [3,2], [2,3])

$\Longrightarrow$ $OK_{[3,2]}, OK_{[2,3]}$, ([3,2] and [2,3] OK)

- Add all them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench

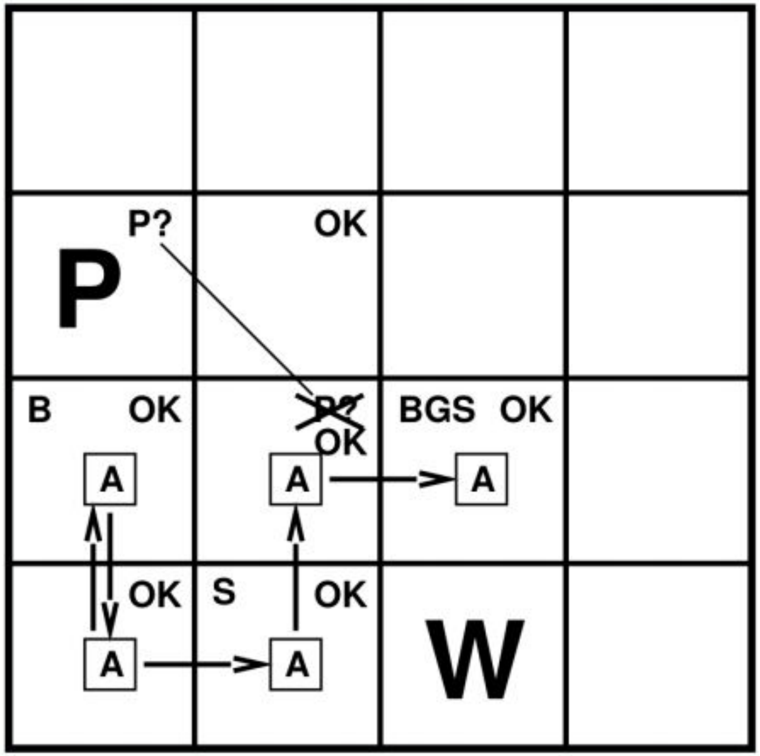
OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

Exploring the Wumpus World

- KB initially contains:
  $G_{[2,3]} \leftrightarrow BGS_{[2,3]}$
- Agent moves to [2,3]
- perceives a glitter: $G_{[2,3]}$

$\Longrightarrow$ $BGS_{[2,3]}$ (bag of gold!)

- Add it them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench
OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

First-order Logic (FOL)

Propositional Logic

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness

First-Order Logic

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  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

First Order Logic

→ provides more "primitives" to express knowledge:

— objects (identity & properties)

— relations among objects (including functions)

Objects: people, houses, numbers, Einstein, Huskers, event, ..

Properties: smart, nice, large, intelligent, loved, occurred, ..

Relations: brother-of, bigger-than, part-of, occurred-after, ..

functions: father-of, best-friend, double-of, ..

Basic elements

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
- Quantifiers:

Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

$$\text{Term} \quad = \quad function(term_1, \ldots, term_n)$$

or constant or variable

— **ground term**: term with no variable

Basic elements

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences

Atomic sentences

state facts

built with terms and predicate symbols

$$\text{Atomic sentence} \quad = \quad predicate(term_1, \ldots, term_n)$$
$$\text{or } term_1 = term_2$$

Examples:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

Complex Sentences

built with atomic sentences and logical connectives

$\neg S$

$S_1 \wedge S_2$

$S_1 \vee S_2$

$S_1 \Rightarrow S_2$

$S_1 \Leftrightarrow S_2$

Examples:

Sibling(KingJohn,Richard) $\Rightarrow$ Sibling(Richard,KingJohn)

Basic elements

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier

Universal quantification

$\forall \; \langle variables \rangle \;\; \langle sentence \rangle$

Example: all dogs like bones $\forall \, x Dog(x) \Rightarrow LikeBones(x)$

Universal quantification

$\forall \langle variables \rangle \ \ \langle sentence \rangle$

Example: all dogs like bones $\forall\, x Dog(x) \Rightarrow LikeBones(x)$

$\forall\, x\ P$ is equivalent to the conjunction of instantiations of P

$$\begin{aligned} & Dog(Indy) \Rightarrow LikeBones(\qquad\qquad) \\ \wedge\quad & Dog(Rebel) \Rightarrow LikeBones(\qquad) \\ \wedge\quad & Dog(Rover) \Rightarrow LikeBones(\qquad) \\ \wedge\quad & \ldots \end{aligned}$$

Universal quantification

$\forall \langle variables \rangle \;\; \langle sentence \rangle$

Example: all dogs like bones $\forall\, x Dog(x) \Rightarrow LikeBones(x)$

$\forall\, x\, P$ is equivalent to the <u>conjunction</u> of <u>instantiations</u> of P

$$
\begin{array}{ll}
 & Dog(Indy) \Rightarrow LikeBones(Indy) \\
\wedge & Dog(Rebel) \Rightarrow LikeBones(Rebel) \\
\wedge & Dog(Rover) \Rightarrow LikeBones(Rover) \\
\wedge & \ldots
\end{array}
$$

Existential quantification

$\exists \langle variables \rangle \;\; \langle sentence \rangle$

Example: some top-student will attend AAAI

$\exists \, x TopStudent(x) \wedge AttendsAAAI(x)$

Pat, Leslie, Chris are top-students

Existential quantification

$\exists \langle variables \rangle \;\; \langle sentence \rangle$

Example: some top-student will attend AAAI

$\exists\, x TopStudent(x) \wedge AttendsAAAI(x)$

Pat, Leslie, Chris are top-students

$\exists\, x\, P$ is equivalent to the disjunction of instantiations of P

$$
\begin{array}{ll}
 & TopStudent(Pat) \wedge AttendsAAAI(Pat) \\
\vee & TopStudent(Leslie) \wedge AttendsAAAI(Leslie) \\
\vee & TopStudent(Chris) \wedge AttendsAAAI(Chris) \\
\vee & \ldots
\end{array}
$$

Example: Family relations (kinship):

– Objects: people

– Properties: gender, married, divorced, single, widowed

– Relations: parenthood, brotherhood, marriage..

Unary predicates: Male, Female

Binary relations: Parent, Sibling, Brother, Child, etc.

Functions: Mother, Father

Examples

Brothers are siblings

. $\forall x, y$ $\Leftrightarrow$

Examples

Brothers are siblings

- $\forall x, y\ Brother(x, y) \Leftrightarrow Sibling(x, y)$

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- $\forall x, y \ Brother(x, y) \Leftrightarrow Sibling(x, y)$

"Sibling" is reflexive

- $\forall x, y \ Sibling(\quad) \Leftrightarrow Sibling(\quad)$

Examples

Brothers are siblings

. $\forall x, y \; Brother(x, y) \Leftrightarrow Sibling(x, y)$

"Sibling" is reflexive

. $\forall x, y \; Sibling(x, y) \Leftrightarrow Sibling(y, x)$

Examples

Brothers are siblings

- $\forall x, y \; Brother(x, y) \Leftrightarrow Sibling(x, y)$

"Sibling" is reflexive

- $\forall x, y \; Sibling(x, y) \Leftrightarrow Sibling(y, x)$

One's mother is one's female parent

- $\forall x, y \qquad \Leftrightarrow (\qquad \wedge \qquad)$

Examples

Brothers are siblings

- $\forall x, y \; Brother(x, y) \;\Leftrightarrow\; Sibling(x, y)$

"Sibling" is reflexive

- $\forall x, y \; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$

One's mother is one's female parent

- $\forall x, y \; Mother(x, y) \;\Leftrightarrow\; (Female(x) \;\wedge\; Parent(x, y))$

Examples

Brothers are siblings

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"Sibling" is reflexive

- $\forall x, y \; Sibling(x, y) \Leftrightarrow Sibling(y, x)$

One's mother is one's female parent

- $\forall x, y \; Mother(x, y) \Leftrightarrow (Female(x) \wedge Parent(x, y))$

A first cousin is a child of a parent's sibling

$\forall x, y \qquad \Leftrightarrow$

$\exists a, b \qquad \wedge \qquad \wedge$

Examples

Brothers are siblings

- $\forall x, y \; Brother(x, y) \Leftrightarrow Sibling(x, y)$

"Sibling" is reflexive

- $\forall x, y \; Sibling(x, y) \Leftrightarrow Sibling(y, x)$

One's mother is one's female parent

- $\forall x, y \; Mother(x, y) \Leftrightarrow (Female(x) \wedge Parent(x, y))$

A first cousin is a child of a parent's sibling

$\forall x, y \; FirstCousin(x, y) \Leftrightarrow$

$\exists a, b \quad \wedge \; Sibling(a, b) \; \wedge$

Examples

Brothers are siblings

- $\forall x, y\; Brother(x, y) \;\Leftrightarrow\; Sibling(x, y)$

"Sibling" is reflexive

- $\forall x, y\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$

One's mother is one's female parent

- $\forall x, y\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \;\wedge\; Parent(x, y))$

A first cousin is a child of a parent's sibling

$$\forall x, y\; FirstCousin(x, y) \Leftrightarrow$$
$$\exists a, b\; Parent(a, x) \wedge Sibling(a, b) \wedge Parent(b, y)$$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

predicate, terms, and equality to form atomic sentences

Father(John)=Henry

Inference rule for FOL

(a) Universal elimination

(b) Existential elimination

(c) Existential introduction

Universal elimination (UE)

Example:

Sentence $= \forall x \; Likes(x, IceCream)$

Variable $= x$, constant $=$ Ben, binding list$=$ {x/Ben}

we can infer: $Likes(Ben, IceCream)$

(A universal sentence is the conjunction of all its possible instantiations)

Universal elimination (UE)

For any sentence α, variable v, and ground term g:

$$\frac{\forall v\ \alpha}{Subst(\{v, g\}), \alpha}$$

Example:

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Variable $= x$, constant $=$ Ben, binding list$=$ {x/Ben}

we can infer: $Likes(Ben, IceCream)$

(A universal sentence is the conjunction of all its possible instantiations)

Existential elimination

Example:

Sentence $= \exists x\; Kill(x, Victim)$

Variable $= x$, constant $=$ Murdered, binding list$=$ {x/Murderer}

we can infer: $Kill(Murderer, Victim)$

warning: Murderer does <u>not</u> appear in KB

e.g., $Kill(Victim, Victim)$ is not a logical consequence of the sentence

Existential elimination

For any sentence α, variable v, and constant symbol term k:

$$\frac{\exists\, v\ \alpha}{Subst(\{v,k\}),\alpha}$$

Example:

Sentence $= \exists\, x\ Kill(x, Victim)$

Variable $= x$, constant $=$ Murdered, binding list$=$ {x/Murderer}

we can infer: $Kill(Murderer, Victim)$

warning: Murderer does <u>not</u> appear in KB

e.g., $Kill(Victim, Victim)$ is not a logical consequence of the sentence

Existential introduction

Example:

Sentence = $Likes(Jerry, IceCream)$

Variable = x, ground term = Jerry

we can infer: $\exists\, x\ Likes(x, IceCream)$

Existential introduction

For any sentence α, variable v, and ground term g,
warning: v does not appear α, g <u>does</u> appear in α

$$\frac{\alpha}{\exists\, v\ Subst(\{g, v\}), \alpha}$$

Example:

Sentence $= Likes(Jerry, IceCream)$

Variable $= x$, ground term $=$ Jerry

we can infer: $\exists\, x\ Likes(x, IceCream)$