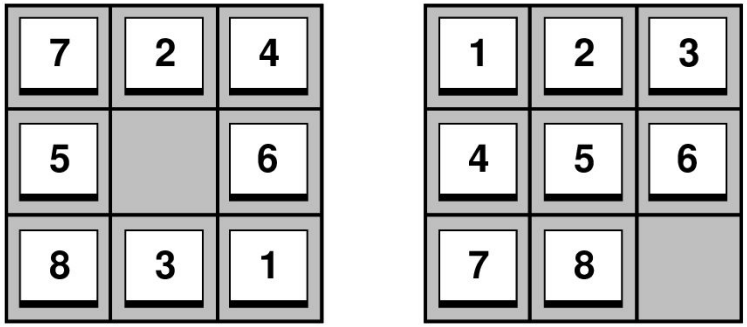
Artificial Intelligence

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

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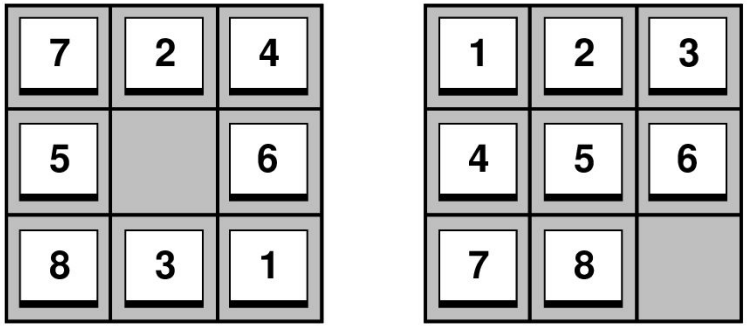

$h_1(S) =$?? 6
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If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1 and is better for search

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

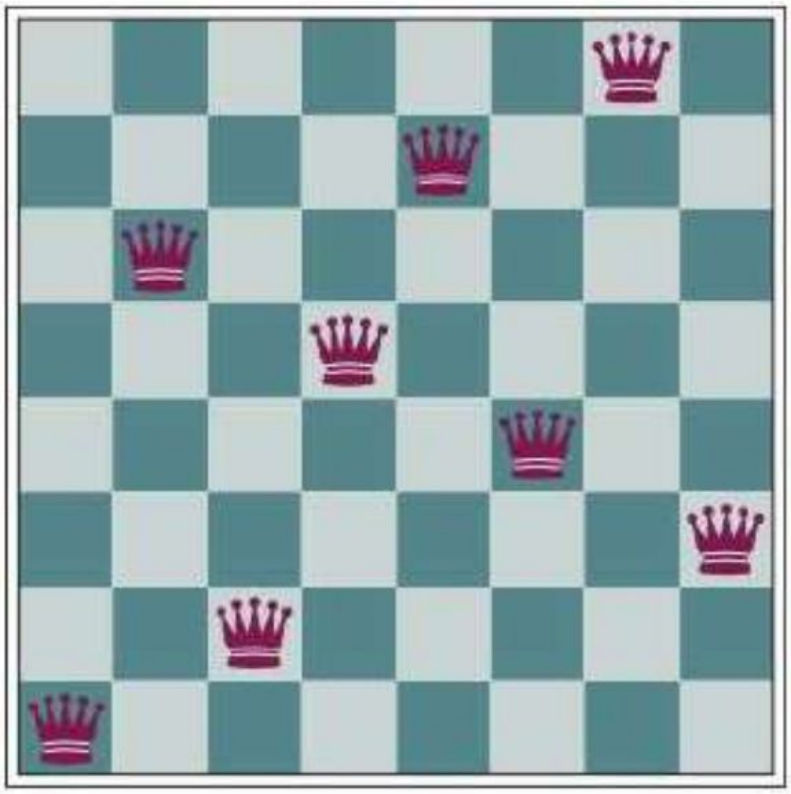
Space?? O(b^m), as in Greedy Best-First — may end up with all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

If h is admissible

Hill climbing Search (greedy local search)

Hill climbing is sometimes called greedy local search because it grabs a good neighbor state without thinking ahead.



Just five steps

Local search

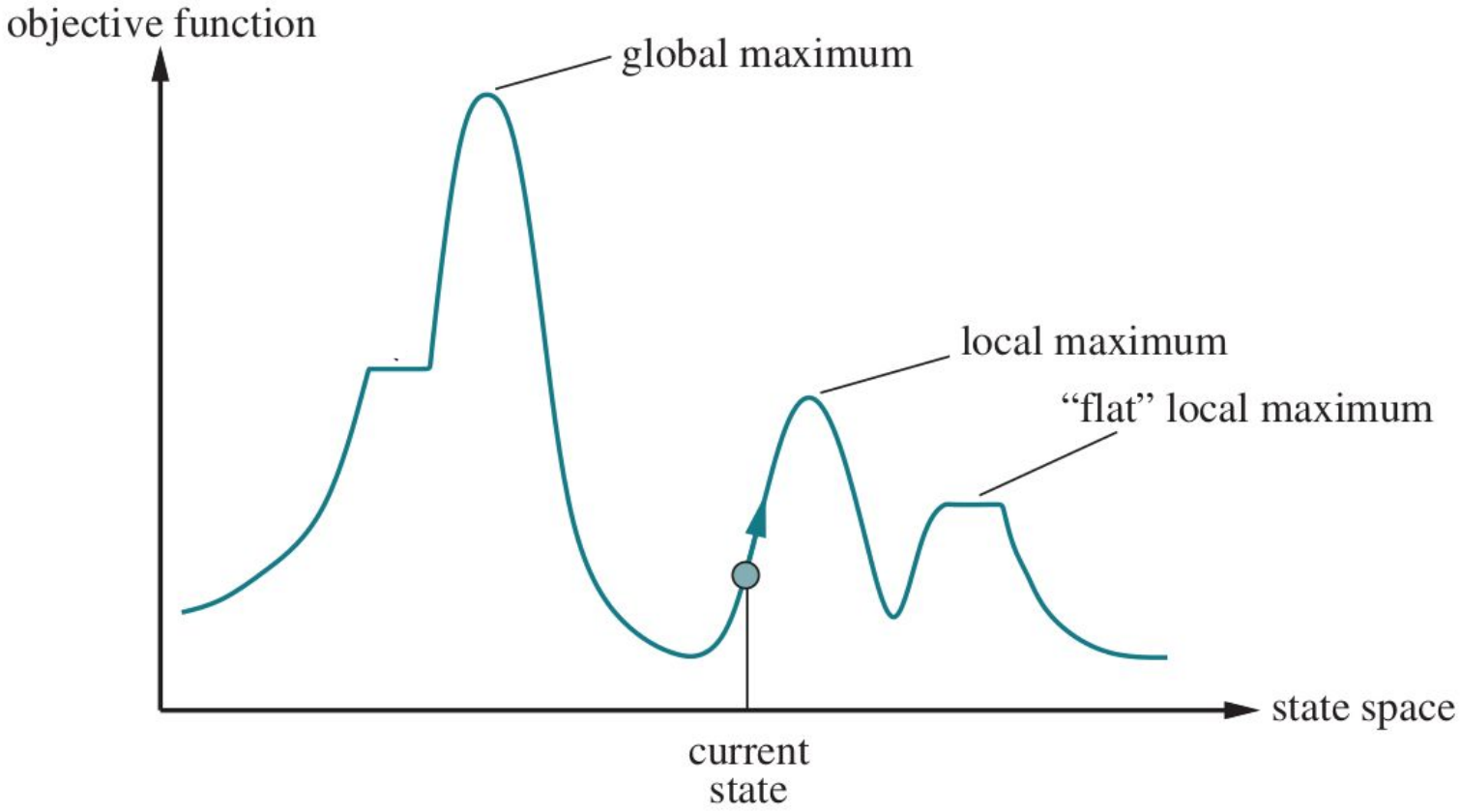
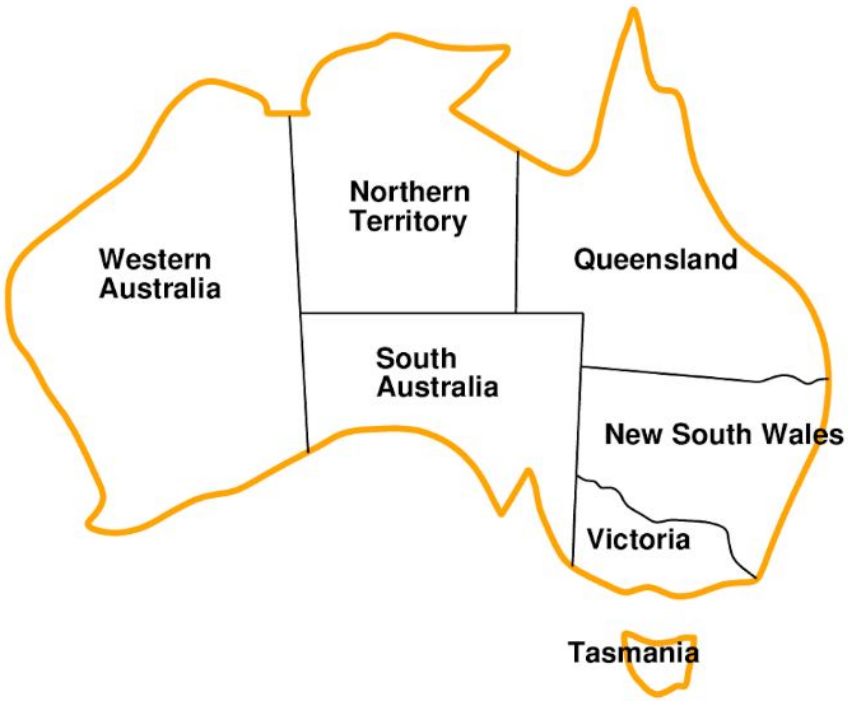


Figure 4.1 A one-dimensional state-space landscape in which elevation corresponds to the objective function. The aim is to find the global maximum.

Constraint Satisfaction Problem

Constraint satisfaction problems (CSPs)

- CSPs are mathematical questions defined as a set of objects whose state must satisfy a number of constraints.



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- Standard search problem considers the states, actions, successor function, and goal test.

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- Standard search problem considers the states, actions, successor function, and goal test.
- In CSP, state is defined by the variables with values from domain.
- Goal test relates to satisfying constraints that conditions allowable combination of values for the subset of variables.

Constraint satisfaction problems (CSPs)

Constraint satisfaction problem (CSP) consists of following three components.

$\mathcal{X}$ is a set of variables, $\{X_1, \ldots, X_n\}$.

$\mathcal{D}$ is a set of domains, $\{D_1, \ldots, D_n\}$, one for each variable.

$\mathcal{C}$ is a set of constraints that specify allowable combinations of values.

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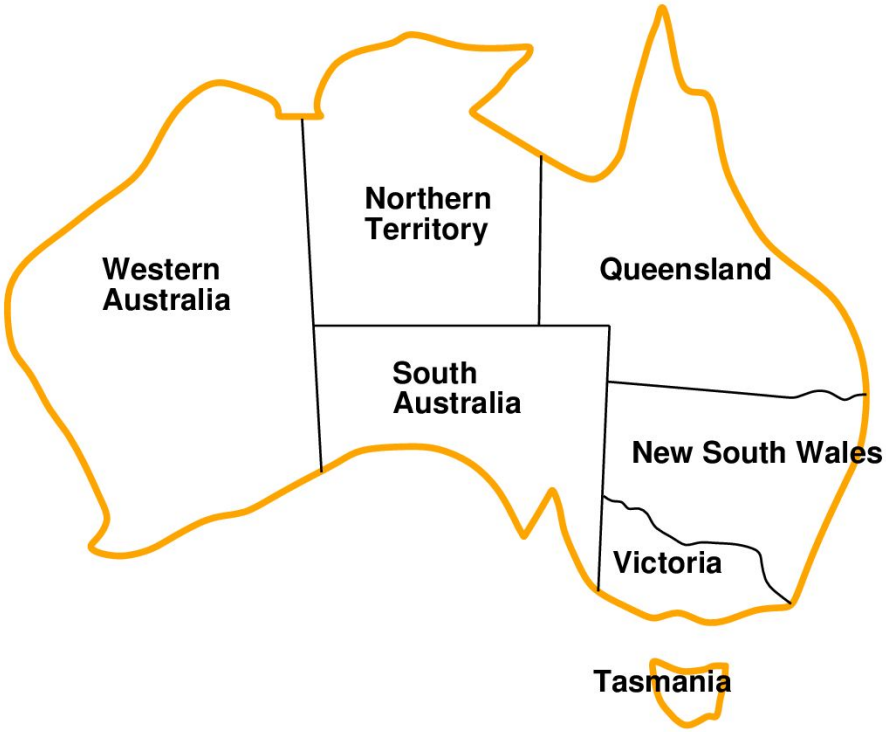
CSPs deal with assignments of values to variables, $\{X_i = v_i, X_j = v_j, \ldots\}$. while satisfying the constraints.

Example: Map coloring

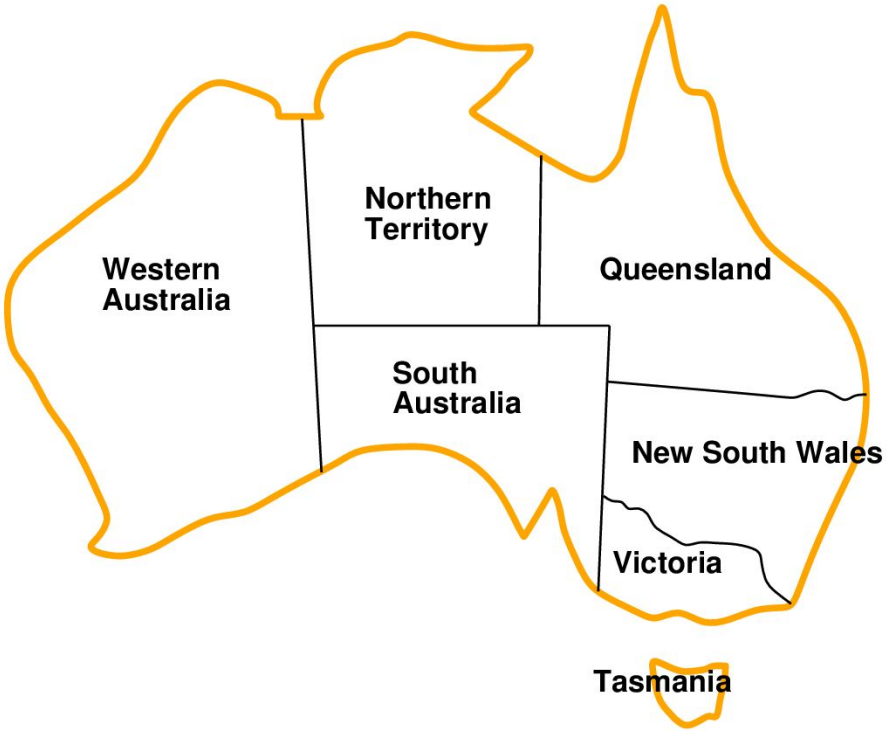


Variables:

Domains:

Constraints:

Example: Map coloring



Variables: MA, NT, Q, NSW, V, SA, T

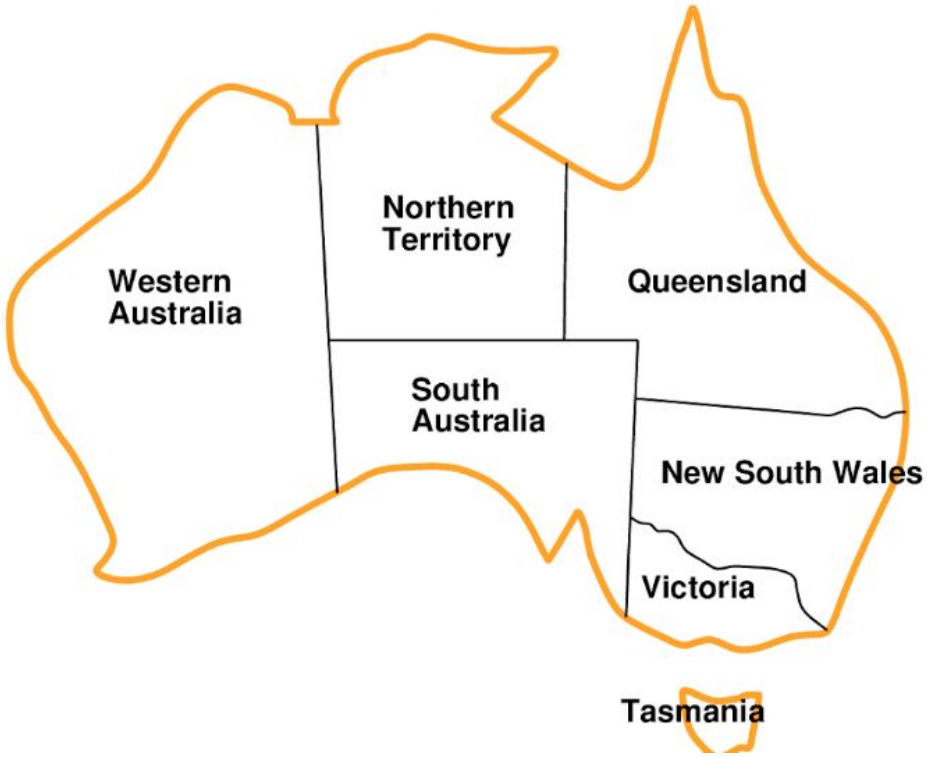
Domains: $D_i = \{r(ed), g(reen), b(lue)\}$

Constraints: adjacent regions must have different colors

e.g., $WA \neq NT$ (if the language allows this), or

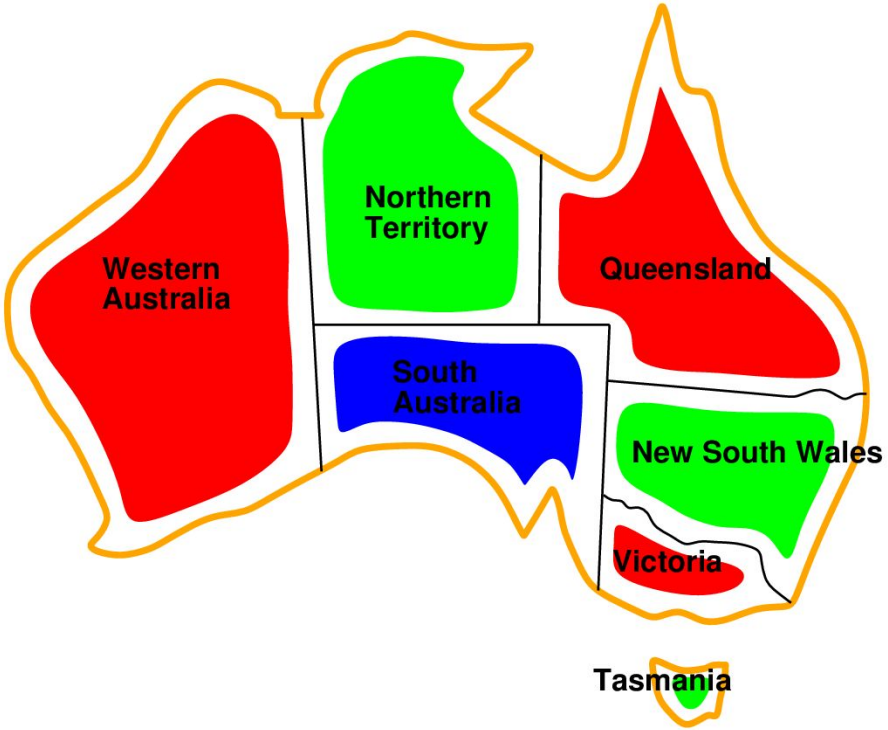
$(WA, NT) \in \{(r, g), (r, b), (g, r), (g, b), \ldots\}$

Example: Map coloring contd.



Solutions are assignments satisfying all constraints,

Example: Map coloring contd.



Solutions are assignments satisfying all constraints,

e.g., $\{WA = r, NT = g, Q = r, NSW = g, V = r, SA = b, T = g\}$

Varieties of constraints

Unary constraints involve a single variable

e.g., $SA \neq g$

Binary constraints involve pairs of variables

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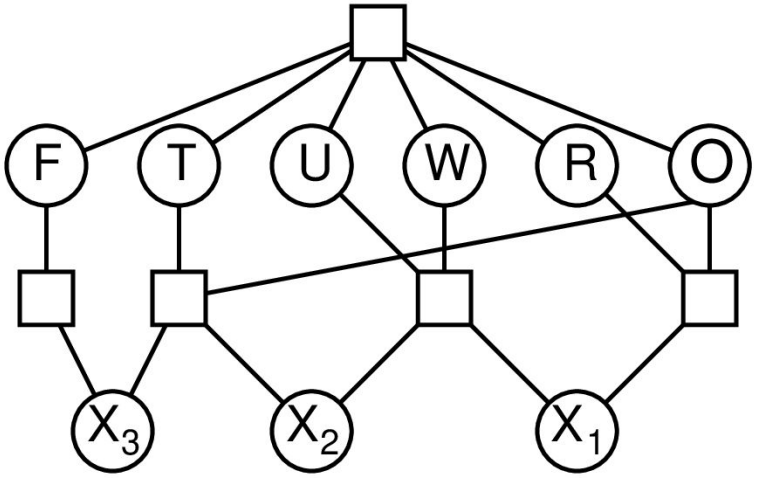
Higher-order constraints involve 3 or more variables

e.g., cryptarithmetic column constraints

Example: Cryptarithmetic

$$\begin{array}{r} \ T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$



Variables: $F,\ T,\ U,\ W,\ R,\ O,\ X_1,\ X_2,\ X_3$

Domain: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Constraints: $alldiff\,(F, T, U, W, R, O)$

$O + O = R + 10 \cdot X_1$

$\ldots$

Varieties of constraints

Unary constraints involve a single variable

e.g., $SA \neq g$

Binary constraints involve pairs of variables

e.g., $SA \neq WA$

Higher-order constraints involve 3 or more variables

e.g., cryptarithmetic column constraints

Preferences are soft constraints

e.g., red is better than $green$

often representable by a cost for each variable assignment

$\rightarrow$ constrained optimization problems

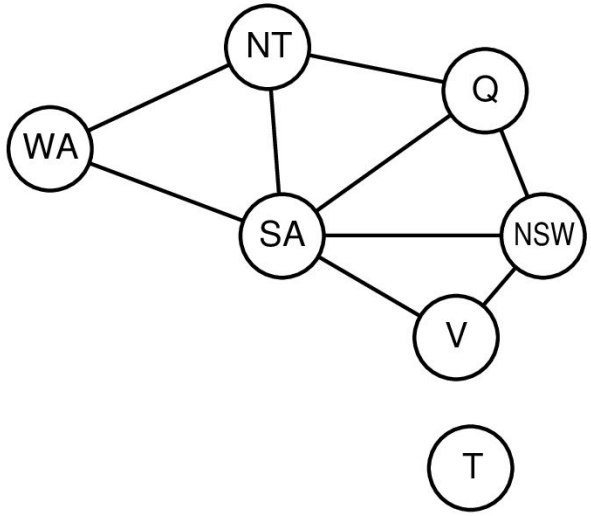
Constraint Satisfaction

- An assignment that does not violate any constraints is called a **consistent** or **legal assignment**.
- A complete assignment is one in which every variable is assigned a value, and a solution to a CSP is a consistent, **complete assignment**.
- A partial assignment is one that leaves some variables **unassigned**, and a **partial solution** is a partial assignment that is consistent.

Constraint graph

Binary CSP: each constraint relates at most two variables

Constraint graph: nodes are variables, arcs show constraints

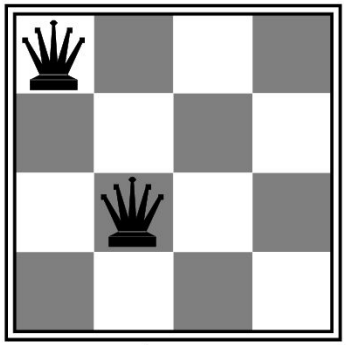


General-purpose CSP methods use the graph structure to speed up search

e.g., Tasmania is an independent subproblem!

Example: 4-Queens as a CSP

Assume one queen in each column. Which row does each one go in?



$Q_1 = 1 \quad Q_2 = 3$

Variables

Domains

Constraints

Example: 4-Queens as a CSP

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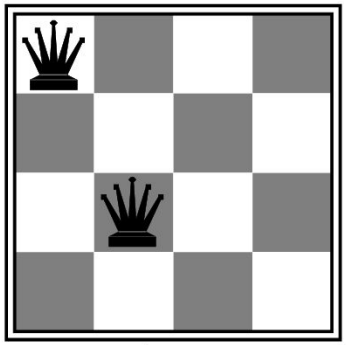

Variables	Q_1, Q_2, Q_3, Q_4	
Domains	$D_i = \{1, 2, 3, 4\}$	
Constraints	$Q_i \neq Q_j$	(cannot be in same row)
	$\lvert Q_i - Q_j \rvert \neq \lvert i - j \rvert$	(cannot be on same diagonal)

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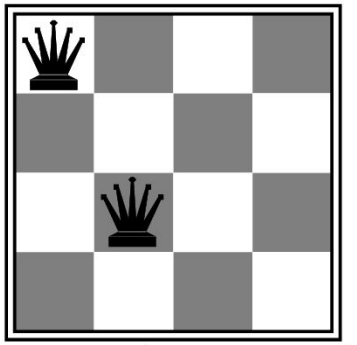

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Translate each constraint into set of allowable values for its variables
E.g., values for (Q_1, Q_2) are $(1,3)$ $(1,4)$ $(2,4)$ $(3,1)$ $(4,1)$ $(4,2)$

Standard search formulation (incremental)

Let's start with a basic, naive approach and then improve it

States are defined by the values assigned so far

Initial state: the empty assignment, $\{\}$

Successor function: assign a value to an unassigned variable that does not conflict with current assignment.
Fail if no legal assignments (not fixable!)

Goal test: the current assignment is complete

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Note:

1. This is the same for all CSPs!
2. Every solution appears at depth n with n variables $\Longrightarrow$ use depth-first search

Backtracking search

Order of variables in variable assignments is irrelevan
i.e., $(WA = r,\ NT = g)$ same as $(NT = g,\ WA = r)$

Only need to consider assignments to a single variable at each node
$\Longrightarrow\ b = d$ and there are d^n leaves

Depth-first search for CSPs with single-variable assignments is called backtracking search

Backtracking search is the basic uninformed algorithm for CSPs

Real World CSPs

- Class Assignment problems: who teaches what class
- Scheduling problems: Which class is offered when and where
- Transportation Scheduling and trips planning
- Many more…

Variable, domain, constraints?

Backtracking Search

Use depth-first-search

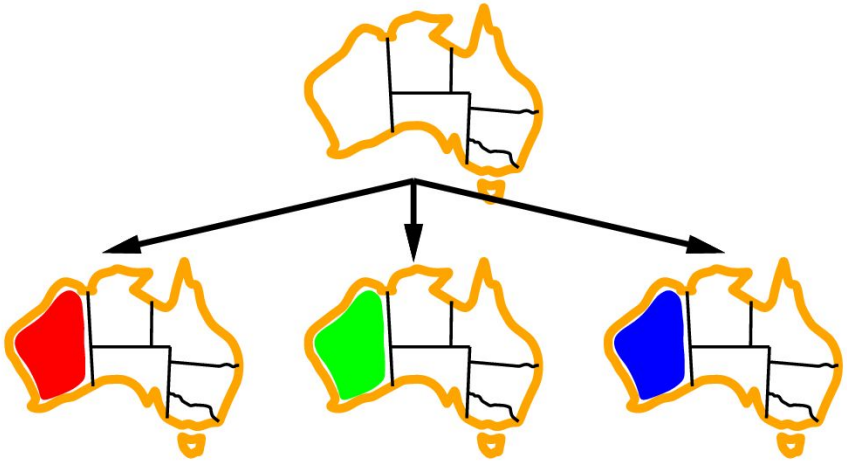
(1) Fix the **order** of assignments

(2) Check for constraint violation and constraint satisfaction when expanding the other nodes

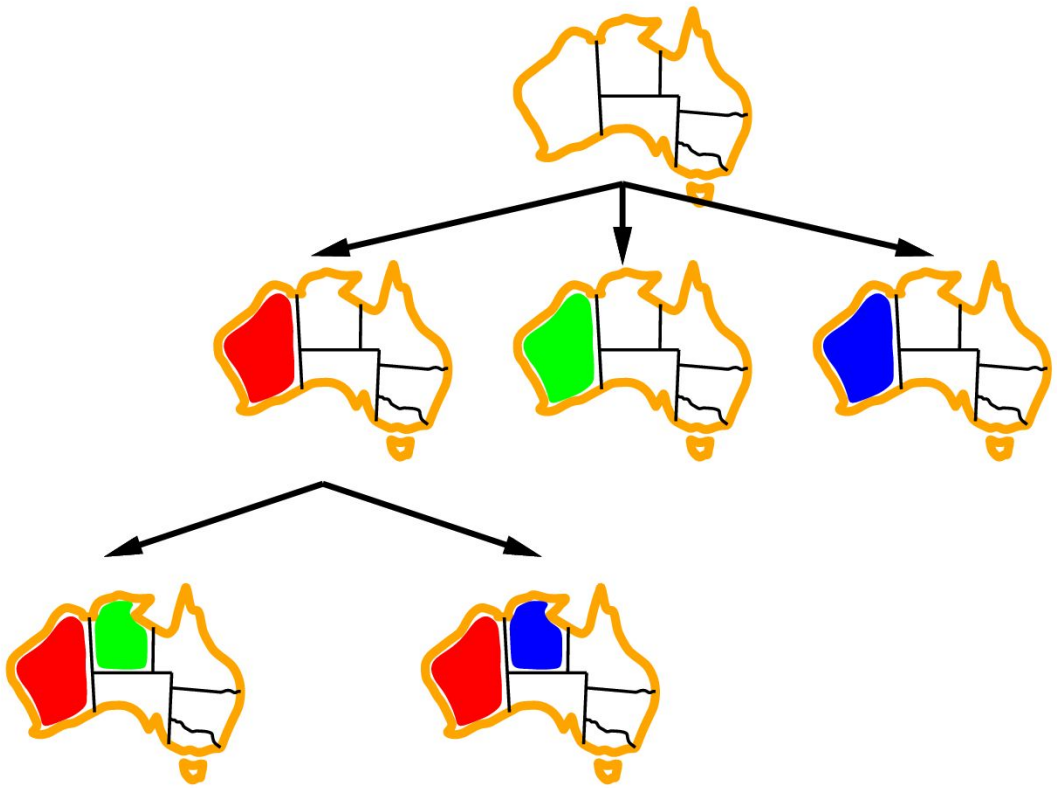
(3) Backtracking is the basic uninformed algorithm for CSPs

Backtracking example

Backtracking example



Backtracking example



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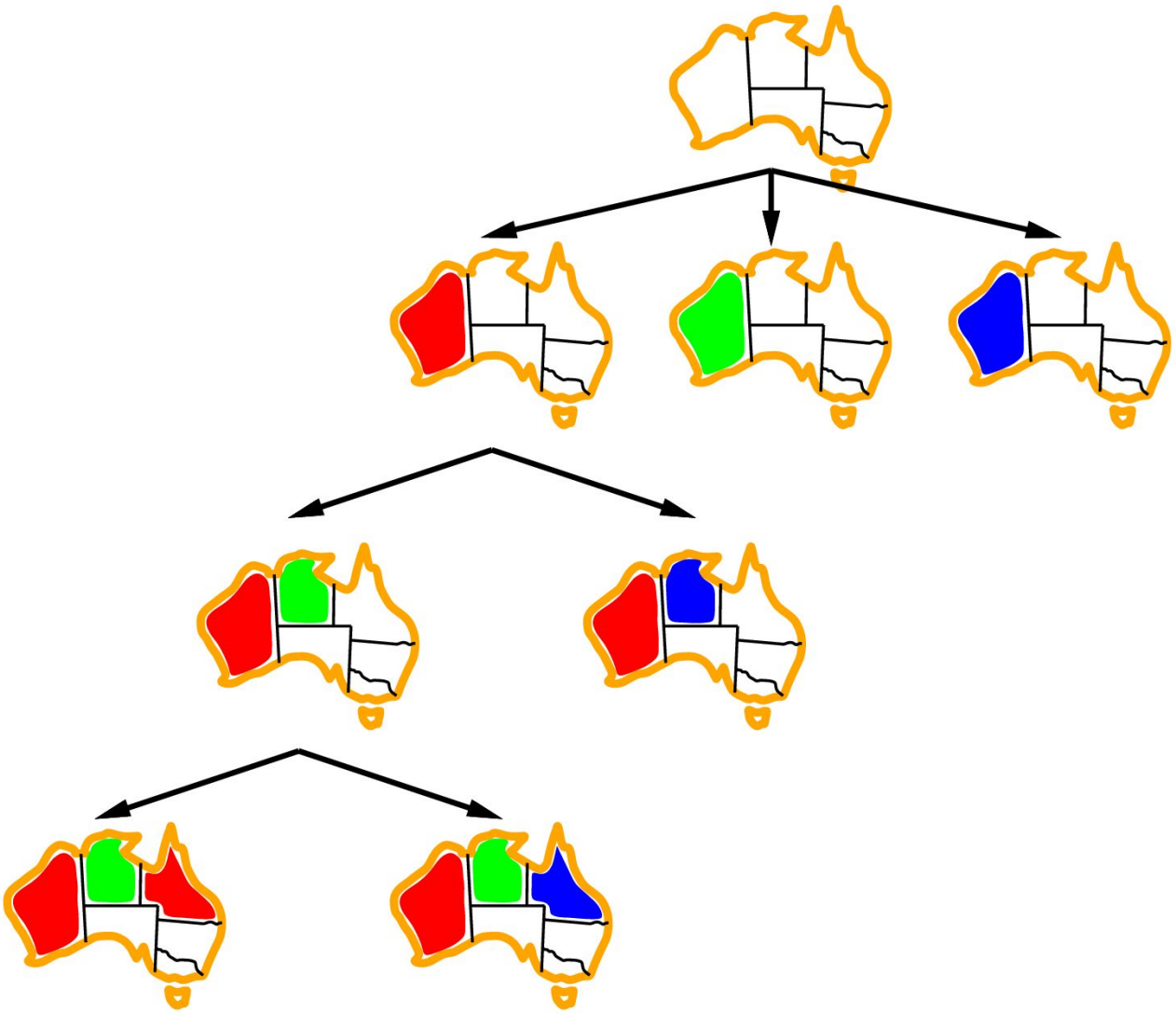

Variable choice heuristics

Minimum remaining values (MRV):

choose the variable with the fewest legal values

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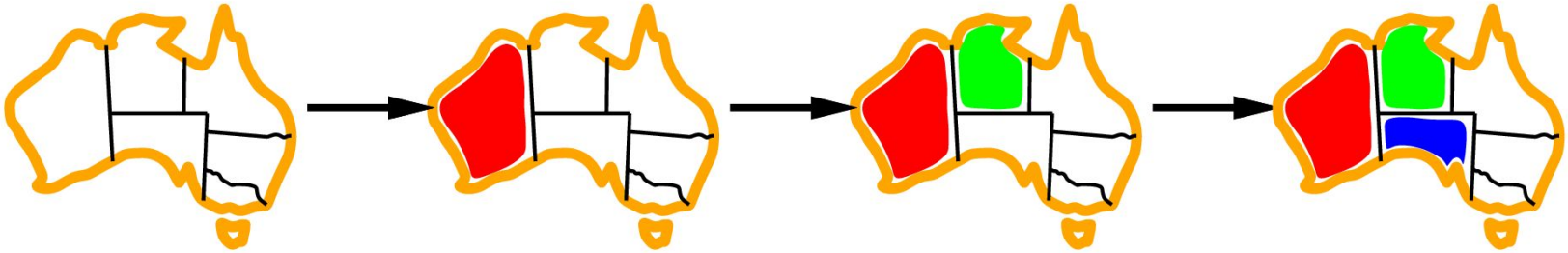

Degree heuristic:

choose the variable with the most constraints on remaining vars

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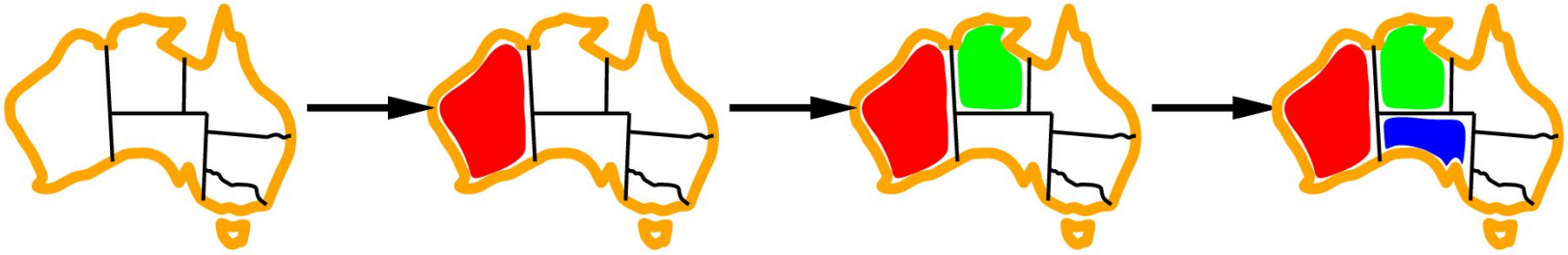

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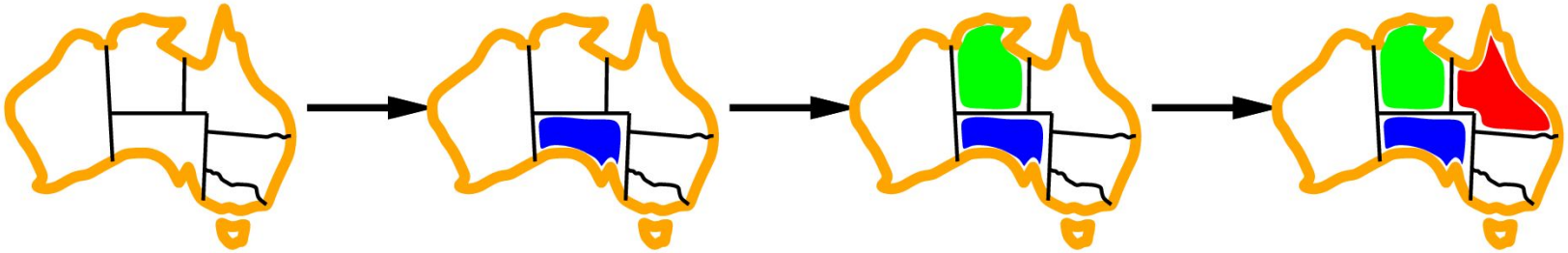

Value choice heuristics

Least constraining value:

For a given a variable, choose the least constraining value: the one that rules out the fewest values in the remaining variables

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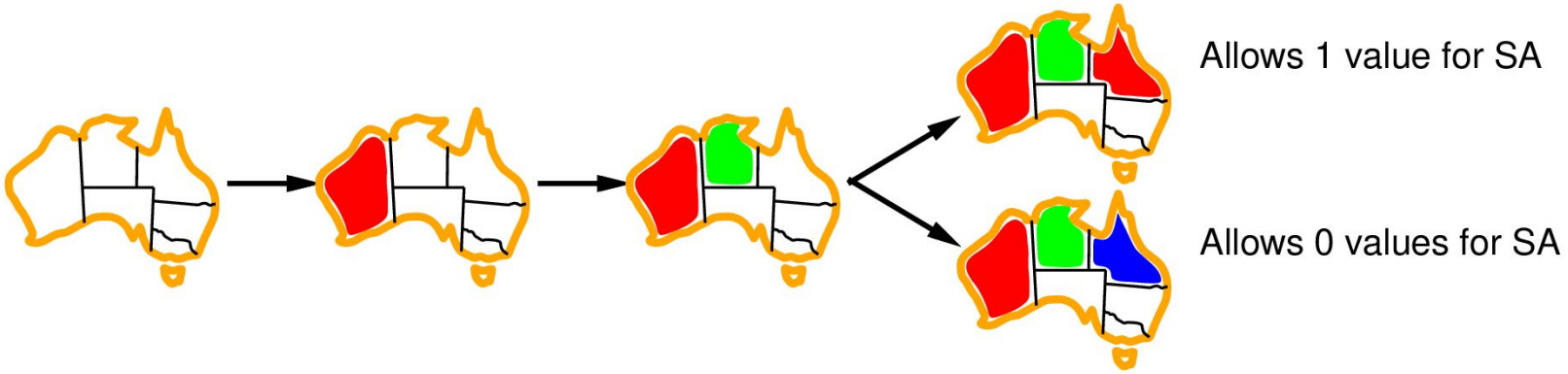
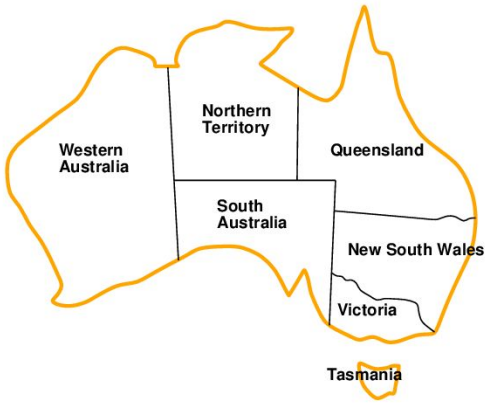


Forward checking

Idea: Keep track of remaining legal values for unassigned variables

Terminate search when any variable has no legal values



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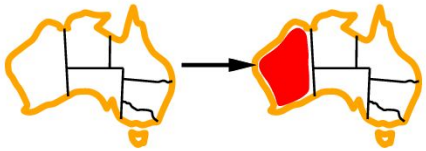

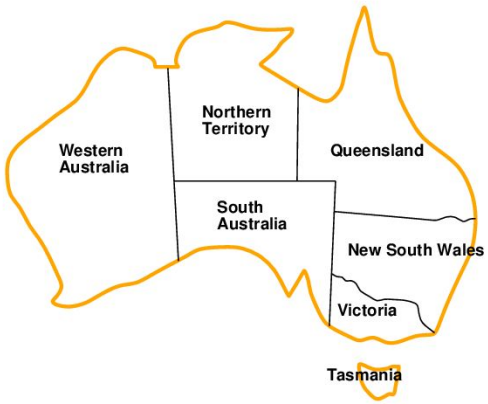

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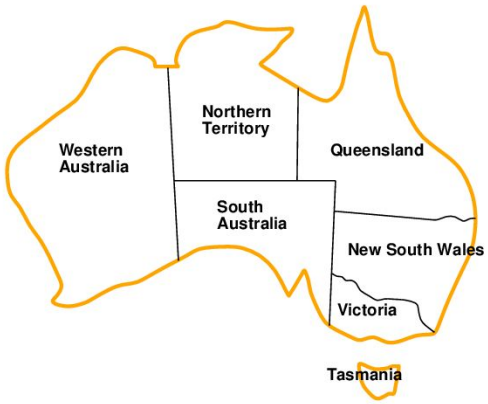

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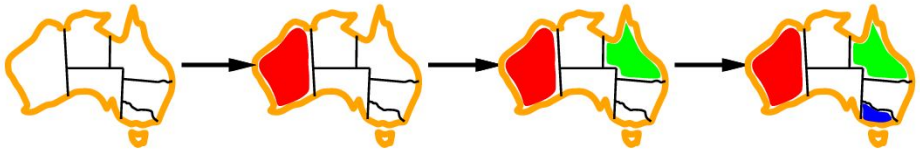
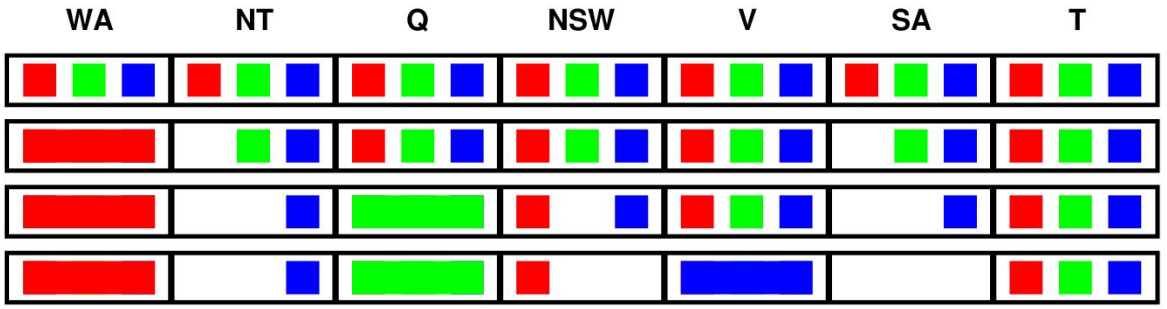
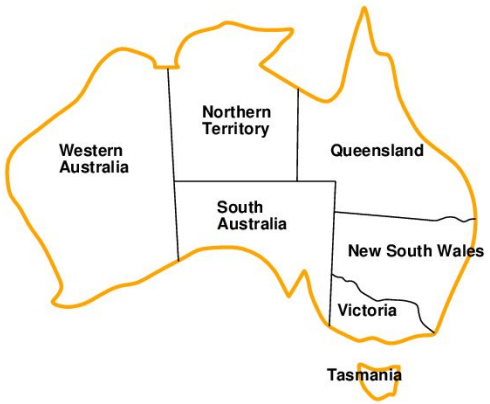
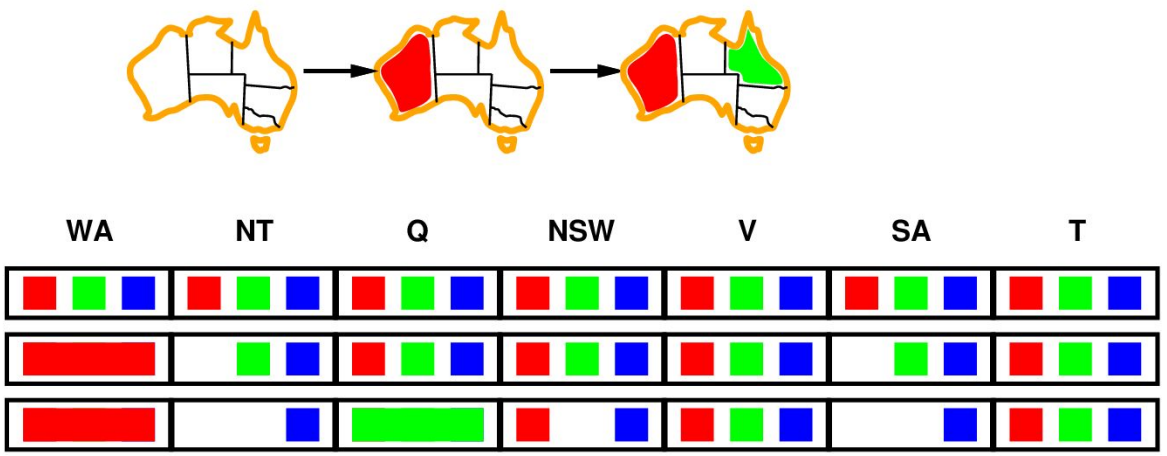






Constraint propagation

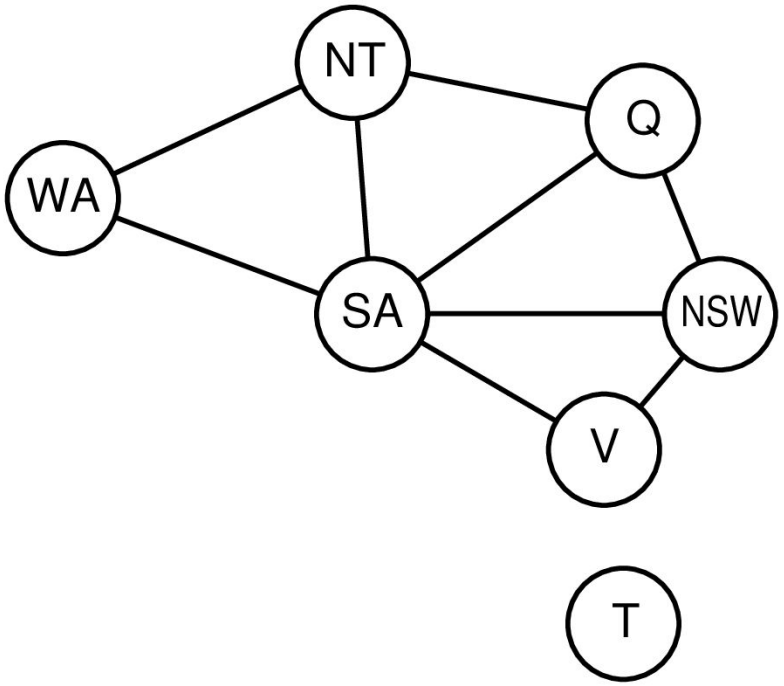
Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:



NT and SA cannot both be blue!

Constraint propagation repeatedly enforces constraints locally

Problem structure

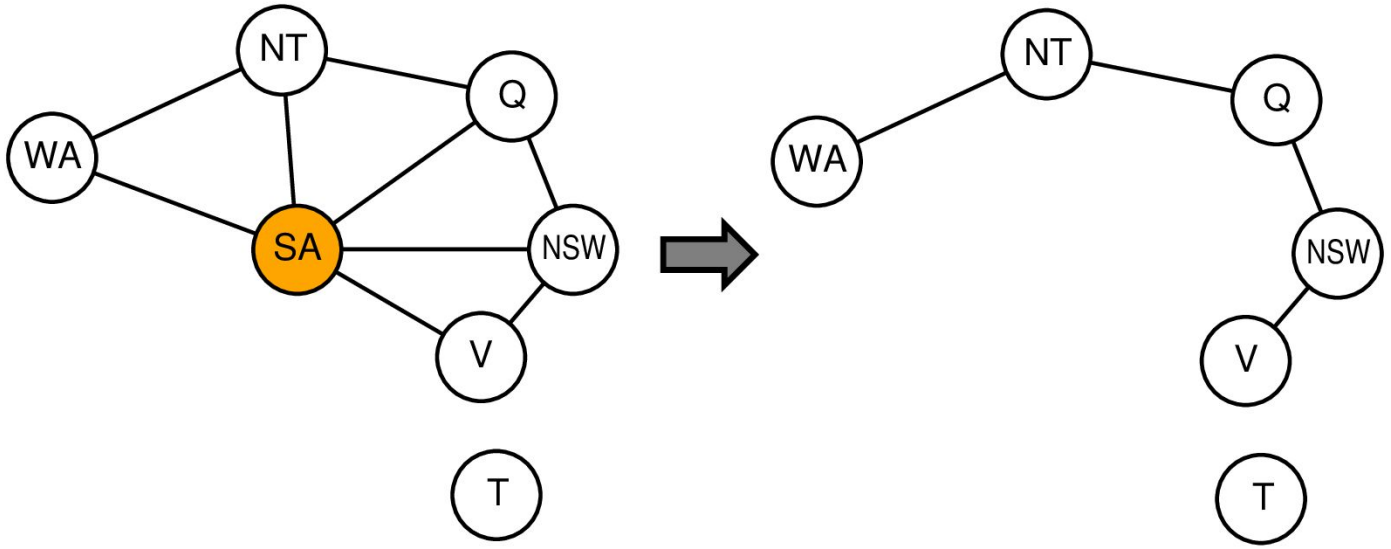


Tasmania and mainland are independent subproblems

Identifiable as connected components of constraint graph

Nearly tree-structured CSPs

Conditioning: instantiate a variable, prune its neighbors' domains



Cutset conditioning: instantiate (in all ways) a set of variables so that the remaining constraint graph is a tree

Constrained optimization problem

Constrained minimization problem

A general constrained minimization problem may be written as follows.

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) = c_i \quad \text{for } i = 1, \ldots, n \quad \text{Equality constraints} \\ & h_j(\mathbf{x}) \geqq d_j \quad \text{for } j = 1, \ldots, m \quad \text{Inequality constraints} \end{aligned}$$

- Where above two constraints are constraints (hard) that required to be satisfied.
- $f(\mathbf{x})$ is the objective function that needs to be optimized subject to the constraints.

Unconstrained optimization problem $\min_x F(x)$ or $\max_x F(x)$

Constrained minimization example

$$\min f(\mathbf{x}) = x_1^2 + x_2^4$$

subject to

$$x_1 \geq 1$$

and

$$x_2 = 1,$$

where $\mathbf{x}$ denotes the vector (x_1, x_2).

Suppose, $(x_1, x_2) = (0, 0)$ and $(x_1, x_2) = (1, 1)$.

Constraint optimization

- The constrained-optimization problem (COP) is a significant generalization of the classic constraint-satisfaction problem (CSP).

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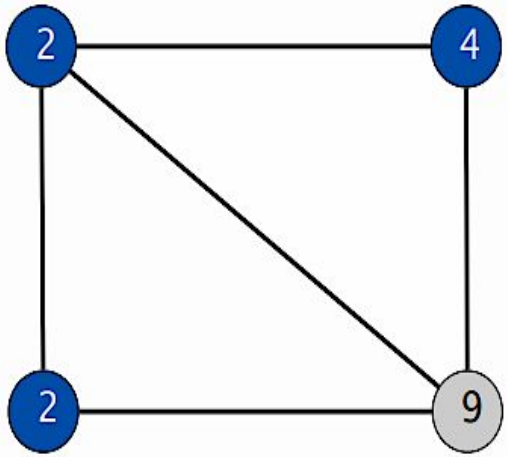
- COP is a CSP that includes an objective function to be optimized.
- COP is the process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.

Constraint optimization

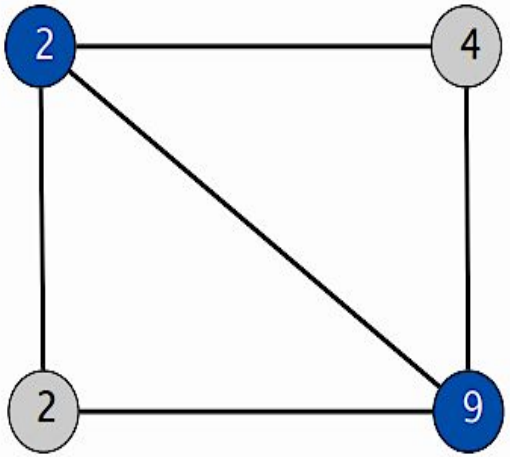
- The constrained-optimization problem (COP) is a significant generalization of the classic constraint-satisfaction problem (CSP).
- COP is a CSP that includes an objective function to be optimized.
- COP is the process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.
- The objective function is either a cost function, which is to be minimized, or a reward function or utility function, which is to be maximized.
- Typical constraints might involve time, money, and resources.

Constrained minimization example

Weighted vertex cover. Given a graph G with vertex weights, find a vertex cover of minimum weight.



weight = 2 + 2 + 4



weight = 11