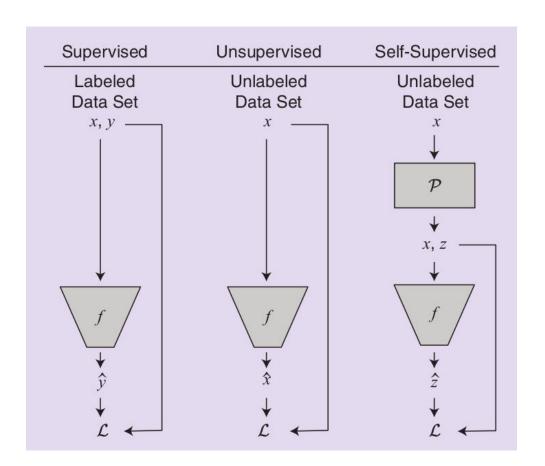
# Self-Supervised Representation Learning (SSRL)

### Overview



# Supervised Model

- Utilizes labeled dataset  $D_t$  for training a predictive model  $f(x) = g_{\phi}(h_{\theta}(x))$ .
- The model comprises a representation extractor  $h_{\theta}$  and a classifier/regression function  $g_{\phi}$ .
- Training aims to minimize a loss function  $\mathcal{L}$ , optimizing parameters  $\theta$  and  $\phi$ .

### Supervised Model

We train this predictive model by minimizing a loss function  $\mathcal{L}$ , such as the negative log likelihood

$$\underset{\theta,\phi}{\operatorname{argmin}} \sum_{\left(x_i^{(t)}, y_i^{(t)}\right) \in D_t} \mathcal{L}\left(g_\phi\left(h_\theta\left(x_i^{(t)}\right)\right), y_i^{(t)}\right). \tag{1}$$

However,  $h_{\theta}$  may have hundreds of millions of parameters, requiring millions of labeled data points in  $D_t$  to fit this correctly.

# **Unsupervised Models**

### Challenges with Large Parameter Spaces:

- $h_{\theta}$  can have hundreds of millions of parameters, requiring vast labeled datasets  $(D_t)$  for fitting.
- Labeled data  $(D_t)$  is often scarce, whereas unlabeled data  $(D_s)$  is abundant in many applications.

### Utilizing Unlabeled Data through Unsupervised Learning:

• Leverages unlabeled data  $(D_s)$  to build generative models or learn compact latent representations.

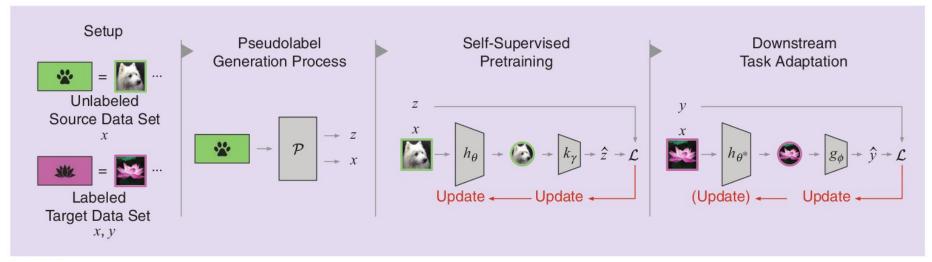
### **Unsupervised Models**

The common unsupervised methods, such as autoencoders and clustering learn compact latent representations. For example, autoencoders often optimize the following reconstruction objective:

$$\underset{\theta,\phi}{\operatorname{argmin}} \sum_{x_i^{(i)} \in D_i} \mathcal{L}\left(g_{\phi}\left(h_{\theta}\left(x_i^{(t)}\right)\right), x_i^{(t)}\right), \tag{2}$$

where  $h_{\theta}(\cdot)$  extracts a compact feature from the input, and  $g_{\phi}(\cdot)$  uses it to reconstruct the original input.

# Self-Supervised Representation Learning (SSRL)



**FIGURE 2.** The self-supervised workflow starts with an unlabeled source data set and a labeled target data set. As defined by the pretext task, pseudolabels are programmatically generated from the unlabeled set. The resulting inputs, x and pseudolabels z, are used to pretrain the model  $k_{\gamma}(h_{\theta}(\cdot))$ —composed of feature extractor  $h_{\theta}$  and output  $k_{\gamma}$  modules—to solve the pretext task. After pretraining is complete, the learned weights  $\theta^*$  of the feature extractor  $h_{\theta}$  are transferred and used together with a new output module  $g_{\phi}$  to solve the downstream target task.

### SSRL

#### Data Sets:

- $\circ$  Labeled target data set,  $D_t$ , for the specific task.
- $\circ$  Larger unlabeled source data set,  $D_s$ , readily available.

### Pseudolabeled Data Set Generation:

- $\circ$  Pretext task creates new pseudolabeled data set,  $ar{D}_s = \{x_i, z_i\}_{i=1}^M = \mathcal{P}(D_s)$ .
- $\circ$  Process  $\mathcal P$  typically involves transformation or masking parameters.
- Repeated at each training epoch's start for fresh sample generation.

### Pretext Model Training:

- Trains model  $k_{\gamma}(h_{\theta}(\cdot))$  to optimize the self-supervised objective on  $\bar{D}_s$ .
- $\circ$  Objective: Minimize the loss  $\mathcal{L}(k_{\gamma}(h_{\theta}(x_i)), z_i)$  across pseudolabeled data.
- $\circ$  Results in optimal parameter estimation  $\theta^*$  without the need for labeled data.

### Input and Pseudolabel Nature:

- $\circ$  Typically, input  $x_i$  is a single data point.
- $\circ$  Pseudolabel  $z_i$  usually represents a class label with scalar value.

### Linear classifier (on downstream dataset)

For linear readout, let  $(\theta, \gamma)$  be the weights of the pretrained model, consisting of a feature extractor,  $h_{\theta}$ , followed by a task-specific head,  $k_{\gamma}$ .

The simplest way to reuse  $h_{\theta}$  for a new task is to replace head with a new one,  $g_{\phi}$ , designed for new task.

This head is then trained with the feature extractor frozen. Given a target data set of N instances,

$$D_t = \left\{x_i^{(t)}, y_i^{(t)}
ight\}_{i=1}^N$$
 , the training objective is

$$\underset{\phi}{\operatorname{argmin}} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}\left(g_{\phi}\left(h_{\theta}\left(x_{i}^{(t)}\right)\right), y_{i}^{(t)}\right) \tag{4}$$

# Pretext tasks

# SSRL in Computer Vision

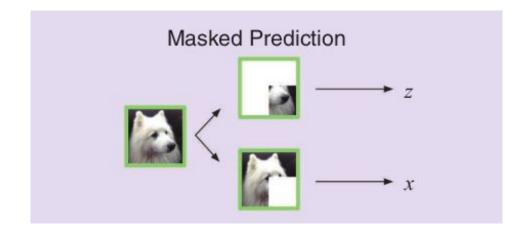
- The aim is to leverage self-supervised learning for representation learning, enabling models to learn meaningful and useful representations from unlabeled data.
- SSRL operates within the computer vision domain by utilizing freely available labels to train deep representations, thereby addressing the scalability bottleneck of supervised learning.

# SSRL in Computer Vision

- SSRL achieves this by training models on pretext tasks that do not necessitate manual annotation.
- Pretext tasks involve learning to predict certain properties or relationships within the data, such as relative position or viewing angle of training images.
- Through training on these pretext tasks, models learn to extract meaningful features from the data without explicit labels.

#### Masked Prediction:

- Model predicts missing information in input data with some elements masked.
- Minimizing reconstruction loss approach
- Examples: missing words in sentences, time slices in speech, or regions in images.



### **Masked Prediction**

# Algorithm 1. The pseudolabel generation process ${\cal P}$ for masked prediction.

```
Input: Unlabeled data set D_s = \{x_i^{(s)}\}_{i=1}^M.

for i from 1 to M do

Generate indices, I, of elements to remove from x_i^{(s)}.

z_i \leftarrow \{x_{i,j}^{(s)}: j \in I\}

x_i \leftarrow \{x_{i,j}^{(s)}: j \notin I\}

end for

Output: \{x_i, z_i\}_{i=1}^M.
```

### Generating pseudolabels for masked prediction

- Input: Start with an unlabeled data set  $D_s = \{x_i^{(s)}\}_{i=1}^M$ , where M is the number of data points.
- Process:
  - For each data point i from 1 to M:
    - Generate a set of indices I, which represent elements to be removed from  $x_i^{(s)}$ .
    - Create the pseudolabel  $z_i$  by selecting elements from  $x_i^{(s)}$  corresponding to indices in I.
    - Define the modified input  $x_i$  by excluding elements from  $x_i^{(s)}$  that are indexed by I.
- Output: Produce a set of pairs  $\{x_i, z_i\}_{i=1}^M$ , where each pair consists of the modified input and its corresponding pseudolabel.

### Objective Function Overview

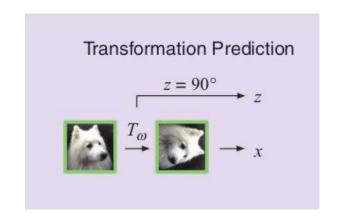
- Demonstrates using real data where a square region of an image is masked.
- Within this region, I denotes the set of indices inside the square mask.
- The masked pixels correspond to  $z_i$ , and pixels outside the masked region are  $x_i$ .
- Minimize a reconstruction loss, typically using mean square error for the masked region prediction.

$$heta^* = rgmin_{ heta, \gamma} rac{1}{|\mathcal{P}(D_s)|} \sum_{(x_i, z_i) \in \mathcal{P}(D_s)} (k_\gamma(h_ heta(x_i)) - z_i)^2.$$

 This setup aims to predict the contents of the masked region accurately leveraging the surrounding pixels' information.

#### **Transformation Prediction:**

- Model predicts applied transformation to input data, aiming for invariant representations.
- Valuable where labeled data is scarce, offering meaningful feature learning without explicit annotations.
- Examples: rotation angle of an image, original order of shuffled frames in a video.



### **Transformation Prediction**

# Algorithm 2. The pseudolabel generation process ${\mathcal P}$ for TP.

```
Input: Unlabeled data set D_s = \{x_i^{(s)}\}_{i=1}^M.

for i from 1 to M do

Sample \omega \sim \Omega

x_i \leftarrow T_\omega(x_i^{(s)})

z_i \leftarrow \omega

end for

Output: \{x_i, z_i\}_{i=1}^M.
```

### Pseudolabel generation process for Transformation Prediction (TP)

- Input: An unlabeled dataset  $D_s = \{x_i^{(s)}\}_{i=1}^M$ , which consists of M samples.
- Process:
  - $\circ$  For each sample *i* from 1 to M:
    - A transformation parameter  $\omega$  is sampled from a predefined set of transformations  $\Omega$ .
    - The transformation  $T_{\omega}$  corresponding to the sampled parameter is applied to the input sample  $x_i^{(s)}$  to generate a transformed sample  $x_i$ .
    - The transformation parameter  $\omega$  is then assigned as the pseudolabel  $z_i$  for the transformed sample  $x_i$ .
- **Output**: The algorithm outputs a set of transformed samples and their corresponding pseudolabels  $\{x_i, z_i\}_{i=1}^{M}$ .

### Learning Objective:

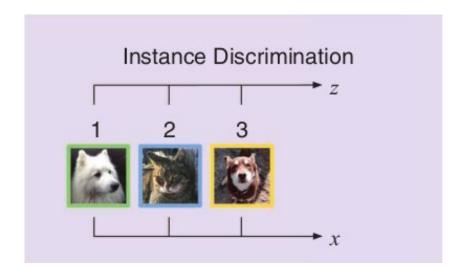
· The objective might include a cross-entropy loss for categorical transformation parameters.

$$heta^* = rgmin_{ heta, \gamma} \sum_{(x_i, z_i) \in \mathcal{P}(D_s)} \mathcal{L}_{CE}(k_\gamma(h_ heta(x_i)), z_i).$$

• The full process involves generating several views of each  $x_i^{(s)}$ , each with a different set of transformation parameters.

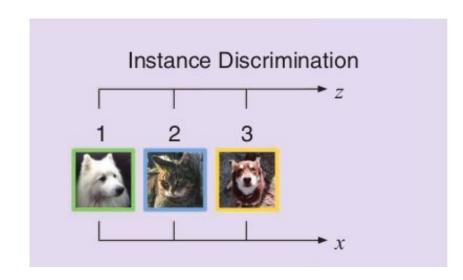
#### Instance Discrimination:

 Model treats each instance in the dataset as its own class and learns to discriminate between them.



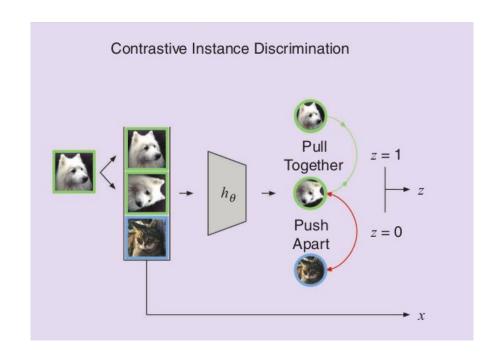
#### **Instance Discrimination:**

- Cross Entropy: Assign one-hot encoding of class label, train with categorical cross-entropy loss.
- Contrastive Learning: Train model to predict
  whether pairs of inputs belong to same or different
  classes, pulling positive pairs closer and pushing
  negative pairs apart.
- Regularization-based Approaches: Use regularization techniques to prevent feature collapse without negative examples.



#### Contrastive Instance Discrimination:

- Instance discrimination that leverages contrastive loss.
- Pulls together same-instance vectors and pushes apart different-instance vectors.



### **Instance Discrimination**

# Algorithm 3. The pseudolabel generation process ${\mathcal P}$ for contrastive-instance discrimination.

```
Input: Unlabeled data set D_s = \{x_i^{(s)}\}_{i=1}^M.
  for i from 1 to M do
       Sample x^a \sim T(x_i^{(s)})
       Sample x^+ \sim T(x_i^{(s)})
       for k from 1 to K do
            Sample i \sim \mathcal{U}(1, M)
                                                               ▶ Pick another raw input.

    Get a random transform

            Sample x_k^- \sim T(x_i^{(s)})
       end for
        x_i \leftarrow \{(x^a, x^+), (x^a, x_1^-), ..., (x^a, x_K^-)\}.
        z_i \leftarrow \{1,0,...,0\}.
   end for
Output: \{x_i, z_i\}_{i=1}^M.
```

#### Pseudolabel Generation for Contrastive-Instance Discrimination

- Input: An unlabeled data set  $D_s = \{x_i^{(s)}\}_{i=1}^M$  .
- Process for each data point i from 1 to M:
  - Generate a transformed version of  $x_i$ , labeled  $x^a$ , using a transformation T.
  - $\circ$  Generate another transformed version of  $x_i$ , labeled  $x^+$ , with the same transformation T.
  - For k from 1 to K (number of negative samples):
    - Randomly select a different data point j from the data set.
    - Generate a transformed version of  $x_j$ , labeled  $x_k^-$ , with the transformation T.
- Create a tuple of transformed versions for  $x_i$ :  $\{(x^a, x^+), (x^a, x_1^-), \ldots, (x^a, x_K^-)\}$ .
- Assign a pseudolabel  $z_i$  with '1' for the positive pair  $(x^a, x^+)$  and '0's for all negative pairs  $(x^a, x_k^-)$ .
- Output: A set of tuples  $\{(x_i, z_i)\}_{i=1}^M$ , each containing an anchor, a positive, and K negative samples with corresponding pseudolabels.

### **Objective Function**

#### Feature Extraction:

 $\circ$  Samples are encoded by a feature extractor to obtain representations:  $r^a=h_ heta(x^a)$ ,  $r^+=h_ heta(x^+)$ ,  $r^-_i=h_ heta(x^-_i)$ .

### Similarity Function:

 $\circ$  A similarity function  $\Phi$  measures the similarity between positive pairs (the anchor with a positive sample) and negative pairs (the anchor with a negative sample).

#### Contrastive Loss:

$$\mathcal{L}_{ ext{con}} = -\mathbb{E} \left[ \log rac{\Phi(r^a, r^+)}{\Phi(r^a, r^+) + \sum_{j=1}^k \Phi(r^a, r_j^-)} 
ight],$$

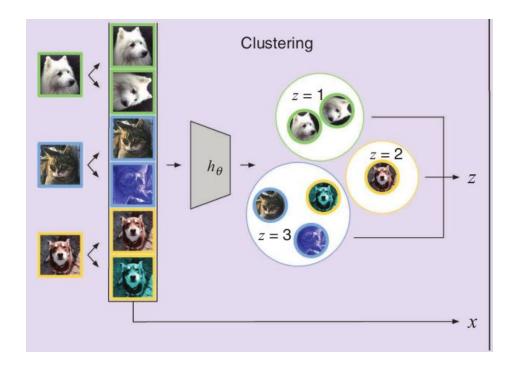
where k different negative samples are contrasted with the anchor.

### Model Update:

$$heta^* = rgmin_{ heta, \gamma} \sum_{(x_i, z_i) \in \mathcal{P}(D_x)} \mathcal{L}_{ ext{con}}(k_{\gamma}(h_{ heta}(x_i)), z_i).$$

### Clustering:

- Model divides training data into groups with high intra-group similarity and low inter-group similarity.
- Methods involve joint learning of feature extractors and clustering.
- Goal: Find meaningful similarities for grouping instances.



# Clustering

# Algorithm 4. The pseudolabel generation process ${\mathcal P}$ for clustering.

```
Input: Unlabeled data set D_s = \{x_i^{(s)}\}_{i=1}^M.
Input: Representations \{r_i\}_{i=1}^M, where r_i \leftarrow h_{\theta}(x_i^{(s)})
Input: Cluster centers \{c_j\}_{j=1}^k, via clustering on \{r_i\}_{i=1}^M.
   for i from 1 to M do
        Sample x_i \sim T(x_i^{(s)})
         z_i \leftarrow \operatorname{argmin}_{j \in [k]} \| c_i - r_i \|
   end for
Output: \{x_i, z_i\}_{i=1}^M.
```

### Pseudolabel generation process for Clustering

#### Inputs:

- $\circ$  Unlabeled data set  $D_s = \{x_i^{(s)}\}_{i=1}^M$ .
- Representations  $\{r_i\}_{i=1}^M$ , where each  $r_i$  is derived from  $h_{\theta}(x_i^{(s)})$ .
- $\circ$  Cluster centers  $\{c_j\}_{j=1}^k$ , obtained through clustering on the representations  $\{r_i\}$ .

#### Process:

- For each data point  $x_i^{(s)}$  in  $D_s$ :
  - Transform  $x_i^{(s)}$  using a transformation function T to get a sample  $x_i$ .
  - Assign a pseudolabel  $z_i$  by finding the nearest cluster center  $c_j$  to the representation  $r_i$ .

#### Output:

 $\circ$  The transformed samples and their corresponding pseudolabels  $\{x_i,z_i\}_{i=1}^{M}$  .

### Objective Function for Clustering

- Involves assigning each input  $x_i$  to a cluster class  $z_i$ .
- The model is optimized using a cross-entropy loss based on these cluster assignments.
- Cross-Entropy Loss Optimization:

$$heta^* = rgmin_{ heta, \gamma} \sum_{(x_i, z_i) \in \mathcal{P}(D_s)} \mathcal{L}_{CE}(k_{\gamma}(h_{ heta}(x_i)), z_i).$$

- After optimizing with cross-entropy loss, the process returns to the clustering step.
- Uses newly updated model representations for reassigning cluster classes, facilitating an iterative refinement of both model parameters and cluster assignments.