Discrete Time Signals (2)

- **In case of Image, a spatial sampling is used in both horizontal and vertical directions. In this case, the horizontal and vertical sampling frequencies are in the <u>cycles per unit of distance</u> in the direction defining the two sampling periods.**
- If n^{th} sample value is complex for one or more value of n, then it will be complex sequence

$$x[n] = x_{real}[n] + jx_{\mathrm{Im}aginary}[n]$$

For Real sequence, $x_{\mathrm{Im}aginary}[n] = 0$

For Imaginary sequence, $x_{real}[n] = 0$

Discrete Time Signals (2)

- $\{x[n]\} = \{\cos 0.3n\}$ is a real sequence
- $\{y[n]\} = \{e^{j0.3n}\}$ is a complex sequence
- We can rewrite

$$\{y[n]\} = \{\cos 0.3n + j\sin 0.3n\} = \{\cos 0.3n\} + j\{\sin 0.3n\}$$

where

$$\{y_{re}[n]\} = \{\cos 0.3n\}$$
$$\{y_{im}[n]\} = \{\sin 0.3n\}$$

An important difference – frequency range

- Consider a frequency $(\omega_0 + 2\pi)$

$$x[n] = Ae^{j(\omega_0 + 2\pi)n} = Ae^{j\omega_0 n}\underbrace{e^{j2\pi n}}_{=1} = Ae^{j\omega_0 n}$$

- More generally $(\omega_0 + 2\pi k), \quad k \varepsilon I$

$$x[n] = Ae^{j(\omega_0 + 2\pi k)n} = Ae^{j\omega_0 n}\underbrace{e^{j2\pi kn}}_{=1} = Ae^{j\omega_0 n}$$

- Same for sinusoidal sequences

$$x[n] = A\cos[(\omega_0 + 2\pi k)n + \phi] = A\cos(\omega_0 n + \phi)$$

- So, only consider frequencies in an interval of 2π such as

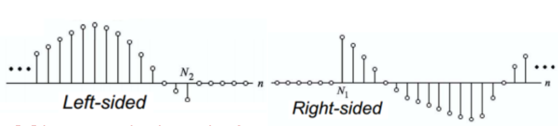
$$-\pi < \omega_0 \le \pi \quad or \quad 0 \le \omega_0 < 2\pi$$

Length of DT Signal (1)
Types of Sequences: Left-and right-sided

- Length of DT sequences: finite or infinite
- Finite-duration or finite-length sequence:
  - Defined in the interval $N_1 \le n \le N_2$, where $N_2 \ge N_1$ are finite
  - Length (duration): $N = N_2 - N_1 + 1$
  - In practical applications the sequences are finite
  - A length-N sequence is referred as an N-point sequence.
  - The Length of a finite-length sequence can be increased by zero-padding, i.e., by appending it with zeros

Length of DT Signal (2)
Types of Sequences: Left-and right-sided

- Infinite-duration or infinite-length sequence:
  - May have a left or right end or neither
  - A right-sided sequence x[n] = 0 for $n<N_1$. If $N_1 \ge 0$, a RSS is called a causal sequence.
  - A left-sided sequence x[n] = 0 for $n>N_2$. If $N_2<0$, a LSS is called an anti-causal sequence.



x[n] has zero-valued samples for $n>N_2$

x[n] has zero-valued samples for $n<N_1$

Length of DT Signal (3):Example

- $x[n] = n^2$, $-3 \le n \le 5$ is a finite-length sequence of length
- y[n] = cos 0.4n is an infinite-length sequence

Size of a DT Signal (1)

- The strength of a discrete-time signal is given by its norm. The L_p norm is defined as

$$L_p\text{-norm}: \quad \|x\|_p = \left(\sum_{n=-\infty}^{\infty} |x[n]|^p\right)^{1/p}, \qquad p \varepsilon I^+$$

- Typical values for p are p = 1, p = 2, and p = ∞.

$$\|x\|_1 = \left(\sum_{n=-\infty}^{\infty} |x[n]|\right)$$ mean absolute value of {x[n]}
Mean absolute value of a length-N sequence $\|x\|_1/N$

$$\|x\|_2 = \left(\sum_{n=-\infty}^{\infty} |x[n]|^2\right)^{1/2}$$ Root mean squared (RMS) value of {x[n]}
$\|x\|_2/\sqrt{N}$ RMS value of a length-N sequence.

$$\|x\|_\infty = \left(\sum_{n=-\infty}^{\infty} |x[n]|^\infty\right)^{1/\infty} = 1$$ Peak absolute value of {x[n]}

$$\|x\|_\infty = |x|_{\max}$$

Size of a DT Signal (2): Example

- Let y[n], 0≤n≤N-1, be an approximation of x[n], 0≤n≤N-1
- An estimate of the relative error is given by the ratio of the L_2-norm of the difference signal and the L_2-norm of x[n]:

$$E_{rel} = \left(\frac{\sum_{n=0}^{N-1} |y[n]-x[n]|^2}{\sum_{n=0}^{N-1} |x[n]|^2}\right)^{1/2}$$

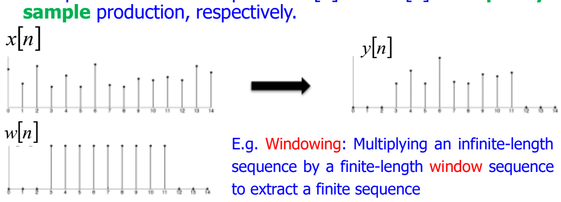
Operations on Sequences (1): Basic Operations

- **Product (Modulation)** operation:
  - Modulator

$$x[n] \longrightarrow \otimes \longrightarrow y[n], \quad w[n] \uparrow$$

$$y[n] = x[n] \times w[n]$$

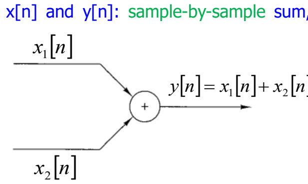
- The product of two sequences x[n] and w[n]: **sample-by-sample** production, respectively.



E.g. Windowing: Multiplying an infinite-length sequence by a finite-length window sequence to extract a finite sequence

Operations on Sequences (2): More operations

- The sum of two sequences x[n] and y[n]: sample-by-sample sum, respectively.
- **Addition** operation:
  - Adder



sequence

$x_1[0]$ $x_1[2]$

$x_1[n] = \{4, 5, -3\}$ at n=0 n=2

$x_2[n] = \{-2, 4, 3\}$ n=0 n=2

y[0] y[2]

$\therefore y[n] = \{2, 9, 0\}$ n=0 n=1 n=2

Operations on Sequences (2): More operations

- Multiplication of a sequence x[n] by a number α: **multiplication of each sample value by α.**
- **Scaling** operation:
  - Scalar

$$x[n] \longrightarrow \triangleright^{a} \longrightarrow y[n] = ax[n]$$

sequence

$x[n] = \{2, -5, 6, 8\}$; $a = 0.1$; $y[n] = \{0.2, -0.5, 0.6, 0.8\}$

at n=0 n=3 at n=0 n=3

Operations on Sequences (3): Time shifting

- **Time-shifting** operation: $y[n] = x[n-N], \quad N \in I$
- If N> 0, it is delaying operation
  - Unit delay

$$x[n] \longrightarrow \boxed{z^{-1}} \longrightarrow y[n]$$

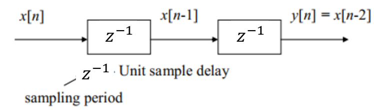
$$y[n] = x[n-1]$$

- If N< 0, it is an advance operation
  - Unit advance

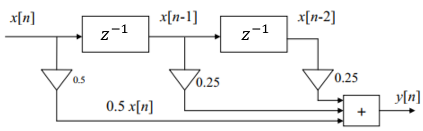
$$x[n] \longrightarrow \boxed{z} \longrightarrow y[n] \qquad y[n] = x[n+1]$$

Operations on Sequences (3): Time shifting

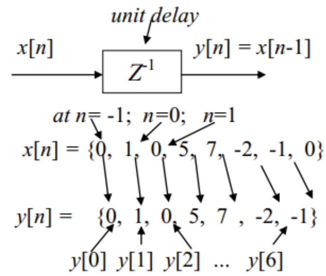
(a) $y[n] = x[n-2]$



(b) $y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-1] + \frac{1}{4}x[n-2]$



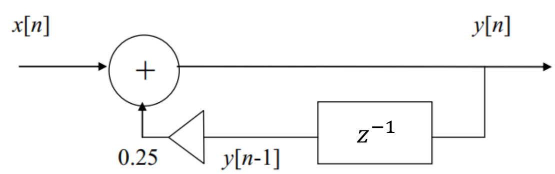
Operations on Sequences (3): Time shifting



$\{ y[n] = x[n-1]; y[0]=x[0-1]=x[-1]; y[1] = x[1-1]=x[0];\ldots\}$

Operations on Sequences (3): Time shifting

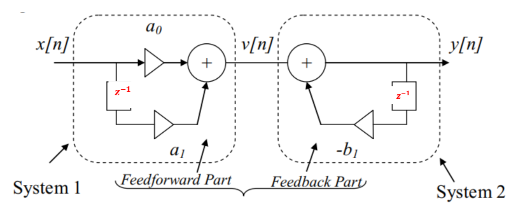
(c) $y[n] = x[n] + 0.25y[n-1]$



Operations on Sequences (3): Time shifting

$y[n] = a_0x[n] + a_1x[n-1] - b_1y[n-1]$

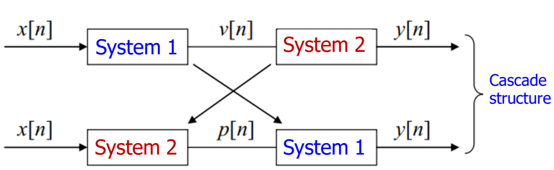
- System Implementation for the Difference equation



- Direct Form I structure

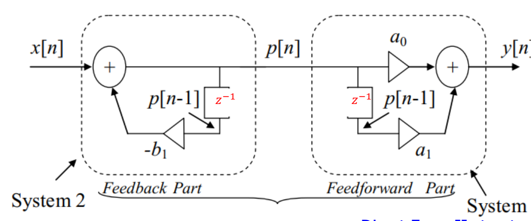
$$v[n] = a_0x[n] + a_1x[n-1] \quad \text{- system 1}$$
$$y[n] = v[n] - b_1y[n-1] \quad \text{- system 2}$$

Operations on Sequences (3): Time shifting



- Without changing the input-output relationship, we can reverse the ordering of the two systems in the cascade representation.

Operations on Sequences (3): Time shifting

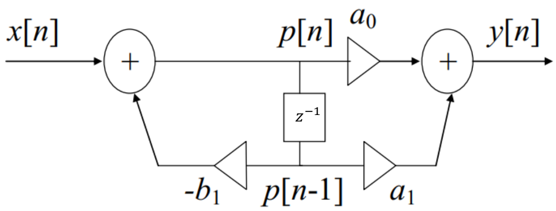


Direct Form II structure

$$p[n] = x[n] - b_1p[n-1]$$

$$y[n] = a_0p[n] + a_1p[n-1]$$

Operations on Sequences (3): Time shifting



Canonical form: DF II

- There is no need for two delay operations; they can be combined into a single delay.
- Since delay operations are implemented with memory in a computer, would require less memory compared to the implementation of Figure.

Operations on Sequences (4): Time reversal

- **Time-reversal (folding)** operation:

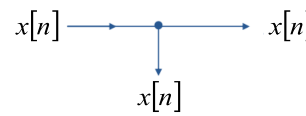
$$y[n] = x[-n]$$

- **Example:**

$$\{g[n]\} = \left\{0, 1+j4, -2+j3, \underset{\uparrow}{4-j2}, -5-j6, -j2, 3\right\}$$

$$\{g[-n]\} = \left\{3, -j2, -5-j6, \underset{\uparrow}{4-j2}, -2+j3, 1+j4, 0\right\}$$

- **Branching operation:** Used to provide multiple copies of a sequence



Sampling Rate Alteration (1): Basic Operations

- Certain operations change the effective sampling rate of sequences by adding or removing samples
- A process to generate a new sequence y[n] with a sampling F'_s rate higher or lower than that of the sampling rate F_s of a given sequence x[n]
- Sampling rate alteration: Number of samples/sec.

$$R = \frac{F_s'}{F_s}$$

  - Up-sampling = adding more samples = interpolation (R>1)
  - Down-sampling = discarding samples = decimation (R<1)

Sampling Rate Alteration (2): Down Sampling

- In **down-sampling** by an integer factor M > 1, every M^th sample of the input sequence is kept and M-1 in-between samples are removed:

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow x_d[n]$$

$$x_d[n] = x[nM]$$

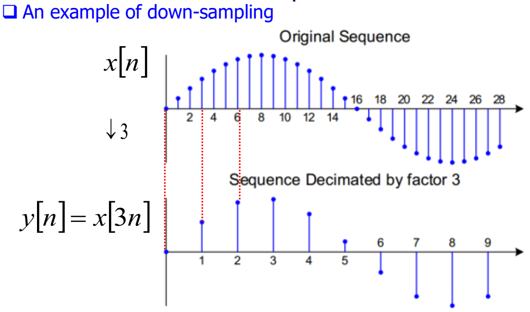
Sampling Rate Alteration (3): Down Sampling Example

- x[n] = [1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0], find the resultant signal when it is down sampled by 3.

- Solution:
  x[n] = [1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0]
  (remember down sampling by 3 means 3-1 = 2 sample points are deleted)
- x_d[n] = [1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0]
  x_d[n] = [1 0 1 1 0 0 1]

Sampling Rate Alteration (5): Down Sampling Example

- An example of down-sampling



$$y[0] = x[0], \quad y[1] = x[3], \quad y[2] = x[6], \text{ etc}$$

Sampling Rate Alteration (6): UpSampling

- Up-sampling is the converse of down-sampling: L-1 zero values are inserted between each pair of original values

$$x_u[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots \\ 0, & otherwise \end{cases}$$

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow x_u[n]$$

Sampling Rate Alteration (7): UpSampling Example

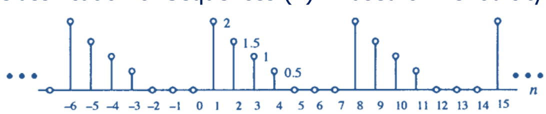
- x[n] = [1 0 1 1 0 0 1], find the resultant signal when it is up sampled by 3.

- Solution:
  x[n] = [1 0 1 1 0 0 1]

  (remember up sampling by 3 means 3-1 = 2 zeros are inserted between the original sample points)

- x_u[n] = [1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 1 0 0]

Classification of Sequences (1): Based on Periodicity



- Periodicity: $x_p[n] = x[n + kN], \quad N \varepsilon I^+ \quad k \varepsilon I \quad \forall n$
- The fundamental period $\mathbf{N_f}$ is the smallest value of **N**, which satisfies the above equation.
- The periodicity of sinusoidal sequence

$$x(n) = A\cos(\omega_0 n + \phi)$$

$$x(n+N) = A\cos(\omega_0 n + \omega_0 N + \phi)$$

If $\omega_0 N = 2\pi k \quad or \quad \frac{\omega_0}{2\pi} = \frac{k}{N} \qquad N, k$: any integer

$x[n]$ is a periodic sequence and its period is

$$N = \frac{2\pi k}{\omega_0} \qquad N_{\min} = \frac{2\pi}{\omega_0}$$

- For the complex exponential, this condition becomes

$$Ae^{j\omega_0 n} = Ae^{j\omega_0 (n+N)} = Ae^{j(\omega_0 n + \omega_0 N)}$$

which requires $\mathbf{\omega_0 N = 2\pi k}$ for some integer k

- The ratio of the given normalized frequency $\mathbf{\omega_0}$ and **2π** must be a rational number. $\frac{\omega_0}{2\pi} = \frac{k}{N}$ k → arbitrary integer

- So

$$x_1[n] = \cos(n\pi/4), \quad period \quad N = 8$$
$$x_2[n] = \cos(3n\pi/8), \quad period \quad N = 16$$

Increasing frequency → increasing period!

- For the discrete-time sinusoidal signal

$$x[n] = A\cos(\omega_0 n + \phi),$$

as ω_0 ↑ from 0 to 2π, x[n] oscillates more and more rapidly.

as ω_0 ↑ from π to 2π, the oscillations become slower.

- Ex.: $x_1[n] = \cos(\omega_o n + \phi) \qquad x_2[n] = \cos(\omega_o(n+N) + \phi)$

$$x_2[n] = \cos(\omega_o n + \phi)\cos\omega_o N - \sin(\omega_o n + \phi)\sin\omega_o N$$
$$= \cos(\omega_o n + \phi) = x_1[n] \quad \textbf{iff} \quad \sin\omega_o N = 0 \ \text{ and } \ \cos\omega_o N = 1$$

These two conditions are met if $\omega_0 N = 2\pi k \quad or \quad \frac{\omega_0}{2\pi} = \frac{k}{N}$

- $x[n] = \cos(0.3\pi n)$

$$\Rightarrow \omega_0 = 0.3\pi$$
$$\Rightarrow \frac{\omega_0}{2\pi} = \frac{0.3\pi}{2\pi} = \frac{3}{20} = \frac{k}{N_0}$$
$$\Rightarrow N_0 = 20 \ \text{samples}$$

Rational number
i.e. Periodic Signal

- $x[n] = \cos(0.5n)$

$$\Rightarrow \omega_0 = 0.5$$
$$\Rightarrow \frac{\omega_0}{2\pi} = \frac{0.5}{2\pi} = \frac{1}{4\pi}$$

Not a Rational number
i.e. Aperiodic Signal

- Sum of two or more periodic sequence → is a periodic sequence
- If $x_a[n]$ and $x_b[n]$ are two periodic sequences with fundamental periods N_a and N_b, then the sequence **$y[n] = x_a[n] + x_b[n]$** is a periodic sequence with fundamental period

$$N = \frac{N_a N_b}{GCD(N_a, N_b)}$$

where GCD is the greatest common divisor of N_a and N_b.

(b) Here we have the sum of two periodic signals,

$$x(n) = \cos(n\pi/12) + \sin(n\pi/18)$$

with the period of the first signal being equal to $N_1 = 24$, and the period of the second, $N_2 = 36$. Therefore, the period of the sum is

$$N = \frac{N_1 N_2}{\gcd(N_1, N_2)} = \frac{(24)(36)}{\gcd(24, 36)} = \frac{(24)(36)}{12} = 72$$

- Notice: Sampling of a periodic CT signal does not guarantee the periodicity of the sampled sequence

- If x[n] is a periodic, complex-valued signal, its real and imaginary parts are also periodic. That is,
- If $x[n] = x_{re}[n] + j\, x_{im}[n]$ is periodic, then $x_{re}[n]$ and $x_{im}[n]$ are periodic.
- Their fundamental periods are related by

$$N = LCM(N_{re}, N_{im})$$

where N_{re} and N_{im} are the fundamental periods of real and imaginary parts of complex-valued signal x[n]

Convolution Sum

- Convolution is the process by which an input interacts with an LTI system to produce an output

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

- Convolution between of an input signal x[n] with a system having impulse response h[n] is given as,

4.Sum 2.Shift

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k]$$

3. Multiply 1.Flip

$$= x[n] * h[n] \qquad , -\infty \le n \le \infty$$

- .

x[k] = [3 1 2] h[k] = [3 2 1]

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

Hint: The value of k starts from **(– length of h + 1)** and continues till **(length of h + length of x – 1)**

Here ***k*** starts from **-3 + 1 = -2** and continues till **3 + 3 – 1 = 5**

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 3 x 1 = 9

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 3 x 1 = 9

y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 3 x 1 = 9

y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

y[3] = 1 x 1 + 2 x 2 = 5

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 3 x 1 = 9

y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

y[3] = 1 x 1 + 2 x 2 = 5

y[4] = 2 x 1 = 2

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 3 x 1 = 9

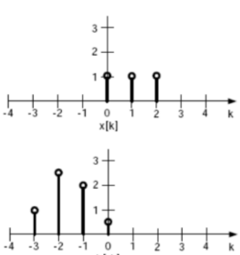
y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

y[3] = 1 x 1 + 2 x 2 = 5

y[4] = 2 x 1 = 2

y[5] = 0 (no overlap)

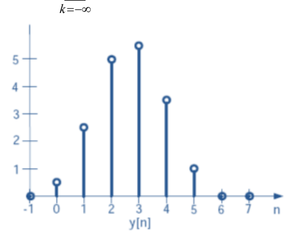
$$\boxed{y[n]=\{9\ \ 9\ \ 11\ \ 5\ \ 2\ \ 0\}}$$

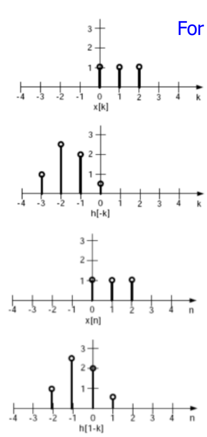


$$x[n]=\{\underset{\uparrow}{1},1,1,0,0,0\}$$

$$h[n]=\{\underset{\uparrow}{0.5},2,2.5,1\}$$

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=x[n]*h[n]$$





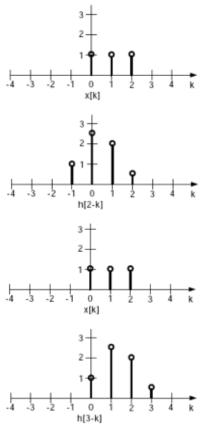
For n=0 y[0] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]

y[0] = 0.5

For n=1:

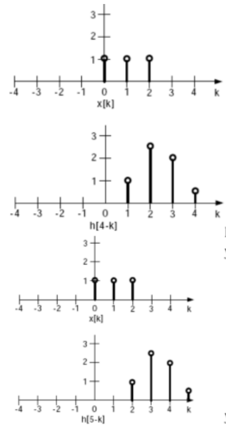
y[1] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]

y[1] = 2 + 0.5 = 2.5



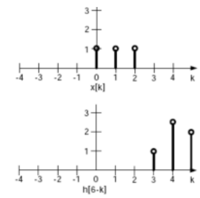
For n=2:
y[2] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]
y[2] = 2.5 + 2 + 0.5 = 5

For n=3:
y[3] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]
y[3] = 1+ 2.5 + 2 = 5.5



For n=4:
y[4] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]
y[4] = 1+ 2.5 = 3.5

For n=5:
y[5] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]
y[5] = 1

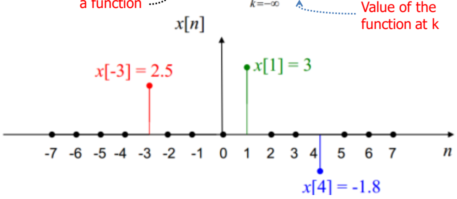


For n=6:
y[6] = h[-4] x[-4] +
h[-3] x[-3] +
h[-2] x[-2] +
h[-1] x[-1] +
h[-0] x[-0] +
h[1] x[1] +
h[2] x[2] +
h[3] x[3] +
h[4] x[4]
y[6] = 0

Impulse Representation of Sequences

- An arbitrary sequence x[n] can be expressed as a superposition of scaled versions of shifted unit impulses, δ[n-k]

A sequence, a function $\rightarrow$ $x[n]=\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$ Value of the function at k



n	x[n]
-4	0
-3	2.5
-2	0
-1	0
0	0
1	3
2	0
3	0
4	-1.8
5	0
6	0

$$x[n]=\boxed{x[-3]\delta[n+3]}+\boxed{x[1]\delta[n-1]}+\boxed{x[4]\delta[n-4]}$$

x[n] = 2.5 δ[n + 3] + 3 δ[n-1] – 1.8 δ[n-4]
= {2.5, 0, 0, <u>0</u>, 3, 0, 0, -1.8} Underlined value at the origin

Properties of LTI systems

- Defined by the impulse response
  - Stable: Bounded-input bounded-output (BIBO) stable

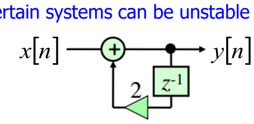
$$S=\sum_{n=-\infty}^{\infty}\left|h[n]\right|<\infty$$

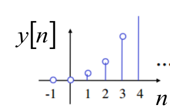
For every bounded input $x[n]<B_x \quad \forall n$

output is also subject to a finite bound, $y[n]<B_y \quad \forall n$

  - causality $h[n]=0, \quad n<0$

- Certain systems can be unstable e.g.





Output grows without limit in some conditions

Properties of LSI systems

- LSI System output y[n], input x[n] convolved with impulse response h[n] → h[n] completely describes system

- Defined by discrete-time convolution
  - Commutative

$$x[n]*h[n]=h[n]*x[n]$$

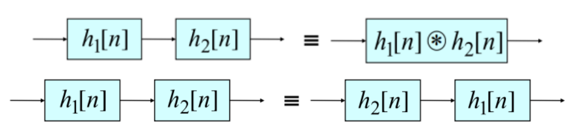
  - Linear

$$x[n]*\left(h_1[n]+h_2[n]\right)=x[n]*h_1[n]+x[n]*h_2[n]$$

Properties of LSI systems (Cont'd)

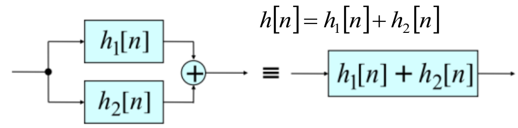
- Cascade connection (Associative property)

$$h[n]=h_1[n]*h_2[n]$$



- Parallel connection (Distributive property)

$$h[n]=h_1[n]+h_2[n]$$



Important Note

- Identity and Shifting Property
  - Unit sample sequence δ[n] is the identity element for convolution i.e.

$$y[n]=x[n]*\delta[n]=x[n]$$

  - On shifting δ[n] by k, the convolution sequence is also shifted by k, i.e.

$$y[n]=x[n]*\delta[n-k]=x[n-k]$$

- Example

$$y[n]=\delta[n-2]*\delta[n-1]=\delta[(n-2)-1]=\delta[n-3]$$

Drill Problem

- Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses

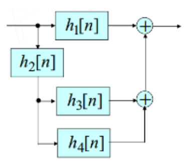
$$h_1[n]=\left(\frac{1}{2}\right)^n u[n],\quad h_2[n]=\left(\frac{1}{4}\right)^n u[n]$$

$$h[n]=h_1[n]*h_2[n]=\sum_{k=-\infty}^{\infty}h_1[k]h_2[n-k]$$

$$h[n]=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k\left(\frac{1}{4}\right)^{n-k}=\left(\frac{1}{4}\right)^n\sum_{k=0}^{\infty}2^k$$

$$=\left(\frac{1}{2}\right)^n\left[2-\left(\frac{1}{2}\right)^n\right],n\geq 0$$

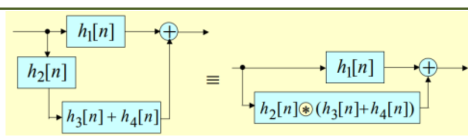
Example



- Determine the impulse response of an interconnection of four DT systems.

$$h_1[n]=\delta[n]+\frac{1}{2}\delta[n-1],\quad h_2[n]=\frac{1}{2}\delta[n]-\frac{1}{4}\delta[n-1]$$

$$h_3[n]=2\delta[n],\quad h_4[n]=-2\left(\frac{1}{2}\right)^n u[n]$$



$$h[n]=h_1[n]+h_2[n]*\{h_3[n]+h_4[n]\}$$
$$=h_1[n]+\underbrace{h_2[n]*h_3[n]}+\underbrace{h_2[n]*h_4[n]}$$

$$h_2[n]\circledast h_3[n]=(\tfrac{1}{2}\delta[n]-\tfrac{1}{4}\delta[n-1])\circledast 2\delta[n]$$
$$=\delta[n]-\tfrac{1}{2}\delta[n-1]$$

$$h[n]=h_2[n]*h_4[n]=\left\{\frac{1}{2}\delta[n]-\frac{1}{4}\delta[n-1]\right\}*\left\{-2\left(\frac{1}{2}\right)^n u[n]\right\}$$

$$h[n]=\frac{1}{2}\delta[n]\times-2\left(\frac{1}{2}\right)^n u[n]-\frac{1}{4}\delta[n-1]\times-2\left(\frac{1}{2}\right)^n u[n]$$

$$=\delta[n]\times-\left(\frac{1}{2}\right)^n u[n]+\frac{1}{2}\delta[n-1]\left(\frac{1}{2}\right)^n u[n]$$

$$=-\left(\frac{1}{2}\right)^n u[n]+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

$$=-\left(\frac{1}{2}\right)^n u[n]+\left(\frac{1}{2}\right)^n u[n-1]$$

$$=-\left(\frac{1}{2}\right)^n\underbrace{\{u[n]-u[n-1]\}}_{\delta[n]}$$

$$=-\left(\frac{1}{2}\right)^n\delta[n]=-\delta[n]$$