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Radix-4 $\quad N = 4^V$

Radix-4 DIT $\longrightarrow$ $x(4n), x(4n+1), x(4n+2), x(4n+3)$

Radix-4 DFT $\longrightarrow$ $X(4K), X(4K+1), X(4K+2), X(4K+3)$

- DIT $\quad N = LM = \frac{N}{4} \cdot 4$

$$X(p,q) = \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} x(l,m)\, e^{-j\frac{2\pi(l+mL)(Mp+q)}{N}}$$

$$X(Mp+q) \Rightarrow \quad n = l + mL, \quad L = \frac{N}{4} \quad M = 4$$

$$X(4p+q) = \sum_{l=0}^{\frac{N}{4}-1} \sum_{m=0}^{3} x(l+mL)\, e^{-j\frac{2\pi lMp}{N}} e^{-j\frac{2\pi lq}{N}} \cancel{e^{-j\frac{2\pi mLMp}{N}}}^{\,1} e^{-j\frac{2\pi mLq}{N}}$$

- $$= \sum_{l=0}^{N/4-1} \sum_{m=0}^{3} x\left(l + \frac{mN}{4}\right) e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}} e^{-j\frac{2\pi mq}{4}}$$

$$= \sum_{l=0}^{N/4-1} x(l)\, e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}} + \sum_{l=0}^{N/4-1} x\left(l+\frac{N}{4}\right) e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}} e^{-j\frac{\pi q}{2}} +$$

$$\sum_{l=0}^{N/4-1} x\left(l+\frac{2N}{4}\right) e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}} e^{-j\frac{2\pi q}{2}} + \sum_{l=0}^{N/4-1} x\left(n+\frac{3N}{4}\right) e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}} e^{-j\frac{\pi 3q}{2}}$$

$$= \sum_{l=0}^{N/4-1} \left[x(l) + x\left(l+\frac{N}{4}\right) e^{-j\frac{\pi q}{2}} + x\left(l+\frac{N}{2}\right) e^{-j\frac{2\pi q}{2}} + x\left(l+\frac{3N}{4}\right) e^{-j\frac{3\pi q}{2}}\right] e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}}$$

$$X(4p+q) = \sum_{l=0}^{N/4-1} \left[x(l) + (-j)^q x\left(l+\frac{N}{4}\right) + (-1)^q x\left(n+\frac{N}{2}\right) + (j)^q x\left(n+\frac{3N}{4}\right)\right] e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi lq}{N}}$$

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$$X(4p) = \sum_{l=0}^{N/4-1} \left[x(l) + x\left(l+\frac{N}{4}\right) + x\left(n+\frac{N}{2}\right) + x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi lp}{N/4}}$$

$$X(4p+1) = \sum_{l=0}^{N/4-1} \left[x(l) - j\, x\left(l+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) + j\, x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi l}{N}}$$

$$X(4p+2) = \sum_{l=0}^{N/4-1} \left[x(l) - x\left(l+\frac{N}{4}\right) + x\left(n+\frac{N}{2}\right) - x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi l(2)}{N}}$$

$$X(4p+3) = \sum_{l=0}^{N/4-1} \left[x(l) + j\, x\left(l+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) - j\, x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi lp}{N/4}} e^{-j\frac{2\pi l(3)}{N}}$$

Frequency DIF

$$N = L \cdot M = \frac{N}{4} \cdot 4$$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nK}{N}}$$

$$X(K) = \sum_{n=0}^{N/4-1} x(n) e^{-j\frac{2\pi nK}{N}} + \sum_{n=N/4}^{N/2-1} e^{-j\frac{2\pi nK}{N}} x(n) + \sum_{n=N/2}^{3N/4-1} x(n) e^{-j\frac{2\pi nK}{N}} + \sum_{n=3N/4}^{N-1} x(n) e^{-j\frac{2\pi nK}{N}}$$

$$m_1 = n - \frac{N}{4} \qquad m_2 = n - \frac{N}{2} \qquad m_3 = n - \frac{3N}{4}$$

$$X(K) = \sum_{n=0}^{N/4-1} x(n) e^{-j\frac{2\pi nK}{N}} + \sum_{m_1=0}^{N/4-1} x\left(m_1+\frac{N}{4}\right) e^{-j\frac{2\pi\left(m_1+\frac{N}{4}\right)K}{N}} + \sum_{m_2=0}^{N/4-1} x\left(m_2+\frac{N}{2}\right) e^{-j\frac{2\pi\left(m_2+\frac{N}{2}\right)K}{N}}$$
$$+ \sum_{m_3=0}^{N/4-1} x\left(m_3+\frac{3N}{4}\right) e^{-j\frac{2\pi\left(m_3+\frac{3N}{4}\right)K}{N}}$$

$$X(K) = \sum_{n=0}^{N/4-1} x(n) e^{-j\frac{2\pi nK}{N}} + \sum_{m_1=0}^{N/4-1} x\left(m_1+\frac{N}{4}\right) e^{-j\frac{2\pi m_1 K}{N}} e^{-j\frac{2\pi \frac{N}{4} K}{N}} + \sum_{m_2=0}^{N/4-1} x\left(m_2+\frac{N}{2}\right) e^{-j\frac{2\pi m_2 K}{N}}$$
$$e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot K} + \sum_{m_3=0}^{N/4-1} x\left(m_3+\frac{3N}{4}\right) e^{-j\frac{2\pi m_3 K}{N}} \cdot e^{-j\frac{2\pi}{N}\frac{3N}{4}\cdot K}$$

$$X(K) = \sum_{n=0}^{N/4-1} x(n) e^{-j\frac{2\pi nK}{N}} + \sum_{n=0}^{N/4-1} x\left(n+\frac{N}{4}\right) e^{-j\frac{2\pi nK}{N}} \left(e^{-j\pi/2}\right)^K + \sum_{n=0}^{N/4-1} x\left(n+\frac{N}{2}\right) e^{-j\frac{2\pi nK}{N}} \left(e^{-j\pi}\right)^K$$
$$+ \sum_{n=0}^{N/4-1} x\left(n+\frac{3N}{4}\right) e^{-j\frac{2\pi nK}{N}} \left(e^{-j\frac{3\pi}{2}}\right)^K$$

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$$X(K) = \sum_{n=0}^{N/4-1} \left[x(n) + (-j)^K x\left(n+\frac{N}{4}\right) + (-1)^K x\left(n+\frac{N}{2}\right) + (j)^K x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi nK}{N}}$$

$$\boxed{X(4K) = \sum_{n=0}^{N/4-1} \left[x(n) + x\left(n+\frac{N}{4}\right) + x\left(n+\frac{N}{2}\right) + x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi nK}{N/4}}}$$

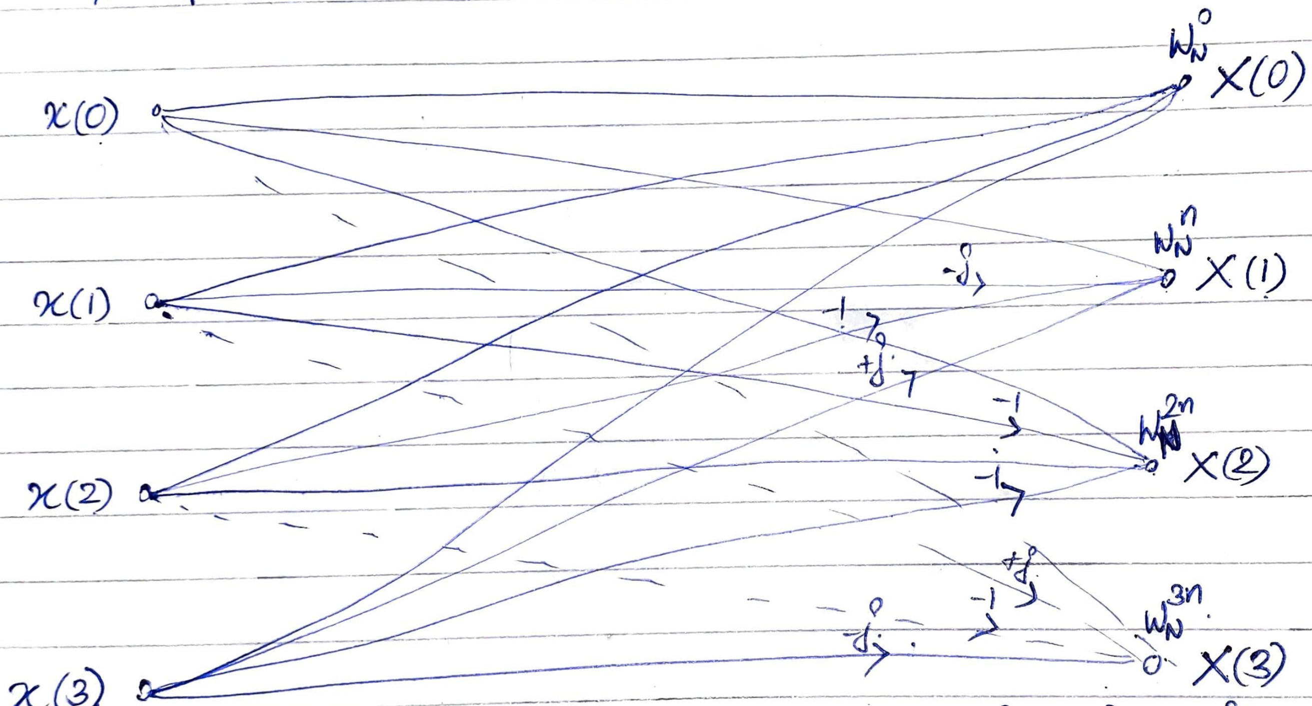
$$\boxed{X(4K+1) = \sum_{n=0}^{N/4-1} \left[x(n) - j x\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) + j x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi nK}{N/4}} e^{-j\frac{2\pi n}{N}}}$$

$$\boxed{X(4K+2) = \sum_{n=0}^{N/4-1} \left[x(n) - x\left(n+\frac{N}{4}\right) + x\left(n+\frac{N}{2}\right) - x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi nK}{N/4}} e^{-j\frac{2\pi n(2)}{N}}}$$

$$\boxed{X(4K+3) = \sum_{n=0}^{N/4-1} \left[x(n) + j x\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) - j x\left(n+\frac{3N}{4}\right) \right] e^{-j\frac{2\pi nK}{N/4}} e^{-j\frac{2\pi n(3)}{N}}}$$

Butterfly Structure of DIF Radix-4

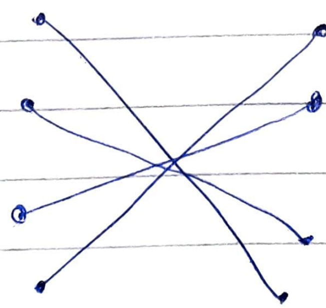
$N = 4 \quad L = N/4 \quad M = 4$

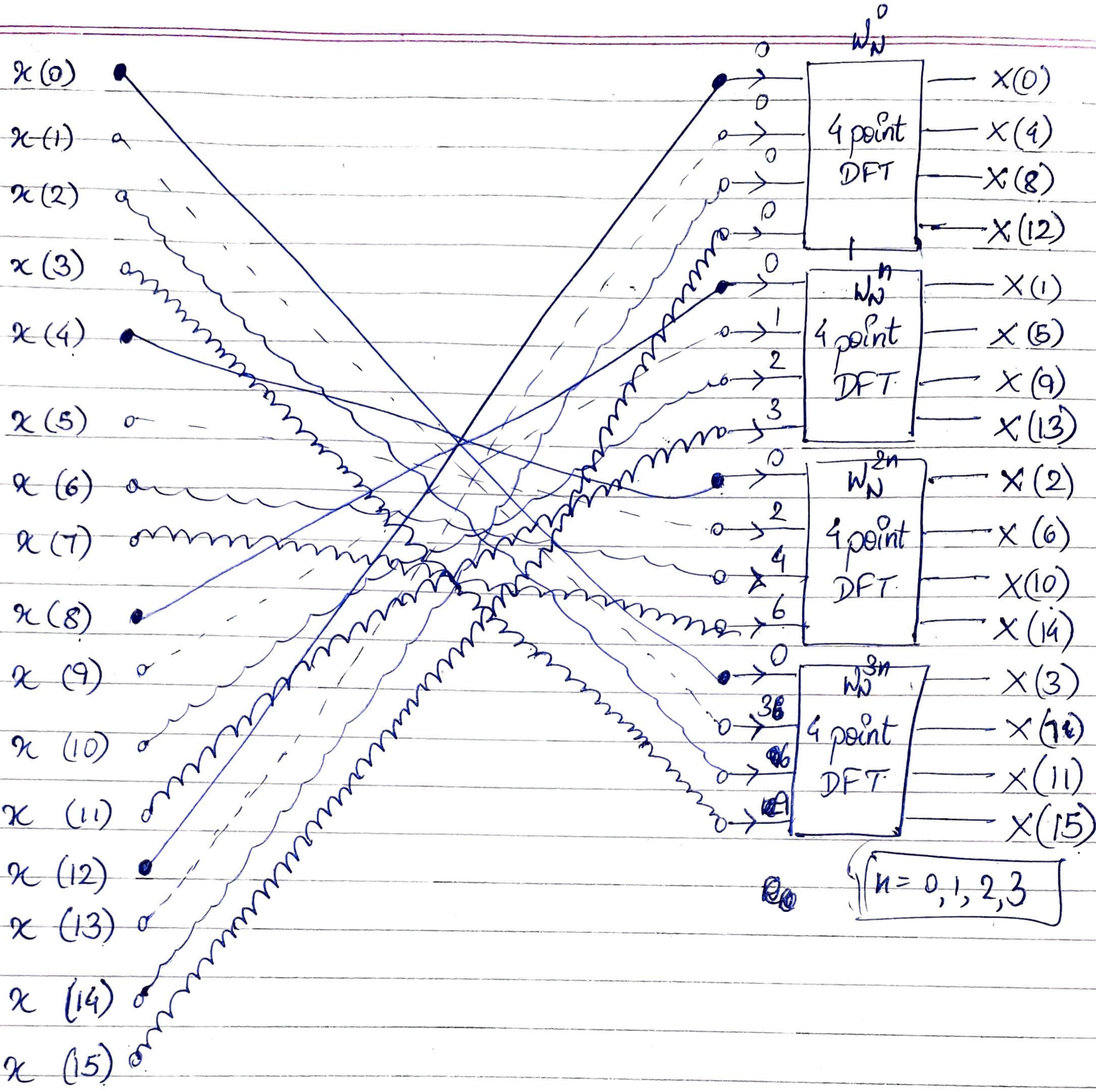


$$W_N^0 = W_N^n = W_N^{2n} = W_N^{3n} = 1$$

$\because n = 0$

DIF Radix 2
Butterfly.





$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

Computational Complexity (Multiplications).

No. of stages $= V = \log_4 N$.

No. of butterflies per stage $= \frac{N}{4}$.

No. of complex multiplication per butterfly $= 3$

Total no. of complex multiplications $= 3\left(\frac{N}{4}\right) \log_4 N$

$$= \frac{3N}{4} \frac{\log_2 N}{\log_2 4}$$

$$= \frac{3N}{8} \log_2 N$$

no. of complex additions per butterfly $= 12$.

Total complex addition $= 12\left(\frac{N}{4}\right) \log_4 N$

$$= \frac{3N}{2} \log_2 N.$$

Radix - 2	Radix 4	
$\frac{N}{2}\log_2 N$	$\frac{3N}{8}\log_2 N$	= Complex multiplication
$\frac{N}{2}(2)\log_2 N$	$\frac{3N}{2}\log_2 N$	= Complex addition

Split Radix FFT

The 'N' can be split up such that factorization is done we get the benefit of radix-2 or radix-4.

Radix - 4 Expressions.

$X(2K) \xleftarrow{\text{Radix-2}}$ Even $X(4K)$

$X(4K+1)$ —— odd

Even $X(4K+2)$

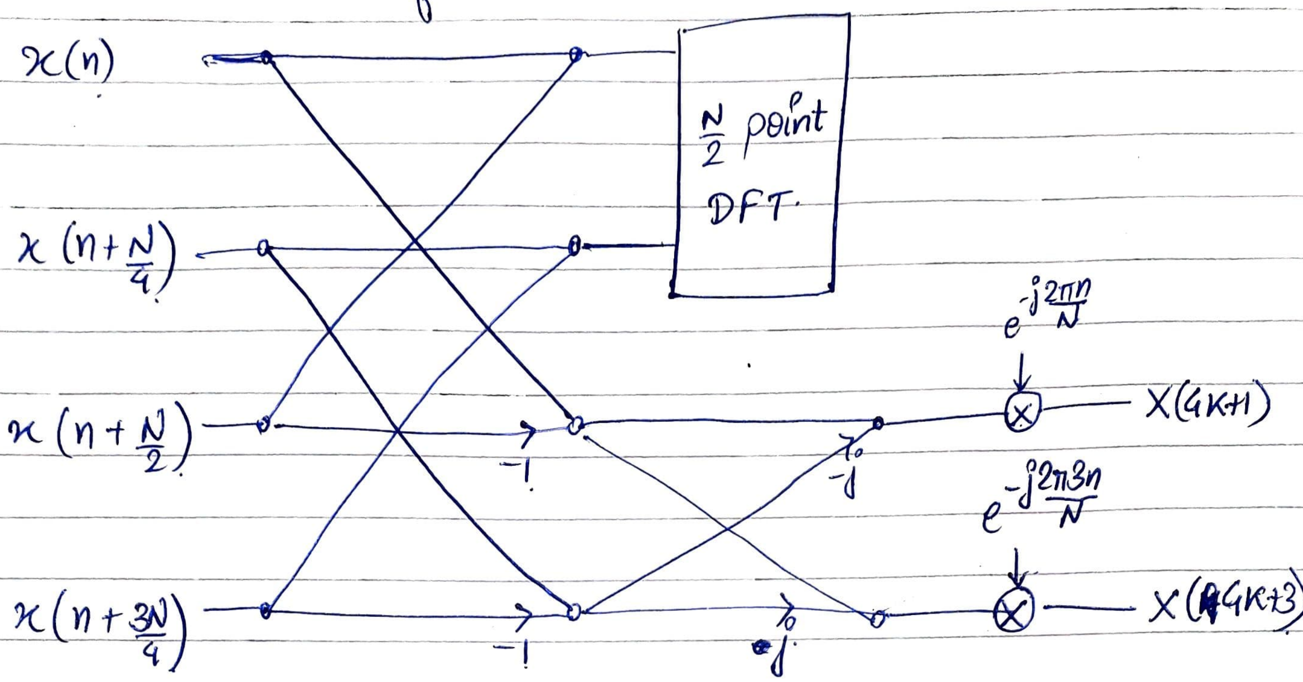
$X(4K+3)$ —— odd

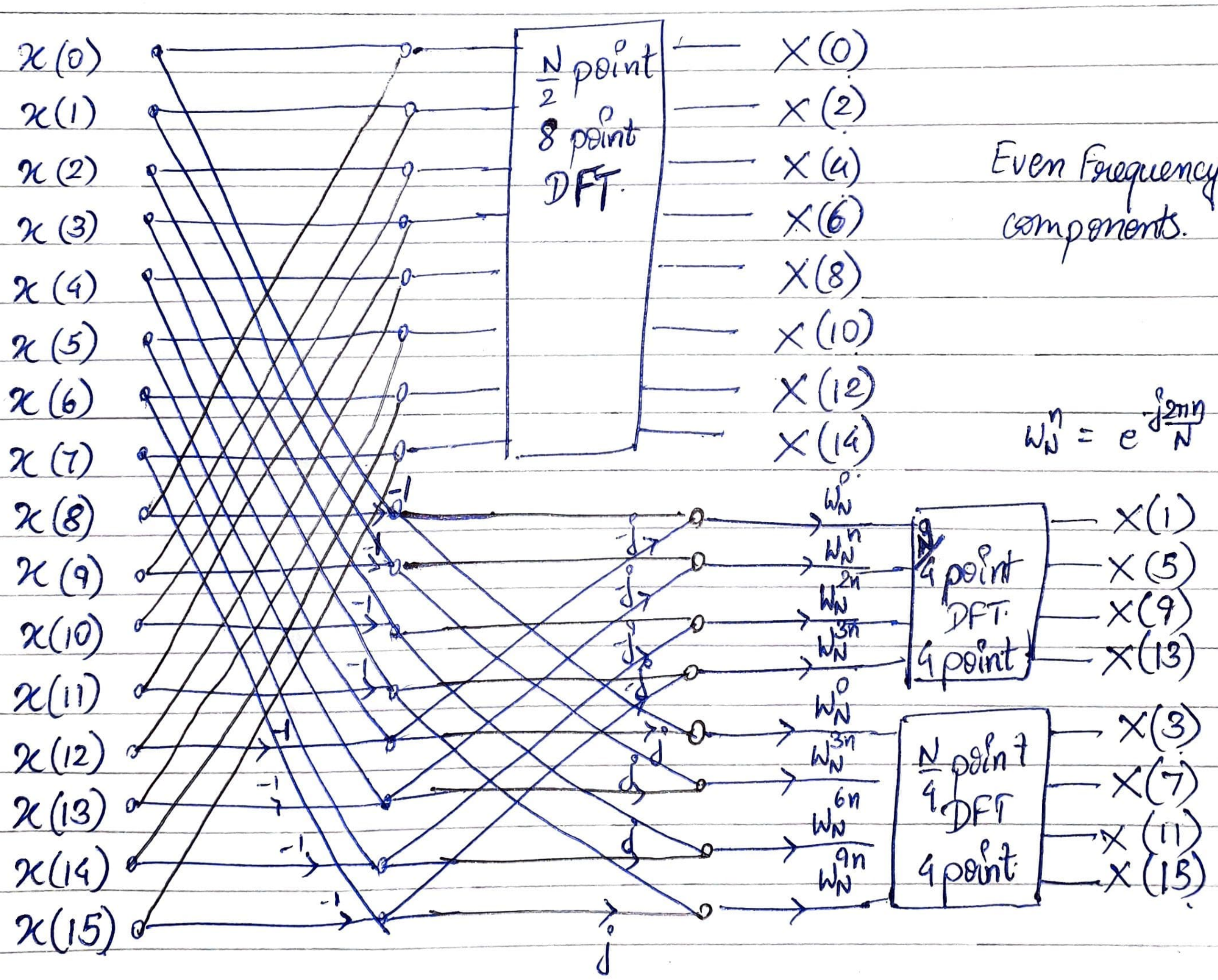
$$X(4K+1) = \sum_{n=0}^{N/4-1}\left[\left\{x(n) - jx\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) + jx\left(n+\frac{3N}{4}\right)\right\}e^{-j\frac{2\pi n}{N}}\right]e^{-j\frac{2\pi nK}{N/4}}$$

$$X(4K+3) = \sum_{n=0}^{N/4-1}\left[\left\{x(n) + jx\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) - jx\left(n+\frac{3N}{4}\right)\right\}e^{-j\frac{2\pi 3n}{N}}\right]e^{-j\frac{2\pi nK}{N/4}}$$

$$X(2K) = \sum_{n=0}^{N/2-1}\left[x(n) + x\left(n+\frac{N}{2}\right)\right]e^{-j\frac{2\pi nK}{N/2}}$$

Split radix butterfly.





$W_N^n = e^{-j\frac{2\pi n}{N}}$