Drill Problem: Digitization (1)

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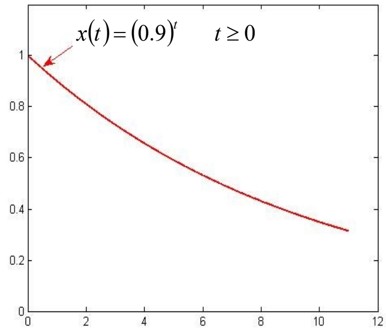

Drill Problem: Digitization (2)

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$$x[n]=\begin{cases}(0.9)^n & n\geq 0\\ 0 & n<0\end{cases}$$

n	$x[n]$ (DT signal)
0	1
1	0.9
2	0.81
3	0.729
4	0.6561
5	0.59049
6	0.531441
7	0.4782969
8	0.43046721
9	0.387420489

Drill Problem: Digitization (3)

$$x[n]=\begin{cases}(0.9)^n & n\geq 0\\ 0 & n<0\end{cases}$$

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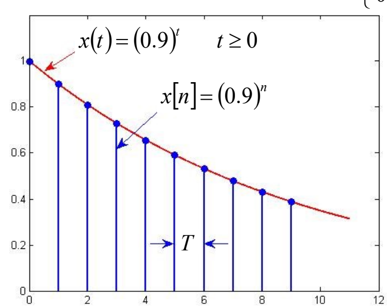

Drill Problem: Digitization (4)

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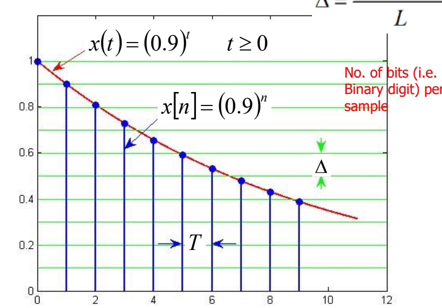
$$\Delta=\frac{x_{\max}-x_{\min}}{L}$$



$L=2^n \quad n=$ No. of bits (i.e. Binary digit) per sample

- Dynamic Range (DR): Capability of transmitting a large Transmission range of signal amplitudes
- DR is the ratio of largest possible magnitude to the smallest magnitude

$$DR=\frac{x_{Max}}{x_{Min}}$$

$$DR=20\log_{10}\frac{x_{Max}}{x_{Min}}$$

Drill Problem: Digitization (5)

$$x[n]=\begin{cases}(0.9)^n & n\geq 0\\ 0 & n<0\end{cases}$$

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n	$x[n]$ (DT signal)	$x_q[n]$ (Rounding)	$e_q[n]=x[n]-x_q[n]$ (Error)
0	1	1.0	0.0
1	0.9	0.9	0.0
2	0.81	0.8	-0.01
3	0.729	0.7	-0.029
4	0.6561	0.7	0.0439
5	0.59049	0.6	0.00951
6	0.531441	0.5	-0.031441
7	0.4782969	0.5	0.0217031
8	0.43046721	0.4	-0.03046721
9	0.387420489	0.4	0.012579511

Drill Problem: Digitization (6)

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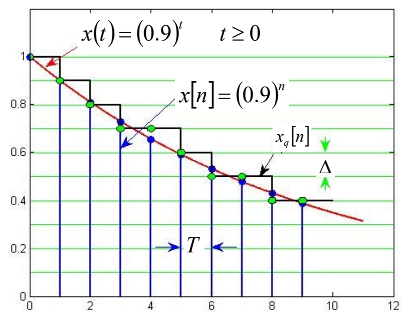

Quantization Error or Noise (3)

- Quantization error/Noise lies in the range (-Δ/2, +Δ/2), the "time average" mean square quantizing error (MSQE) from quantization is

$$\bar{q}^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\,dq = \frac{\Delta^2}{12} = \frac{(2m_p/L)^2}{12} = \frac{m_p^2}{3L^2}$$

Mean square value (time average) or Average power of the Quantizing noise

$$N_q = \bar{q}^2 = \frac{\Delta^2}{12} = \frac{m_p^2}{3L^2}$$

NOTE: The quantization noise (Nq) is proportional to the stepsize (Δ). Reducing the stepsize reduces the quantization noise. OR by increasing the quantization levels.

- N_q is proportional to the fluctuation of the error signal, called quantization noise. $m_q(t) = m(t) + q(t)$
- Quantization SNR: <u>SNR is an important measure of the distortion induced by the quantization process</u>. Larger SQNR, better quality

$$SNR_q = \frac{S_0}{N_q} = \frac{\overline{m}^2(t)}{\Delta^2/12} = 3L^2\frac{\overline{m}^2(t)}{m_p^2}$$

Signal (message) power S_0 is proportional to the square of m(t), thus

$$S_0 = \overline{m}^2(t)$$

Quantization Error or Noise (4)

- To measure of the quality of received signal (that is, the ratio of the strength of the received signal S_0 relative to the strength of the error N_q due to quantization).
- SNR

$$SNR_q = \frac{S_0}{N_q} = \frac{\overline{m}^2(t)}{\Delta^2/12} = 3L^2\left\{\frac{\overline{m}^2(t)}{m_p^2}\right\}$$

Quantization SNR depends on the power of the input signal.

$$SNR_q \propto \overline{m}^2(t)$$

$$SNR_q(dB) = 10\log_{10}(L^2) + 10\log_{10}\left\{\frac{3\overline{m}^2(t)}{m_p^2}\right\}$$

$$SNR_q(dB) = 10\log_{10}(2^{2n}) + 10\log_{10}\left\{\frac{3\overline{m}^2(t)}{m_p^2}\right\}, \qquad L = 2^n$$

$$SNR_q(dB) = 20n\log_{10}(2) + 10\log_{10}\left\{\frac{3\overline{m}^2(t)}{m_p^2}\right\}$$

Each additional bit reduces the quantization error by about 6 dB

$$SNR_q(dB) = 6.02n + 10\log_{10}\left\{\frac{3\overline{m}^2(t)}{m_p^2}\right\}$$

As n increases 1 bit, SNR increases ≈6 dB.

- SNR (in dB) of a Quantizer increases linearly and depends on n. If n increases 1 bit, SNR increases by 6 dB. To increase SNR of a Quantizer means more bits are required and therefore either a higher bandwidth or a longer time period is required to transmit the PCM. **This relationship shows that each added binary digit increases the SNR ratio by 6 dB.**

Practice Problem

- A digital communications link carries binary-coded words representing samples of an input signal x(t) = 3cos600πt + 2cos1800πt, the link is operated at 10,000 bits/s and each input sample is quantized into 1024 different levels.
  a) What is the sampling frequency Fs?
  b) What is the Nyquist rate for the signal x(t) ?
  c) What are the frequencies in the resulting discrete-time signal x[n]?
  d) What is the resolution Δ ?

a) Fs = 10k/log(1024) = 1k
b) Nq = Fmax*2 = 2*900 = 1800
c) f1 = 300, f2 = 900. No aliasing
   x[n] = 3cos(600npi/Fs) + 2cos(1800npi/Fs)
   f1 = 0.3, f2 = 0.5
d) delta = (xmax - xmin)/(1024)
   = (3-2)/1024 = 1/1024

Example (1)

- Given a 3-bit ADC channel accepts analog input ranging from 0 to 5 volts, determine the following:
  a) number of quantization levels
  b) step size of the quantizer or resolution
  c) quantization level when the analog voltage is 3.2 volts
  d) binary code produced by the ADC