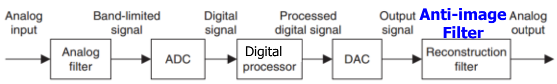
Basic Concepts of Digital Signal Processing (1)

- The idea of processing signals digitally with the availability of computers.



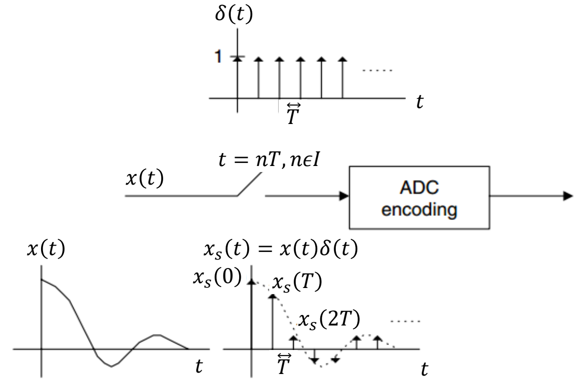
Digital Signal Processing Scheme

Anti-aliasing filter (a LPF that rejects high frequencies that cause aliasing)

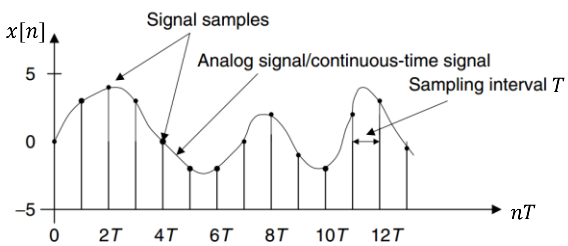
Anti-image filter (a reconstruction LPF that smooths the recovered sample-and-hold voltage levels to an analog signal)

Computer / microprocessor / micro controller/ etc.

Digitization: Sampling and Quantization



Digitization: Sampling and Quantization



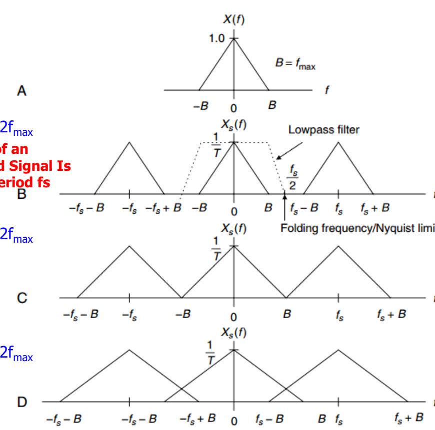
$$X_s(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f - nf_s),$$

$$X_s(f) = \cdots + \frac{1}{T}X(f+f_s) + \frac{1}{T}X(f) + \frac{1}{T}X(f-f_s) + \cdots$$

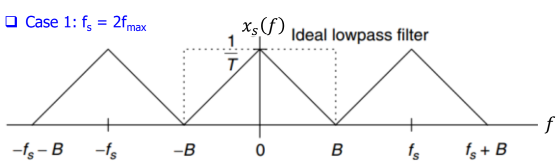
- Case 1: $f_s \geq 2f_{max}$

The Spectrum of an Ideally Sampled Signal Is Periodic with Period fs

- Case 2: $f_s = 2f_{max}$
- Case 3: $f_s < 2f_{max}$



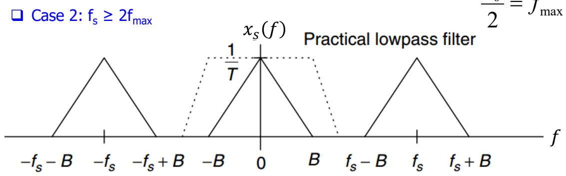
- Case 1: $f_s = 2f_{max}$



- Nyquist frequency is equal to the maximum frequency of the analog signal.

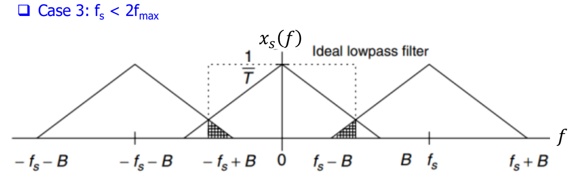
$$\frac{f_s}{2} = f_{max}$$

- Case 2: $f_s \geq 2f_{max}$



- Practical lowpass reconstruction (anti-image) filter can be designed to reject all the images and achieve the original signal spectrum.

- Case 3: $f_s < 2f_{max}$



$$f_{alias} = f_s - f$$

Drill Problem

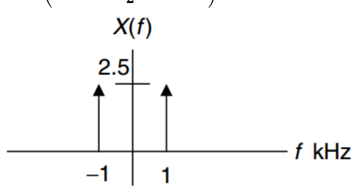
- An analog signal

$$x(t)=5\cos(2\pi\times 1000t),\quad t\ge 0$$

is sampled at a rate of 8 kHz,

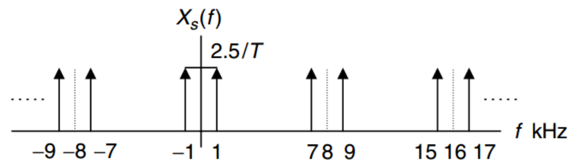
a) Plot the spectrum of the x(t).
b) Plot the spectrum for the sampled signal i.e. x[n] from 0 to 20 kHz.

$$5\cos(2\pi\times 1000t)=5\left(\frac{e^{j2\pi\times 1000t}+e^{-j2\pi\times 1000t}}{2}\right)=2.5\left(e^{j2\pi\times 1000t}+e^{-j2\pi\times 1000t}\right)$$

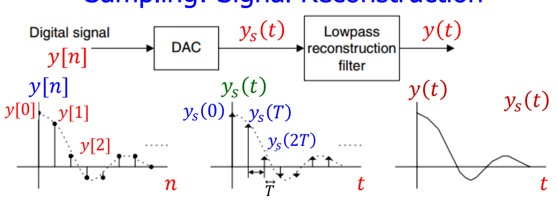


Drill Problem (Cont'd)

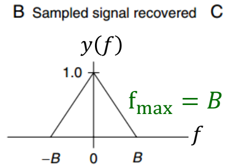
- After the analog signal is sampled at the rate of 8kHz, the sampled signal spectrum and its replicas centered at the frequencies $\pm nf_s$ (-∞<n<∞), each with the scaled amplitude being 2.5/T = 2.5 fs,



Sampling: Signal Reconstruction



A Digital signal processed B Sampled signal recovered C Analog signal recovered



D Recovered signal spectrum

Example (2)

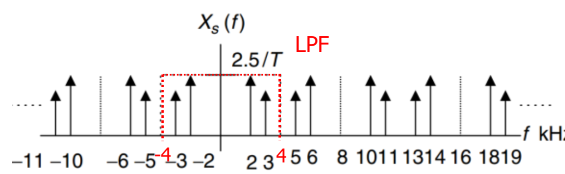
- An analog signal

$$x(t)=5\cos(2\pi\times 2000t)+3\cos(2\pi\times 3000t),\quad t\ge 0$$

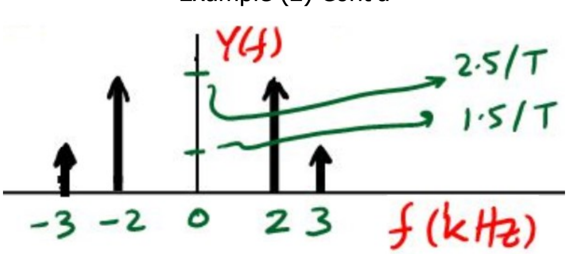
is sampled at a rate of 8kHz,

a) Plot the spectrum of the sampled signal i.e. x[n] up to 20 kHz.
b) Sketch the recovered analog signal spectrum if an ideal LPF with a cut-off frequency of 4 kHz is used to recover the original signal

$$x(t)=\frac{3}{2}e^{-j2\pi\cdot 3000t}+\frac{5}{2}e^{-j2\pi\cdot 2000t}+\frac{5}{2}e^{j2\pi\cdot 2000t}+\frac{3}{2}e^{j2\pi\cdot 3000t}.$$



Example (2) Cont'd



$$x(t)=5\cos(2\pi\times 2000t)+3\cos(2\pi\times 3000t),\quad t\ge 0$$

Aliasing: Time Domain

(Sampling of Sinusoidal signals)

$$x(t)=\sin(\omega t)=\sin(2\pi ft)$$

Digital Frequency

$$x[n]=x(nT_s)=\sin(2\pi fnT_s)=\sin\left(2\pi\left\{\frac{f}{f_s}\right\}n\right)$$

- The sinusoidal signal repeats every 2πm radians.

$$x[n]=\sin\left(2\pi\left\{\frac{f}{f_s}\right\}n\right)=\sin\left(2\pi\left\{\frac{f}{f_s}\right\}n+2\pi m\right)=\sin\left(2\pi\left\{f+\frac{m}{nT_s}\right\}nT_s\right)$$

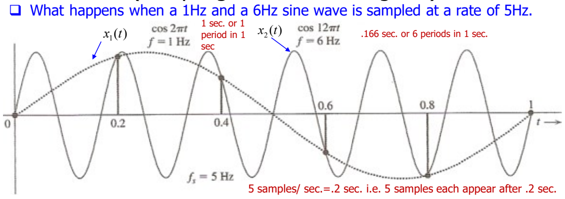
$$x[n]=\sin\left(2\pi\left\{\underbrace{f+\frac{m}{nT_s}}_{f}\right\}nT_s\right)\Rightarrow\quad f=f+\frac{k}{T_s}=f+mf_s\qquad m\varepsilon I$$

$$|\omega_a|=|\omega-m\omega_s|,\quad -\frac{\omega_s}{2}\le\omega\le\frac{\omega_s}{2};\quad m\in I\quad i.e.\pm 1,\pm 2,...$$

- If a periodic signal of frequency f i.e. sampled at f_s, and another signal with frequency f+mfs, i.e. also sampled at f_s, The samples of all these signals will be the same. SAME DT SIGNAL RESULTS OF SAMPLING

Drill Problem: Spectral Folding Effect of Aliasing
(Sampling of Sinusoidal signals)

- What happens when a 1Hz and a 6Hz sine wave is sampled at a rate of 5Hz.



- If a CT signal of frequency ω Hz is sampled at ω_s samples/sec, then sampled version would appear as samples of a CT sinusoid of frequency ω_a in the band 0 to $\omega_s/2$, where:

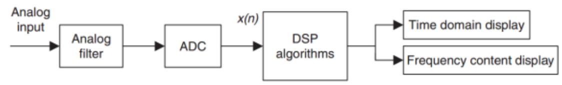
$$|\omega_a| = |\omega - m\omega_s|, \quad -\frac{\omega_s}{2} \le \omega \le \frac{\omega_s}{2}; \quad m \in I \quad i.e. \pm 1, \pm 2, ...$$

Aliases:

$$f_1 + mf_s = 6Hz, 12Hz, 16Hz, ...$$
$$mf_s - f_1 = 4kHz, 9kHz, 14kHz, ...$$
$$f_2 + mf_s = 11Hz, 16Hz, 21Hz, ...$$
$$mf_s - f_2 = 1Hz, 4Hz, 9Hz, ...$$

- The folding frequency $f_s/2$ = 2.5Hz. Hence sinusoids below 2.5 Hz(frequency within the fundamental band) will not be aliased and above 2.5 Hz will be aliased.

Signal Frequency (Spectrum) Analysis



Single tone: 1000 Hz

Double tone: 1000 Hz and 3000 Hz