Lecture – 9A, 9B and 9C

Energy Resources, Economics and Environment

Non Renewable Resource Economics

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"Contemplation of the world's disappearing supplies of minerals, forests, and other exhaustible assets had led to demands for regulation of their exploitation. The feeling that these products are now too cheap for the good of future generations, that they are being selfishly exploited at too rapid a rate, and that in consequence of their excessive cheapness they are being produced and consumed wastefully has given rise to the conservation movement."

Non Renewable Resource Economics

Harold Hotelling 'The Economics of Exhaustible Resources' published in 1931, Journal of Political Economy

$\Pi_t = p_t q_t - C q_t$

Where Π_t is the profit in period t

p_t - market price of resource

q_t - output of resource in period t

C – Marginal cost of resource (assumed constant)

Context

- Mine Owners Problem
- Firm has rights to a fixed stock pf homogenous resource Q_{max}
- Extract at a constant cost per unit C
- Maximise discounted value of profit subject to

$$\sum_{t=0}^{T} q_t = Q_{\max}$$

Assumptions

- T – time horizon – exogenously determined
- Form the Lagrangian and differentiate to get the optimum

Formulation

$$\sum_{t=0}^{T} \frac{\pi_t}{(1+d)^t}$$

$$\sum_{t=0}^{T} \frac{(\mathrm{P}_{\mathrm{t}} q_t - C q_t)}{(1+d)^t}$$

Subject to

$$\sum_{t=0}^{T} q_t = Q_{\max}$$

Formulation

$$L=\sum_{t=0}^{T}\frac{(\mathrm{P_t}q_t-Cq_t)}{(1+d)^t}+\lambda\left[Q_{\max}-\sum_{t=0}^{T}q_t\right]$$

$$\frac{\partial L}{\partial q_t}=\frac{(\mathrm{P_t}-C)}{(1+d)^t}-\lambda=0 \qquad t=0,1,\ \ldots\ldots T$$

$$\lambda=\frac{(\mathrm{P_t}-C)}{(1+d)^t} \qquad t=0,1,\ \ldots\ldots T$$

Formulation

$$P_t = C + \lambda(1+d)^t$$

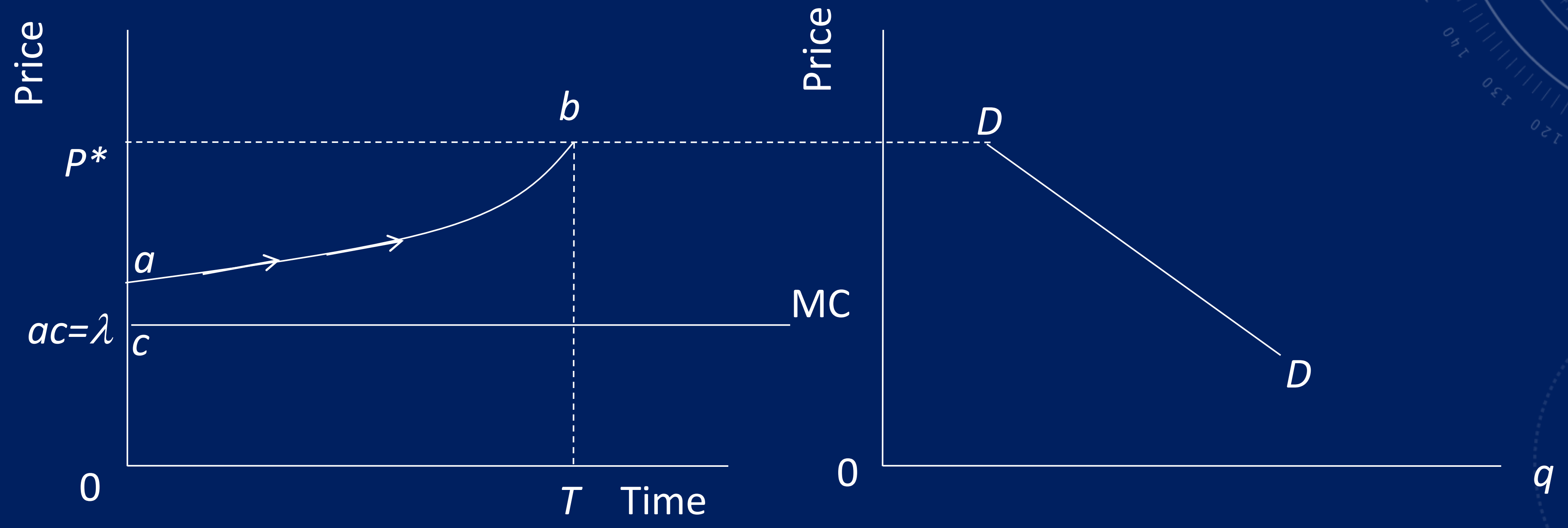
Price = marginal extraction cost + user cost

$$\frac{R_{t+1} - R_t}{R_t} = \frac{\Delta R_t}{R_t} = d$$

$$R_t = P_t - C$$

R_t Revenue in time t per unit

Price Trend



Hotelling Rule

Market price of a resource net extraction cost must rise at a rate equal to the rate of interest

Arbitrage Rule

$$y(t) = rp(t) - \dot{p}$$

$$\dot{p} = dp(t)/dt$$

y (t) = 0 for non renewable resource

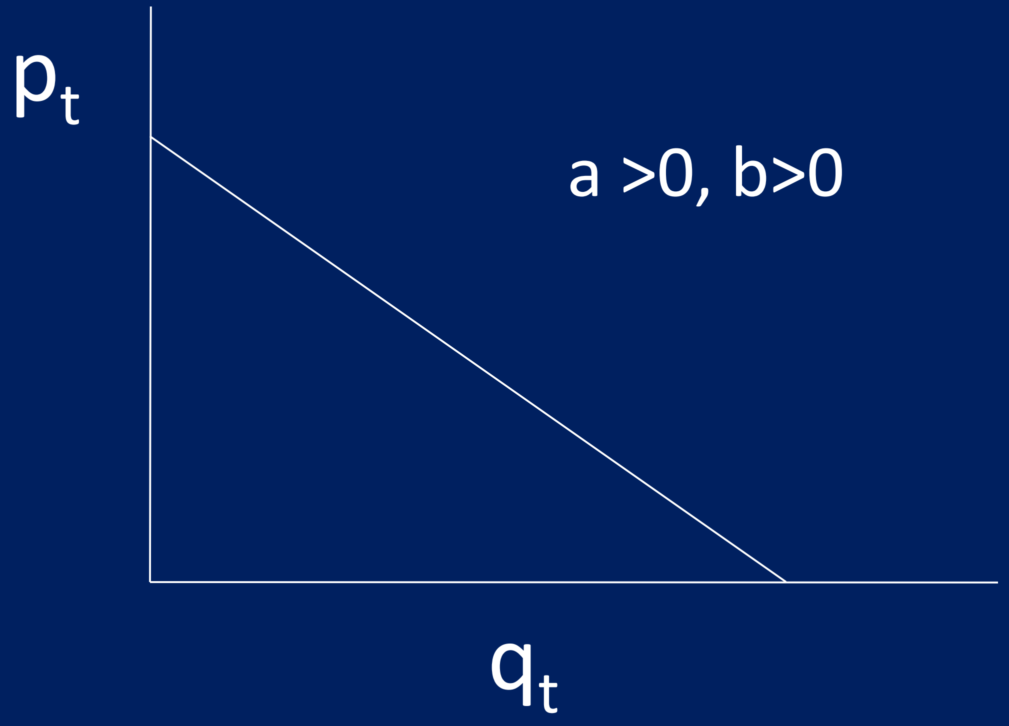
$$\dot{p}\Big/p(t) = r$$

$$p(t) = p(0)e^{rt}$$

$$p(0) = p(t)e^{-rt}$$

Inverse Demand Curve

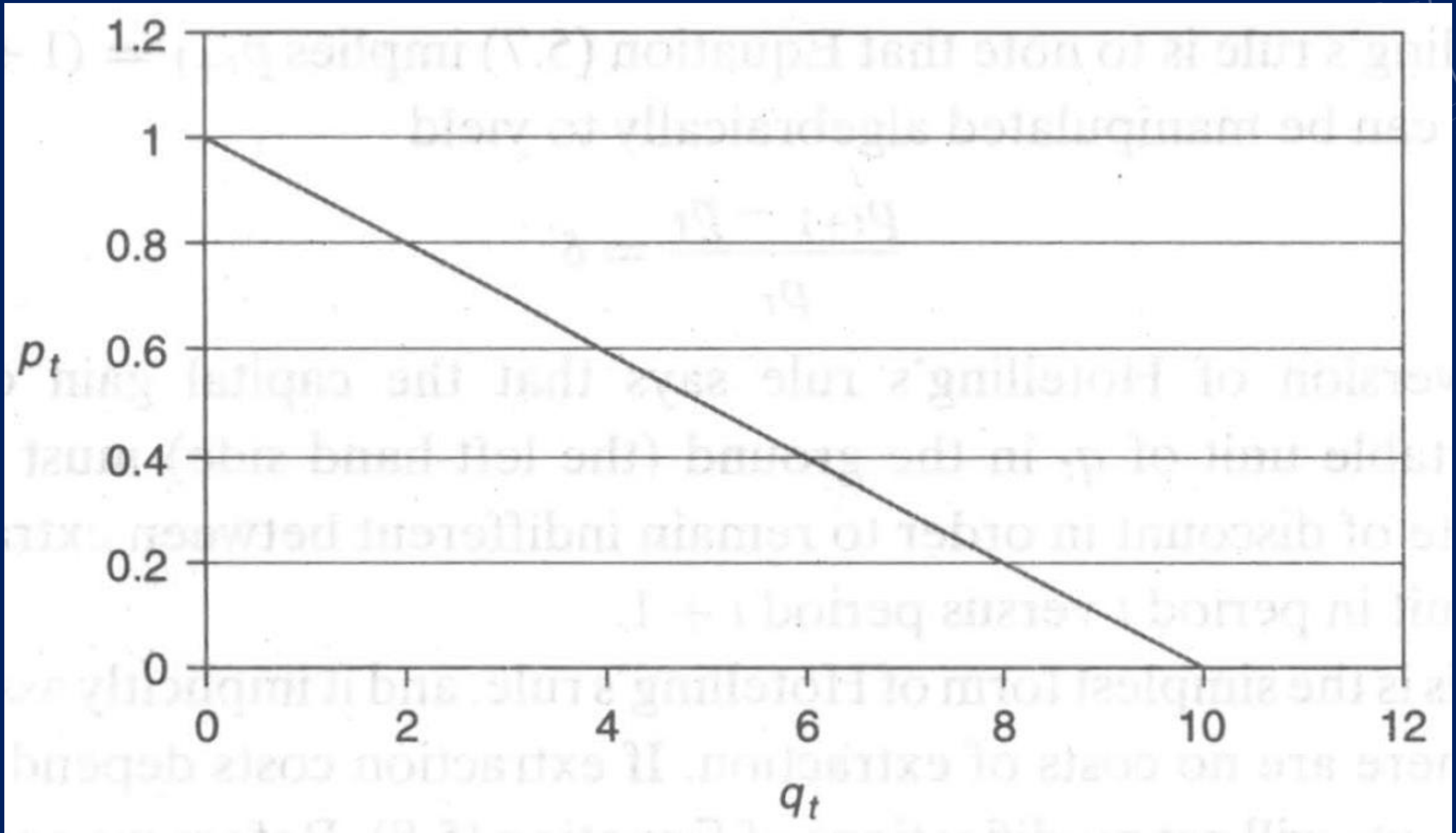
Linear $p_t = a - bq_t$



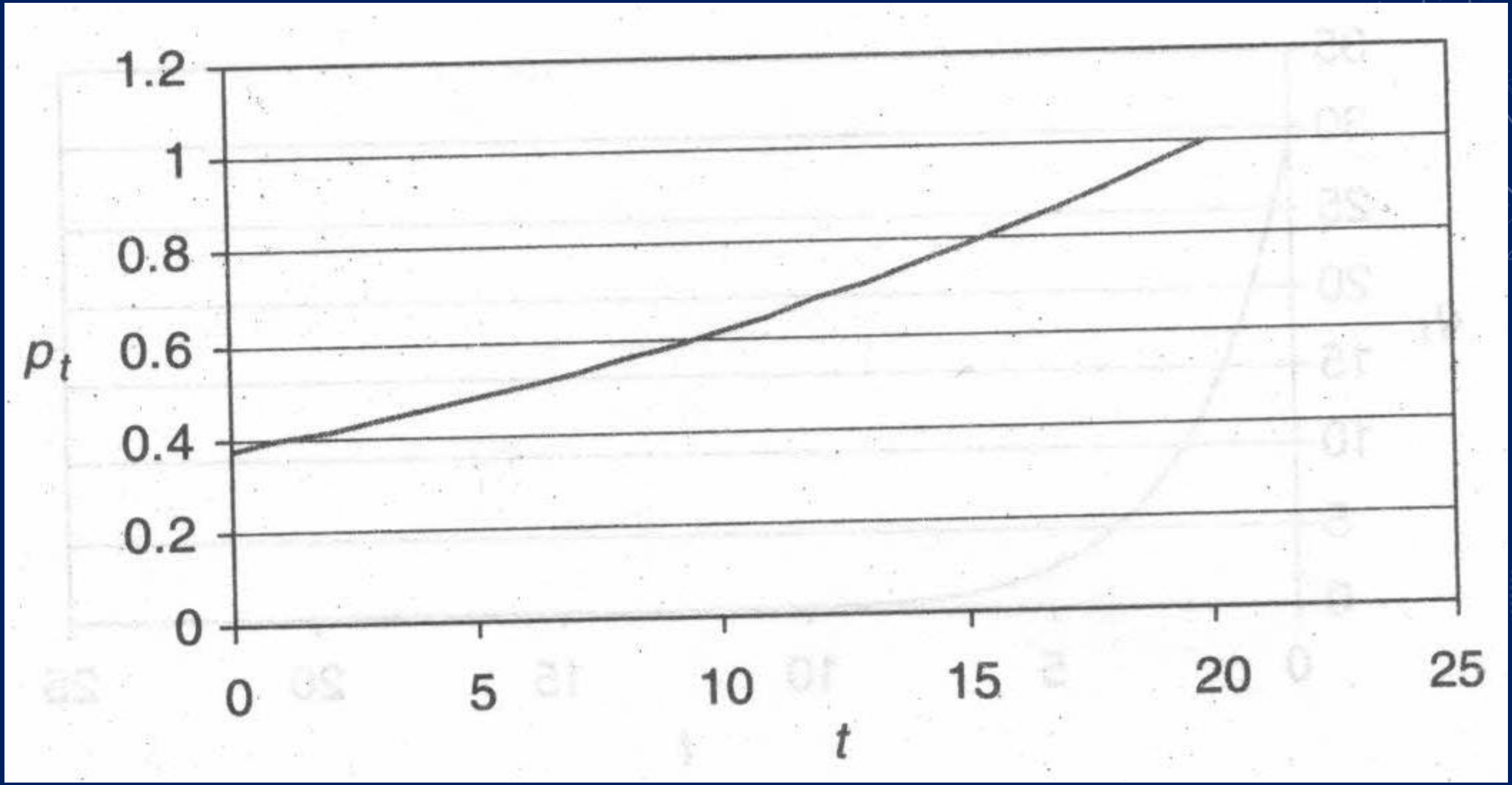
Price Elasticity of Demand

$$\eta_t = \left| \frac{\dfrac{dq_t}{q_t}}{\dfrac{dp_t}{p_t}} \right|$$

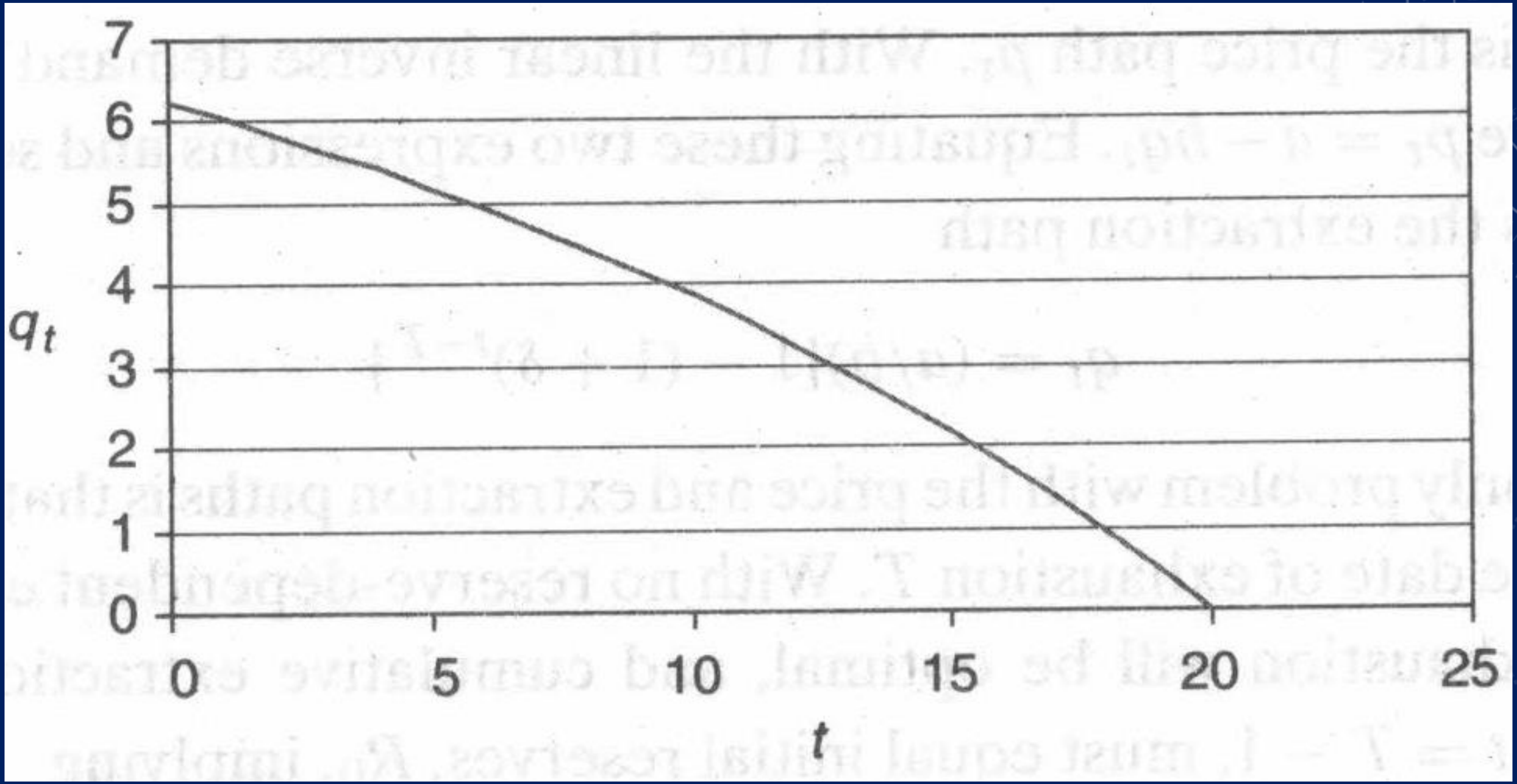
Inverse Demand Curve - Linear



Linear model - Price trend



Linear model - qt trend



$$P_t = a - bq$$

$$\text{at}\ P_t = a\ ,\ q_t = 0$$ Superabundant substitute

$$R_T = q_T = 0$$ resource at time T

Date of exhaustion T is unknown and must be determined along the competitive extraction and price paths

$$P_T = a = P_0(1+d)^T$$

$$P_0 = \frac{a}{(1+d)^T}$$

$$P_t = P_0(1+d)^t$$

$$\boxed{P_t = a(1+d)^{t\text{-}T}}$$

Price path

$$a - bq = a(1+d)^{t-T}$$

$$bq = a\left[1-(1+d)^{t-T}\right]$$

$$q_t = \frac{a}{b}\left[1-(1+d)^{t-T}\right]$$

$$\sum_{t=0}^{T-1} q_t = \sum_{t=0}^{T-1}\left(\frac{a}{b}\right)\left[1-(1+d)^{t-T}\right]$$

$$= R_0$$

$$R_0 = \frac{a}{b}\sum_{t=0}^{T-1}\left[1-(1+d)^{t-T}\right]$$

Constant Elasticity Demand

Constant elasticity inverse demand curve is given by

$$P_t = a q_t^{-b}$$

$$= \frac{a}{q_t^{b}} \qquad a > 0 \qquad b > 0$$

$$\frac{dP_t}{dq_t} = a(-b) q_t^{-b-1}$$

$$= \frac{-ab}{q_t^{b+1}}$$

Elasticity

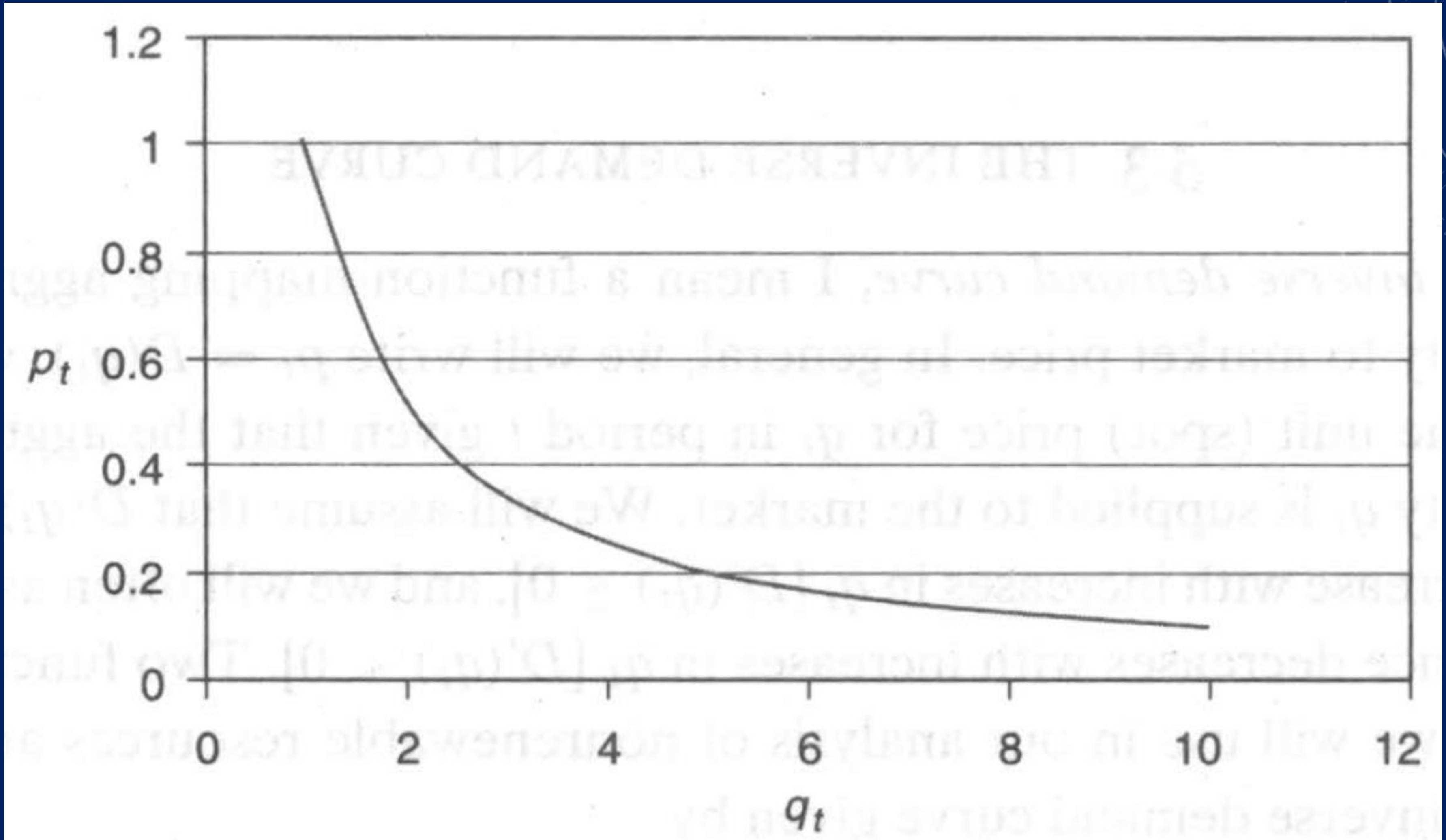
$$\eta_T = \left| \frac{P_t}{q_t} \frac{1}{\left(\frac{dp_t}{dq_t} \right)} \right|$$

$$= \left| \frac{P_t}{q_t} \frac{q_t^{b+1}}{(-ab)} \right|$$

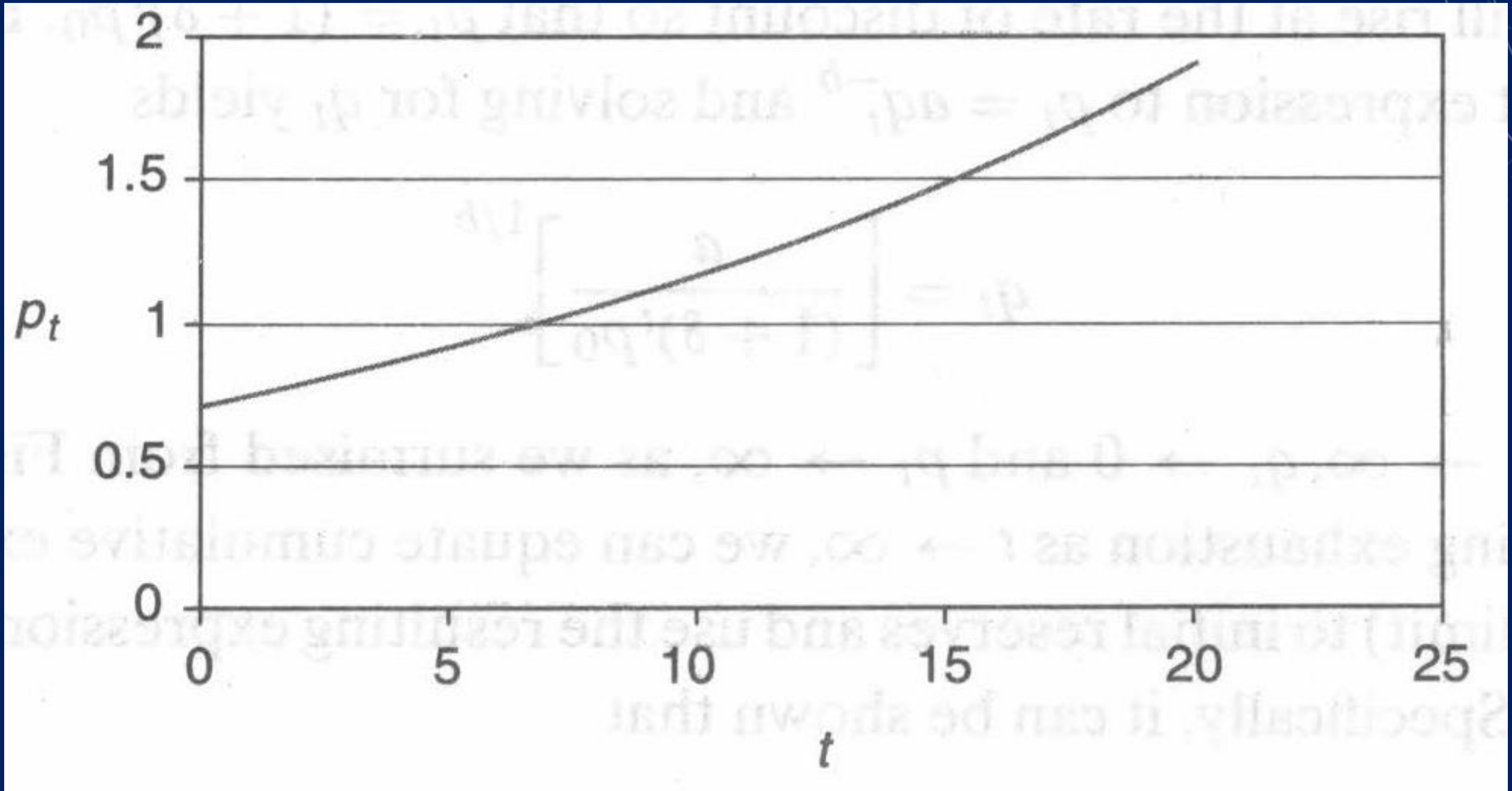
$$=\left|\frac{aq_t^{b}}{q_t}\,\frac{q_t^{b+1}}{(-ab)}\right|$$

$$=\left|\frac{1}{-b}\right|=\frac{1}{b}$$

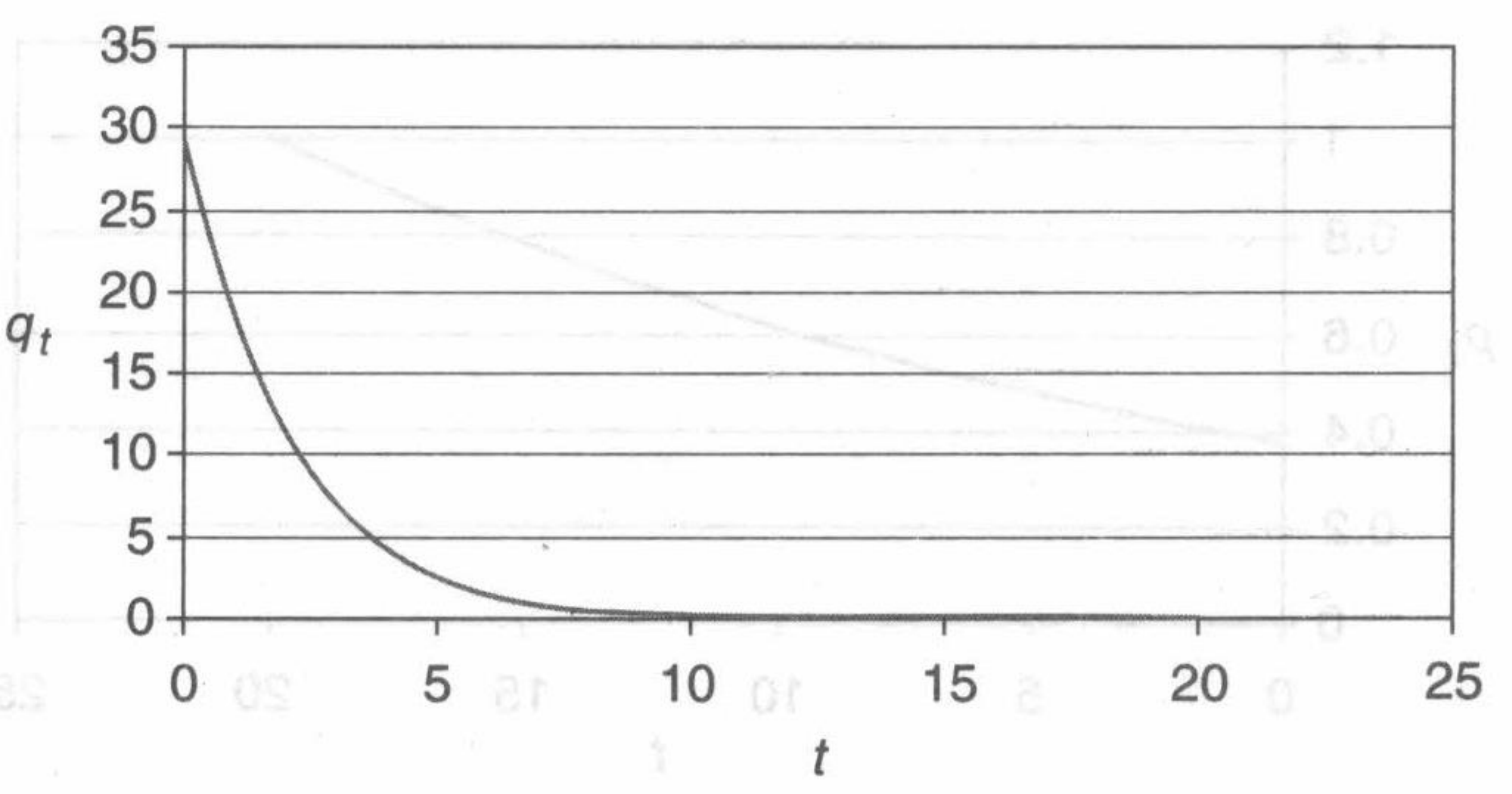
Inverse Demand Curve – Constant Elasticity



Constant Elasticity- Price trend



Constant Elasticity- qt trend



Constant elasticity demand curve

$$P_t = aq_t^b$$

$$a > 0 \text{ and } b \geq 0 \text{ are paramete}$$

Competitive extraction and price paths for the constant elasticity inverse demand curve – made difficult by lack of a choke-off price. No finite terminal price.

$$P_t = (1+d)^t P_0$$

$$P_t = aq_t^b$$

$$\frac{a}{q_t^b} = (1+d)^t P_0$$

$$q_t^b = \frac{a}{P_0(1+d)^t}$$

$$q_t = \left[\frac{a}{(1+d)^t P_0}\right]^{\frac{1}{b}}$$

As $t \to \infty,\ q_t \to 0$ and $P_t \to \infty$

Assuming exhaustion as $t \to \infty$

$$\sum_{t=0}^{\infty}\left[\frac{a}{(1+d)^t P_0}\right]^{\frac{1}{b}}$$

$$=\left[\frac{a(1+d)}{P_0}\right]^{\frac{1}{b}}\left[\frac{1}{(1+d)^{1/b}-1}\right]^{\frac{1}{b}}=R_0$$

$$\left[\frac{a(1+d)}{P_0}\right]^{\frac{1}{b}}=R_0\left[(1+d)^{\frac{1}{b}}-1\right]$$

$$\left[\frac{a(1+d)}{P_0}\right] = \left[R_0\left[(1+d)^{1/b} - 1\right]\right]^b$$

$$P_0 = \frac{a(1+d)}{\left[R_0\left[(1+d)^{\frac{1}{b}} - 1\right]\right]^b}$$

Monopoly

Consider the situation of a monopolist
Assume an inverse demand curve that is linear

$$P_t = a - bq_t \qquad P_t = D(q_t)$$

$$\pi_t = P_t q_t = aq_t - bq_t^2$$

Maximise present value of revenue
Schedule extraction to equate discounted marginal revenue

$$MR = \frac{\partial \pi_t}{\partial q_t} = a - 2bq_t$$

$$MR_t = (1+d)^t MR_0$$

$$MR_t = a = P_T \text{ when } q_T = 0 \text{ when } t = T$$

$$MR_0 = a(1+d)^{-T}$$

$$MR_t = a(1+d)^{t-T}$$

$$= a - 2bq_t$$

$$a(1+d)^{t-T} = a - 2bq_t$$

$$2bq_t = a - a(1+d)^{t-T}$$

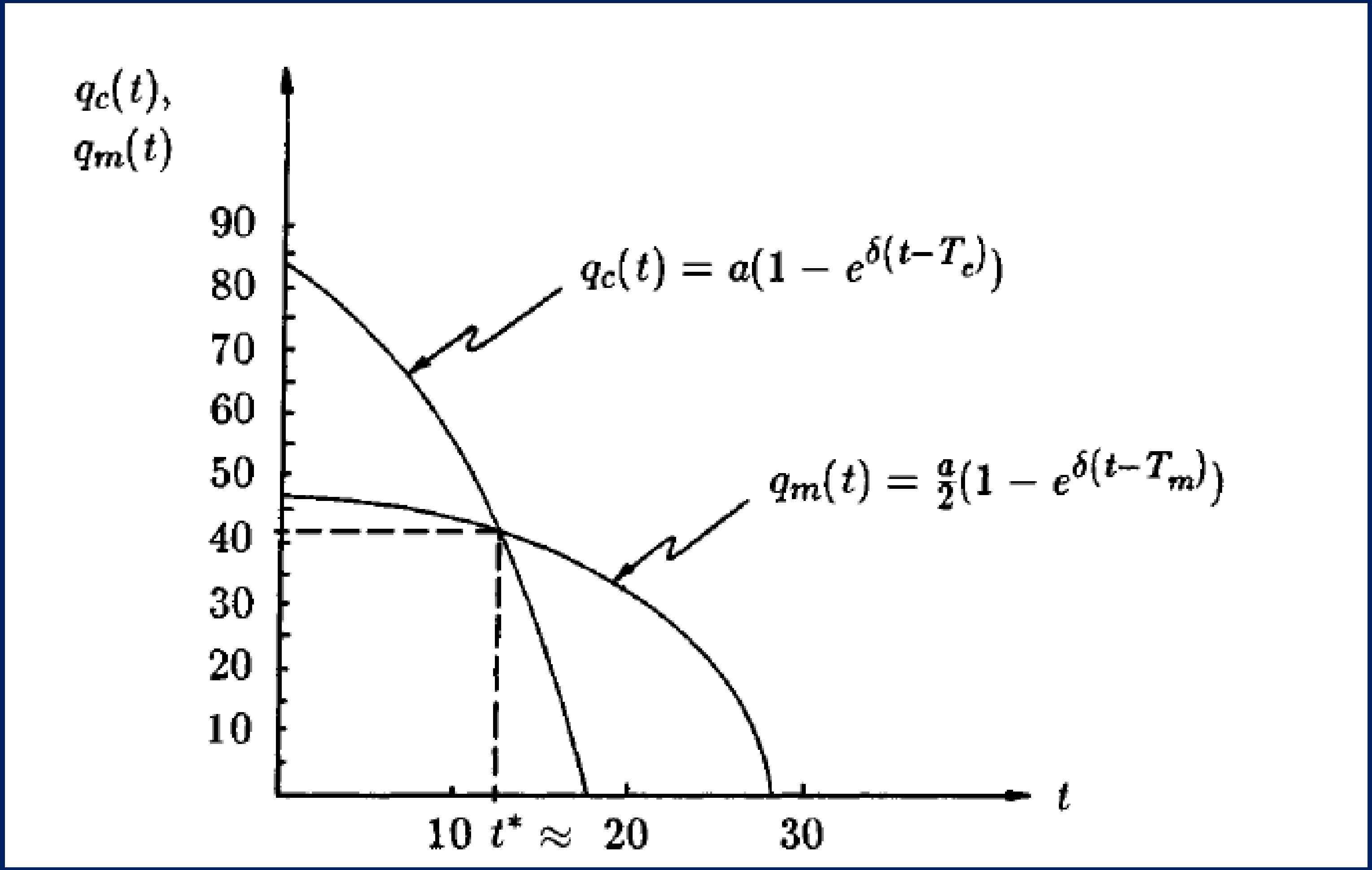
$$\sum_{t=0}^{T-1} \frac{a}{2b}\left[1 - (1+d)^{t-T}\right] = R_0$$

$$R_0 = \frac{a}{2b}T - \frac{a}{2b}\frac{1}{a}\left[1 - \frac{1}{(1+d)^T}\right]$$

$$\frac{2bR_0}{2b} = T - \frac{1}{d}\left[1 - \frac{1}{(1+d)^T}\right]$$

$$T = \frac{2bR_0}{a} + \frac{1}{d}\left[1 - \frac{1}{(1+d)^T}\right]$$

Competition vs Monopoly



Tutorial problem

1. The inverse demand function for a fossil fuel is:

$P_t = 1 – 0.1q_t$, Assume that the costs of extraction are zero. The initial reserves are $R_0=75$ and $d=5\%$ (repeat with $d =10\%$).

a) What is the price elasticity of demand for this function when $q_t =5$?

b) Determine the time path of extraction for a mining industry under pure competition.

c) When does the resource get exhausted?

d) Would the time path of extraction for a monopolistic mining industry be different.

Explain your answer qualitatively

References

- H. Hotelling, ‘The Economics of Exhaustible Resources’ , Journal of Political Economy, 1931
- J.Sweeney , Economic Theory of Depletable Resources, Chapter 17, Handbook of Natural Resource and Energy Economics, Vol 3. Elsevier
- Conrad, Jon M. Clark, Colin Whitcomb EBK2080 Natural resource economics : notes and problems (e-book) Cambridge University Press, 1987.