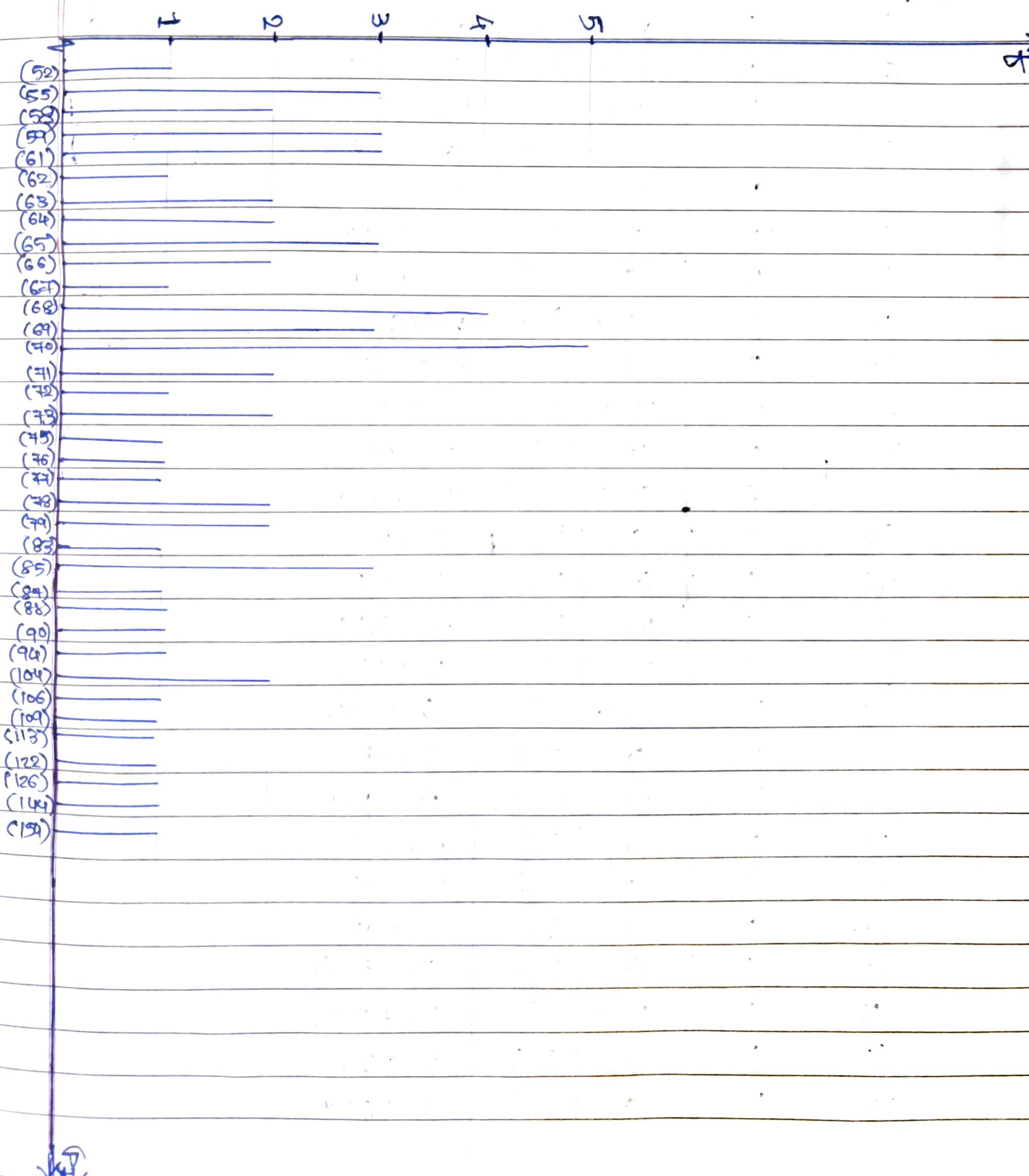
Assignment - 2

Q1) Pixel Intensity: PI and Frequency: f

PI	52	55	58	59	61	62	63	64	65	66	67	68	69	70	71	72	73	75	76
f	1	3	2	3	3	1	2	2	3	2	1	4	3	5	2	1	2	1	1

conti...

PI	77	78	79	83	85	87	88	90	94	104	106	109	113	122	126	144	154
f	1	2	2	1	3	1	1	1	1	2	1	1	1	1	1	1	1



Q2)

PI	f_i	PDF $\left(\frac{f_i}{N}\right)$	CDF $\left(\sum_{j=0}^{i} \text{PDF}(f_j)\right)$
52	1	0.0156	0.0156
55	3	0.0469	0.0625
58	2	0.0313	0.0938
59	3	0.0469	0.1406
61	3	0.0469	0.1875
62	1	0.0156	0.2031
63	2	0.0313	0.2344
64	2	0.0313	0.2656
65	3	0.0469	0.3125
66	2	0.0313	0.3438
67	1	0.0156	0.3594
68	4	0.0625	0.4219
69	3	0.0469	0.4688
70	5	0.0781	0.5469
71	2	0.0313	0.5781
72	1	0.0156	0.5938
73	2	0.0313	0.6250
75	1	0.0156	0.6406
76	1	0.0156	0.6563
77	1	0.0156	0.6719
78	2	0.0313	0.7013
79	2	0.0313	0.7344
83	1	0.0156	0.75
85	3	0.0469	0.7969
87	1	0.0156	0.8125
88	1	0.0156	0.8281
90	1	0.0156	0.8438
94	1	0.0156	0.8594
104	2	0.0313	0.8906
106	1	0.0156	0.9063
109	1	0.0156	0.9219

contin... Q2)

PI	f_i	PDF $\left(\frac{f_i}{N}\right)$	CDF $\left(\sum_{j=0}^{i} PDF(f_j)\right)$
113	1	0.0156	0.9375
122	1	0.0156	0.9531
126	1	0.0156	0.9688
144	1	0.0156	0.9844
154	1	0.0156	1
	N=64		

Q3)

Intensity (I)	0	1	2	3
Frequency (f)	2	5	5	4

$\Rightarrow$ $N = (2+5+5+4) = 16$ ($\because$ No. of pixels)

$\therefore$ PDF & CDF for each intensity level

I	f_i	PDF $\left(\frac{f_I}{N}\right)$	$CDF(I) = \sum_{j=0}^{I}(PDF(j))$	i
0	2	0.125	0.125	0
1	5	0.3125	0.4375	2
2	5	0.3125	0.75	4
3	4	0.25	1	6

$\therefore$ Histogram equalization:

$$i = round((CDF(I) - CDF_{min}) \times (L-1))$$
$$= round((CDF(I) - 0) \times (8-1))$$
$$= round((CDF(I) - 0)(7))$$

$\Rightarrow I = 0$: $i = round((0.125 - 0) \times 7) = 1$

$\Rightarrow I = 1$: $i = round((0.4375 - 0) \times 7) = 3$

$\Rightarrow I = 2$: $i = round((0.75 - 0) \times 7) = 5$

$\Rightarrow I = 3$: $i = round((1 - 0) \times 7) = 7$

Original Image $\Rightarrow$ Equalized Image

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 3 & 3 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 3 & 5 \\ 7 & 7 & 5 & 3 \\ 5 & 3 & 3 & 1 \\ 3 & 5 & 7 & 5 \end{bmatrix}$$

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Q4)

Intensity (i)	Frequency (f_i)	Ideal frequency (I_f)
0	3	5.33
128	8	5.33
255	5	5.33
	N = 16	

∴ After applying uniformity

⇒ Ideal frequency for each intensity level

$$\therefore = \frac{N}{\text{No. of Intensity levels}} = \frac{16}{3} = 5.33$$

⇒ $i = 0$: $f_i = 3$, $I_f = 5.33$

0 appears less frequently than it would be in perfectly uniform distribution. ($f_i < I_f$)

⇒ $i = 128$, $f_i = 8$, $I_f = 5.33$

128 appears more frequently than it should be in uniform distribution. ($f_i > I_f$)

⇒ $i = 255$, $f_i = 5$, $I_f = 5.33$

f_i is close to I_f. As there is small difference between f_i & I_f. It indicates that intensity 255 is close to expected frequency in uniform distribution.

Q5)

Intensity (i)	PDF ($\frac{f_i}{N}$)	CDF ($\sum_{j=5}^{i} PDF(j)$)	f_i	i'
5	0.25	0.25	4	0
6	0.25	0.5	4	4
7	0.25	0.75	4	8
8	0.25	1	4	11

∴ Histogram equalization new intensity levels (i'):

⇒ $i' = \text{round}\left((CDF(i) - CDF_{min}) \times (L-1)\right)$:

⇒ $L = 2^4 = 16$, $CDF_{min} = 0$

⇒ $i' = \text{round}\left((CDF(i) - 0)(15)\right)$

∴ for $i = 5$: $i' = \text{round}((0.25 - 0) \times 15) = 4$

∴ for $i = 6$: $i' = \text{round}((0.5 - 0) \times 15) = 8$

∴ for $i = 7$: $i' = \text{round}((0.75 - 0) \times 15) = 11$

∴ for $i = 8$: $i' = \text{round}((1 - 0) \times 15) = 15$

⇒ equalized image:
$$\begin{bmatrix} 4 & 4 & 8 & 8 \\ 4 & 4 & 8 & 8 \\ 11 & 11 & 15 & 15 \\ 11 & 11 & 15 & 15 \end{bmatrix}$$

⇒ original image (was):
$$\begin{bmatrix} 5 & 5 & 6 & 6 \\ 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 \\ 7 & 7 & 8 & 8 \end{bmatrix}$$

Q6) • Sobel filters: Sobel Operators are characterised by their use of weighted average in gradient calculation, giving more importance to pixels closer to the center of the kernel. They generally have a center row/column with zero values and 2 rows or columns with positive & negative weights symmetrically around it.

$$G_{x_1} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad G_{x_2} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

• Prewitt filters: Prewitt operators are simpler compared to Sobel filters and use uniform weights. The gradients are computed using a simple difference with no weighting beyond basic differences.

$$G_{x_2} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad G_{y_2} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- Construction differences:
  - → Sobel Filters: Employ weighted gradients, where the central row or column has higher weights and weights decrease linearly as you move away from the center. It provides better edge detection with more emphasis on central pixels.
  - → Prewitt Filters: Use uniform weights for calculating gradients. The gradients are based on simple differences without weighing, which makes them simpler but less sensitive to edge variations compared to Sobel filters.

Q7)

$$Im = \begin{bmatrix} 50 & 50 & 50 \\ 100 & 100 & 100 \\ 150 & 150 & 150 \end{bmatrix}$$

∴ Sobel filter gradient calculation:

$$\Rightarrow Gradient_x = \begin{bmatrix} 50 & 50 & 50 \\ 100 & 100 & 100 \\ 150 & 150 & 150 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= 400$$

$$\Rightarrow Gradient_y = \begin{bmatrix} 50 & 50 & 50 \\ 100 & 100 & 100 \\ 150 & 150 & 150 \end{bmatrix} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= 0$$

$$\Rightarrow \text{Gradient magnitude} = \sqrt{(Gradient_x)^2 + (Gradient_y)^2}$$

$$= \sqrt{0^2 + 400^2} = 400$$

~~Prewitt~~

∴ Prewitt filter gradient calculation:

$$\Rightarrow Gradient_x = \begin{bmatrix} 50 & 50 & 50 \\ 100 & 100 & 100 \\ 150 & 150 & 150 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = 300$$

$$\Rightarrow Gradient_y = \begin{bmatrix} 50 & 50 & 50 \\ 100 & 100 & 100 \\ 150 & 150 & 150 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \text{Gradient magnitude} = \sqrt{(Gradient_x)^2 + (Gradient_y)^2}$$
$$= \sqrt{300^2 + 0} = 300$$

- Sobel filter (400) produces a greater magnitude than Prewitt filter (300).
- Sobel filter ~~produces~~ produces stronger response than Prewitt filter because it uses weighted averaging scheme that emphasizes central pixels more than prewitt filter which makes sobel filter better at detecting edges in noisy images or with varying edge strength.

Whereas, Prewitt filter provides a simpler, uniform gradient computation. It is more straightforward but might be less sensitive to edge strength variations compared to filter.

Q8) ~~Sobel~~ Image Patch = $\begin{bmatrix} 50 & 51 & 52 \\ 50 & 50 & 50 \\ 49 & 48 & 47 \end{bmatrix}$

∴ Sobel filter :

$$\Rightarrow Gradient_x = \begin{bmatrix} 50 & 51 & 52 \\ 50 & 50 & 50 \\ 49 & 48 & 47 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = -12$$

$$\Rightarrow Gradient_y = \begin{bmatrix} 50 & 51 & 52 \\ 50 & 50 & 50 \\ 49 & 48 & 47 \end{bmatrix} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \text{Gradient magnitude} = \sqrt{(Gradient_x)^2 + (Gradient_y)^2}$$
$$= \sqrt{(-12)^2 + 0^2}$$
$$= 12$$

∴ Prewitt Filter :

$\Rightarrow$ $Gradient_x = \begin{bmatrix} 50 & 51 & 52 \\ 50 & 50 & 50 \\ 49 & 48 & 47 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = -9$

$\Rightarrow$ $Gradient_y = \begin{bmatrix} 50 & 51 & 52 \\ 50 & 50 & 50 \\ 49 & 48 & 47 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 0$

$\Rightarrow$ Gradient magnitude $= \sqrt{(Gradient_x)^2 + (Gradient_y)^2}$
$= \sqrt{(-9)^2 + 0^2} = 9$

- Sobel filter is more robust to noise due to its weighted averaging, which helps in smoothing out noise and focus on actual edge. Whereas, Prewitt filter is less robust to noise because it uses uniform weights, which makes it more sensitive to noise in image.