MTH-222, MTH-6031: Probability and Statistics

Tutorial # 4 (Random Variable, pmf,pdf)

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1. Consider a binomial random variable X with parameters n and p. Let k^* be the largest integer that is less than or equal to $(n+1)p$. Show that the PMF $f_X(k)$ is monotonically nondecreasing with k in the range from 0 to k^*, and is monotonically decreasing with k for $k \geq k^*$.

2. Let X be the geometric RV with PMF
$$P(X = k) = p(1-p)^k, k = 0, 1, 2, 3, \cdots$$
Compute $P(n \leq X \leq N)$, where $n, N(N > n)$ are positive integers?

3. Does the function $f_\theta(x) = \theta^2 x e^{-\theta x}$ if $x > 0$, and $= 0$ if $x \leq 0$, where $\theta > 0$, define a PDF? if X is an RV with PDF $f_\theta(x)$, find $P(X \geq 1)$.

4. Suppose X a continuous random variable such that $P(X \geq x) = 1$ if $x < 0$, and $= (1 + x/\lambda)^{-\lambda}$, for $x \geq 0$, $\lambda > 0$ is a constant. Find the density function of X.

5. Let X be a Poisson random variable with parameter λ. Show that the PMF $f_X(k)$ increases monotonically with k up to the point where k reaches the largest integer not exceeding λ, and after that point decreases monotonically with k.

6. If a discrete random variable X taking values on $\mathbb{N}$ satisfies
$$P(X > n + m | X > m) = P(X > n), m, n \text{ nonnegative integers} \tag{1}$$
Then show that X is a geometric random variable.

7. Let X be an RV defined on $[0, 1]$. If $P\{x < X \leq y\}$ depends only on $y - x$ for all $0 \leq x \leq y \leq 1$, then show that X is $U[0, 1]$.

8. Let X be a random variable taking values in $[0, \infty)$ such that it satisfies memoryless property
$$P(X > r + s | X > s) = P(X > r), \text{ for all } r, s \geq 0. \tag{2}$$
Then show that there exists a constant $\lambda > 0$ such that $X \sim \exp(\lambda)$.