MTH-222, MTH-6031: Probability and Statistics

Tutorial # 6 (Multiple Random Variables)

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1. Let T be a closed triangle in the plane with vertices $(0,0),(0,\sqrt{2})$, and $(\sqrt{2},\sqrt{2})$. Let $F(x,y)$ denote the area of the intersection of T with $\{(x_1,x_2) : x_1 \le x,\ x_2 \le y\}$. Show that F defines a joint DF in the plane, and find its marginal DFs.

2. Let X and Y be random variables with the joint pmf $P\{X=i, Y=j\} = \frac{1}{N^2}, i,j = 1,2,\cdots,N$. Find (a) $P\{X \ge Y\}$ (b) $P\{X = Y\}$.

3. Let X and Y have a joint density f that is uniform over the interior of the triangle with vertices at $(0,0),(2,0)$, and $(1,2)$. Find $P(X \le 1 \text{ and } Y \le 1)$.

4. Let X and Y be random variables with joint pdf given by
$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda y}, & \text{if } 0 \le x \le y \\ 0, & \text{otherwise.} \end{cases}$, where $\lambda > 0$.
Find the marginal pdfs of X and Y. Also find the joint distribution function of X, Y.

5. Let X, Y be continuous random variables with joint density function
$$f_{X,Y}(x,y) = \begin{cases} e^{-y}(1-e^{-x}) & \text{if } \ 0 < x < y < \infty \\ e^{-x}(1-e^{-y}) & \text{if } \ 0 < y \le x < \infty \end{cases}$$
Show that X and Y are identically distributed.

6. Is the function
$$f(x,y,z,u) = \begin{cases} \exp(-u), & 0 < x < y < z < u < \infty \\ 0 & \text{otherwise} \end{cases}$$
a joint density function? If so, find $P(X \le 7)$, where (X,Y,Z,U) is a random vector with density f.

7. Let (X,Y) have joint density function f and joint distribution function F. Suppose that
$$f(x_1,y_1)f(x_2,y_2) \le f(x_1,y_2)f(x_2,y_1)$$
holds for $x_1 \le a \le x_2$ and $y_1 \le b \le y_2$. Show that $F(a,b) \le F_X(a)F_Y(b)$.

8. Suppose (X,Y,Z) are jointly distributed with density
$$f(x,y,z) = \begin{cases} g(x)g(y)g(z), & x>0, y>0, z>0 \\ 0 & \text{otherwise} \end{cases}$$
Find $P(X > Y > Z)$. Here g is probability density function on $\mathbb{R}$.