MTH-222, MTH-6031: Probability and Statistics

Tutorial # 3 (Conditional Probability, Independence)

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1. Show the formula $P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$, which gives the probability that exactly one of the events A and B will occur. [Compare with the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, which gives the probability that at least one of the events A and B will occur.]

2. Let A and B be events with $P(A) > 0$ and $P(B) > O$. We say that an event B suggests an event A if $P(A|B) > P(A)$, and does not suggest event A if $P(A|B) < P(A)$.

   (a) Show that B suggests A if and only if A suggests B.

   (b) Assume that $P(B^c) > 0$. Show that B suggests A if and only if B^c does not suggest A.

3. Let A, B, C be independent events and satisfies $P(A \cap B) > 0$. Show that $P(C|A \cap B) = P(C)$.

4. Assume that the events A_1, A_2, A_3, A_4 are independent and that $P(A_3 \cap A_4) > 0$. Show that $P(A_1 \cup A_2 | A_3 \cap A_4) = P(A_1 \cup A_2)$.

5. Let A, B, C events such that $P(A \cap B \cap C) > 0$ and $P(C|A \cap B) = P(C|B)$. Show that $P(A|C \cap B) = P(A|B)$.

6. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $A, B, C \in \mathcal{F}$ with $P(B) > 0$ and $1 > P(C) > 0$. If B and C are independent show that

$$P(A|B) = P(A|B \cap C)P(C) + P(A|B \cap C^c)P(C^c). \tag{1}$$

7. Let $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and $P(i) = \dfrac{1}{8}$ for $i = 1, \cdots, 8$. Consider the events $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ and $C = \{1, 3, 5, 7\}$. Show that A, B, C are independent.