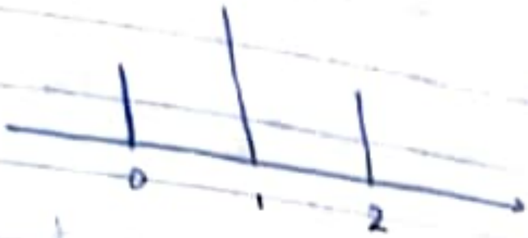

Example:- Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$

$P(\omega_i) = 1/3 \quad \forall \omega_i \in \Omega$

Define $X(\omega_1) = 1, \ X(\omega_2) = 2, \ X(\omega_3) = 3$

$Y(\omega_1) = 2, \ Y(\omega_2) = 3, \ Y(\omega_3) = 1$

$Z(\omega_1) = 3, \ Z(\omega_2) = 1, \ Z(\omega_3) = 2.$

Similarly.

$P(X=1) = P(\omega_1) = 1/3$	$P(Y=1) = 1/3$	$P(Z=1) = 1/3$
$P(X=2) = P(\omega_2) = 1/3$	$P(Y=2) = 1/3$	$P(Z=2) = 1/3$
$P(X=3) = P(\omega_3) = 1/3$	$P(Y=3) = 1/3$	$P(Z=3) = 1/3$

X, Y, & Z have the same pmf

Q1 To show that

$$P((A\cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A\cap B)$$

LHS $= P(A\cap B^c) + P(A^c \cap B)$ by additivity

$= P(A \setminus (A\cap B)) + P(B \setminus (A\cap B))$

$= P(A) + P(B) - 2P(A\cap B)$

Q2. $P(A) > 0$, $P(B) > 0$, Given.

B suggest A if $P(A|B) > P(A)$.

B does not suggest A if $P(A|B) < P(A)$

(a) B suggest A $\iff P(A|B) > P(A)$

$\iff P(A\cap B)/P(B) > P(A)$

$\iff P(A\cap B) > P(A)P(B)$

$\iff P(A\cap B)/P(A) > P(B)$

$\iff P(B|A) > P(B)$

$\iff$ A suggest B

(b) $P(B') > 0$

To show that B suggest A $\Leftrightarrow$ B' does not suggest A

$P(B) + P(B^c) = 1$

$\Leftrightarrow P(A) \times P(B) + P(B^c) \times P(A) = P(A)$

by total probability theorem

$P(A) = P(A\cap B) + P(A\cap B^c)$

$\Leftrightarrow P(A)P(B) + P(B^c)P(A) = P(A\cap B) + P(A\cap B^c)$

$\Leftrightarrow P(A)P(B) - P(A\cap B) = P(A\cap B^c) - P(A)P(B^c)$

$$\Leftrightarrow P(B)\Big(\underbrace{P(A) - \frac{P(A\cap B)}{P(B)}}_{<0}\Big) = P(B^c)\Big(\underbrace{\frac{P(A\cap B^c)}{P(B^c)} - P(A)}_{<0}\Big)$$

$\Leftrightarrow \cancel{P(B)}\ P(A) - P(A|B) < 0 \Leftrightarrow P(A|B^c) - P(A) < 0$

$\downarrow$

this is true

Q3 $$\frac{P(C\cap(A\cap B))}{P(A\cap B)} = \frac{P(A)P(B)P(C)}{P(A)P(B)}$$

($\because$ old proof)

Q.4. $P(A_1 \cup A_2 | A_3 \cap A_4) = P(A_1 \cup A_2)$

$$\text{LHS} = \frac{P((A_1\cup A_2)\cap(A_3\cap A_4))}{P(A_3\cap A_4)}$$

$= \cancel{P((A_1 \cup A_2 \cap A_3)\cup(}$

$$= \cancel{P(A_1\cup}\ \frac{P((A_1\cap A_3\cap A_4)\cup(A_2\cap A_3\cap A_4))}{P(A_3\cap A_4)}$$

$$= \frac{P(A_1\cap A_3\cap A_4) + P(A_2\cap A_3\cap A_4) - P(A_1\cap A_2\cap A_3\cap A_4)}{P(A_3\cap A_4)}$$

$$= \frac{P(A_1)\cancel{P(A_3)P(A_4)} + P(A_2)\cancel{P(A_3)P(A_4)} - P(A_1)P(A_2)\cancel{P(A_3)P(A_4)}}{\cancel{P(A_3)P(A_4)}}$$

HANKA

$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= P(A_1 \cup A_2)$

6 $P(A|B) = P(A|B\cap C)P(C) + P(A|B\cap C^c)\,P(C^c)$

$$RHS = \frac{P(A\cap B\cap C)}{P(B\cap C)}\,P(C) + \frac{P(A\cap B\cap C^c)}{P(B\cap C^c)}\,P(C^c)$$

$P(C) < 1$ $\Rightarrow P(C^c) > 0$ $P(B\cap C) > 0$

$$= \frac{P(A\cap B\cap C)}{P(B)} + \frac{P(A\cap B\cap C^c)}{P(B)}$$

$= \cancel{P(A\cap C|B) + P(A\cap}$

$$= \frac{P(A\cap B\cap C) + P(A\cap B\cap C^c)}{P(B)}$$

(conditional version of total probability theorem)

$$= \frac{P(A\cap B)}{P(B)}$$

$= P(A|B)$

Alternate :- $P(A|B) = P(A|B\cap C)\,P(C|B) + P(A|B\cap C^c)\,P(C^c|B)$

Recall that given a discrete r.v. X, we define the pmf $f: \mathbb{R} \to \mathbb{R}$ $f_X(x) = P(X=x)$

Probabilities of pmf:- let f be a pmf of a discrete r.v., then it has following properties.

(i) $f(x) \geq 0 \quad \forall x \in \mathbb{R}$.

(ii) $\{x \in \mathbb{R} : f(x) > 0\}$ is finite or countably infinite subset of $\mathbb{R}$.

(iii) $\sum_{x \in R_x} f(x) = 1$