MATH-221: Probability and Statistics

Tutorial # 1 (Countable & uncountable sets, Probability Measure)

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1. If A is a finite set and B is a countably infinite set then show that $A \cup B$ is countably infinite.

2. If $A_1, A_2, \cdots$ be countably infinite events then show that $\bigcup_{n=1}^{\infty} A_n$ is countably infinite.

3. A box has 10 balls numbered $1, 2, \cdots, 10$. A ball is picked 'at random' and then a second ball is picked 'at random' from the remaining 9 balls. Find the sample space and probability law corresponding to the above random experiment.

4. Let A, B, C be events such that $P(A) = 0.7, P(B) = 0.6, P(C) = 0.5, P(A \cap B) = 0.4, P(A \cap C) = 0.3, P(C \cap B) = 0.2$ and $P(A \cap B \cap C) = 0.1$. Find $P(A \cup B \cup C), P(A^c \cap C)$ and $P(A^c \cap B^c \cap C^c)$.

5. Prove or disprove: If $P(A \cap B) = 0$ then A and B are mutually exclusive events.

6. Does there exists a probability measure (or function) P such that the events A, B, C satisfies $P(A) = 0.6, P(B) = 0.8, P(C) = 0.7, P(A \cap B) = 0.5, P(A \cap C) = 0.4, P(C \cap B) = 0.5$ and $P(A \cap B \cap C) = 0.1$?

7. For events $A_1, A_2, \cdots, A_n$, show that $P(A_1 \cap A_2 \cap \cdots \cap A_n) \geq \sum_{i=1}^{n} P(A_i) - n + 1$.

8. Let $\Omega = \mathbb{N}$. Define a set function P as follows: For $A \subset \Omega$,
$$P(A) = \begin{cases} 0 & \text{if} \quad A \text{ is finite} \\ 1 & \text{if} \quad A \text{ is infinite} \end{cases}.$$
Is P a probability measure (or function)?

9. Let $A_1, A_2, \cdots$ be a sequence of events then show that
$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} P(A_n).$$