MTH-222, MTH-6031: Probability and Statistics

Tutorial # 2 (Probability spaces, Conditional Probability, Total Probability Theorem)

---

1. Write down the expression in set notation corresponding to each of the following events.
   (a) The event which occurs if exactly one of the events A, B occurs.
   (b) The event which occurs if none of the events A, B, C occurs.
   (c) The event which occurs if exactly two of the events A, B, C occurs.

2. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

3. Let Ω be a nonempty set and P be a function from set of subsets of Ω to $[0, 1]$ such that
   (a) $P(\Omega) = 1$.
   (b) For A and B disjoint, $P(A \cup B) = P(A) + P(B)$.
   (c) If (A_n) is a decreasing sequence of events such that $\bigcap_{n=1}^{\infty} A_n = \emptyset$, then
   $$\lim_{n\to\infty} P(A_n) = 0.$$
   Show that P is a probability measure.

4. Let $A_1, A_2, \cdots$ be a sequence of events such that $P(A_i \cap A_j) = 0$ whenever $i \neq j$. Show that $P\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k)$.

5. A fair die is rolled twice. Let X and Y be the result of the 1st and the 2nd roll, respectively. Determine the conditional probability $P(A|B)$, where
   $$A = \{\max\{X, Y\} = 3\}, B = \{\min\{X, Y\} = 2\}.$$

6. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05. What is the probability of no aircraft presence and a false alarm? What is the probability of aircraft presence and no detection?

7. Suppose there are three chests each having two drawers. The first chest has a gold coin in each drawer, the second chest has a gold coin in one drawer and a silver coin in the other drawer, and the third chest has a silver coin in each drawer. A chest is chosen at random and a drawer opened. If the drawer contains a gold coin, what is the probability that the other drawer also contains a gold coin?

8. **Conditional version of the total probability theorem.** Let $\{A_1, A_2, \cdots, A_N\}$ be a partition of Ω. Also let B be an event such that $P(A_i \cap B) > 0$ for all i. Then for any event A, show that $P(A|B) = \sum_{i=1}^{N} P(A_i|B)P(A|A_i \cap B)$.