MTH-222, MTH-6031: Probability and Statistics

Tutorial # 7 (Joint distributions, Independent random variables)

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1. Let X and Y be continuous random variables with joint pdf f. Find the joint pdf of (i) $X-2$ and $2Y+1$ (ii) X^2 and Y^2.

2. Let X and Y be continuous random variables with joint pdf f. Find the pdf of (i) XY (ii) $X-Y$ (iii) $|Y-X|$.

3. Suppose X and Y be random variables that assume four values $1,2,3,4$. Their joint probabilities are given by the following table.

X \ Y	1	2	3	4
1	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	0
2	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
3	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
4	0	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$

Find the pmf of $X+Y$.

4. Let $X \sim$ uniform$[0,1]$ and $Y \sim$ Bernoulli(0.5), and assume that X, Y are independent. Then determine the joint cumulative distribution function of X and Y.

5. Show that the sum of two independent Poisson random variables with parameters μ and λ respectively, is Poisson with parameter $\mu+\lambda$.

6. Let X and Y be independent Poisson random variables with parameters λ_1, λ_2 respectively. Find $P(Y=m|X+Y=n)$ for $m=0,1,2\cdots,n$.

7. Let X and Y be independent and identically distributed random variables such that $P(X=0)=P(X=1)=\dfrac{1}{2}$. Show that X and $|X-Y|$ are independent.

8. Let $X \sim B\left(5, \dfrac{1}{2}\right)$ and $Y \sim U(0,1)$ are independent. Then compute $\dfrac{P(X+Y \leq 2)}{P(X+Y \geq 5)}$.

9. Let X and Y have the joint density f given by $f(x,y)=\dfrac{\sqrt{3}}{4\pi}e^{-\frac{x^2-xy+y^2}{2}}, x,y \in \mathbb{R}$. Show that X and Y are dependent.

10. Suppose X and Y are two independent random variables such that $EX^4=2, EY^2=1, EX^2=1$, and $EY=0$. Compute var(X^2Y).